

# Critical Terrain Slope Calculation for Locating Small Hydropower Plants

C. Vrekos, C. Evangelides, N. Samarinas, G. Arampatzis

**Abstract**—As known, the water energy is a renewable and clean source of energy. Energy production from hydropower has been the first, and still is today a renewable source used to generate electricity. The optimal location and sizing of a small hydropower plant is a very important issue in engineering design which encourages investigation. The aim of this paper is to present a formula that can be utilized for locating the position of a small hydropower plant although there is a high dependence on economic, environmental, and social parameters. In this paper, the economic and technical side of the problem is considered. More specifically, there is a critical terrain slope that determines if the plant should be located at the end of the slope or not. Of course, this formula can be used for a first estimate and does not include detailed economic analysis. At the end, a case study is presented for the location of a small hydropower plant in order to demonstrate the validity of the proposed formula.

**Keywords**—Critical terrain slope, economic analysis, hydropower plant locating, renewable energy.

## I. INTRODUCTION

NOWADAYS, a crucial issue on earth is water resources management. Furthermore, rivers are renewable resources through the water cycle and electricity can be generated without polluting the environment, which is very important since there is an increase of electricity demand. Significant amount of energy is being produced even today from fossil fuels, which are limited [1]. Increasing conventional fossil fuel-based energy generation contributes significantly to environmental related problems, both locally and globally, like climate change from greenhouse gas emissions [2], [3]. So, the need of proper water resources management combined with the imperative environment protection leads to the construction of small hydropower plants (SHP) and new studies are emerging. Small-scale hydropower is one of the most economical and environmental friendly technology to be considered for electricity production [4]. Multiple proposed projects for drinking water and irrigation systems can take into advantage installation of hydro schemes, too [2].

Finding the optimum location and size of SHP is a very critical problem because it affects the efficiency of the project

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and the cost effectiveness of the investment. In this paper, we present a methodology for a first estimate for the placement of a SHP, considering technical and economic criteria without taking into consideration environmental and social, which sometimes may influence the project. In addition, a detailed economic analysis is not required, which is time and money consuming.

Worldwide, there is no consensus on the definition of SHP, and the limits of power production are different globally. The large majority of SHP are run-of-river type, meaning that they have either zero or very little water storage capability [5]. This is much different in design, appearance and impact from conventional large hydroelectric projects. In these projects, power is generated from flowing water and available head [6]. The turbine only produces power when the water is available from the river [5].

In a typical small hydropower scheme, the water is diverted by the diversion weir-intake. The weir is constructed across the river, which maintains a continuous flow through the intake. Then, the water is conveyed through a diversion canal (headrace) to the forebay in which the water is slowed down sufficiently in order for the suspended particles to settle down before entering the turbine. A pressure pipe, known as a penstock, conveys the water from the forebay to the turbine into the powerhouse to generate electricity. All diverted water returns to the river stream below the power house by tailrace channel, whereas the environmental impact is minor [2], [5].

These projects, according to manometric head, are classified into the following three categories [5], [6]:

- Low head: 2 to 30 m.
- Medium head: 30 to 100 m.
- High head: 100 m and above.

Generally, the further the hydropower plant is located from forebay, the applied manometric head increases and consequently the output power and inflows. On the other hand, it causes an increase of the total cost due to penstock length and electromechanical equipment. In some cases when the terrain slope is small, locating the hydropower plant further from the start of the slope, it is not economically advantageous. There is a critical terrain slope that determines if the plant should be located at the end of that slope or not.

## II. SMALL HYDROPOWER PLANTS

Hydropower is the power derived from the natural flow of water [7]. Therefore, a hydropower plant converts the dynamic energy of water into mechanical shaft power, by turbine, and then in electrical by generator.

The scope of constructing a SHP is to produce electric

energy and the power available is proportional to the product of head and discharge. The most important parameter is the output power. This power is given by the formula [8]:

$$P = \frac{\gamma \cdot Q \cdot H_{man} \cdot n}{1000} = \frac{\rho \cdot g \cdot Q \cdot H_{man} \cdot n}{1000} = 9.81 \cdot Q \cdot H \cdot n \quad (1)$$

where,  $P$  (kW) is the produced power,  $\gamma$  ( $\text{N} \cdot \text{m}^{-3}$ ) specific weight of water, which is given by the equation  $\gamma = \rho \cdot g$ , where  $\rho$  ( $1000 \text{ kg} \cdot \text{m}^{-3}$ ) is the density of water and  $g$  ( $9.81 \text{ m} \cdot \text{s}^{-2}$ ) is the acceleration of gravity,  $Q$  ( $\text{m}^3 \cdot \text{s}^{-1}$ ) is the discharge through the turbine,  $H_{man}$  (m) is the manometric head and  $n$  (pure number  $\leq 1$ ) the efficiency of the system turbine-generator-transformer.

The location of the hydropower plant is critical in order to maximize the profit. The net profit from the production of electricity is annual inflows minus annual outflows:

$$Inflows_{annual} = P \cdot C_{kWh} \cdot t \cdot \alpha \quad (2)$$

$$Outflows_{annual} = (C_{pen} \cdot L_{pen} + C_{E/M} + A) \cdot \varepsilon \quad (3)$$

where,  $P$  (kW) is the produced power,  $C_{kWh}$  (€) is the cost of kWh,  $t$  (hours) is the annual time that the plant is working,  $\alpha$  (pure number  $\leq 1$ ) is a coefficient because the output power is not maximum all the time while the plant is working (0.7 is a satisfying value),  $C_{pen}$  (€) is the cost of the penstock per meter,  $L_{pen}$  (m) is the length of the penstock,  $C_{E/M}$  (€) is the cost of electromechanical equipment, which includes mainly the cost of turbine, generator, transformer and control system, and is given by (4) [9]:

$$C_{E/M} = 20570 \cdot P^{0.7} \cdot H_{man}^{-0.35} \quad (4)$$

$A$  is standard costs which we consider that they do not change too much by different positions,  $\varepsilon$  (pure number  $\leq 1$ ) is the depreciation rate which is [10]:

$$\varepsilon = \frac{T}{[1 - (1+T)^{-N}]} \quad (5)$$

where,  $T$  (pure number  $\leq 1$ ) is the interest rate and  $N$  (years) is years of depreciation.

### III. CRITICAL SLOPE

Based on the above, the inflows depend on the manometric head which is contained in the power formula. If we consider that the terrain slope ( $s$ ) remains constant, then the manometric head can be estimated by the following equation:

$$H_{man} = s \cdot L_{pen} - \Delta h + h \quad (6)$$

where,  $s$  (pure number  $\leq 1$ ) is the terrain slope,  $\Delta h$  (m) is the total energy loss of penstock and  $h$  (m) the water level at the forebay. It is important to mention that linear energy losses at penstock ( $\Delta h$ ) are calculated by the Darcy-Weisbach equation [10]:

$$\Delta h = f \cdot \frac{L_{pen}}{D_{in}} \cdot \frac{u^2}{2 \cdot g} \quad (7)$$

where,  $D_{in}$  (m) is the inner diameter,  $f$  (pure number  $\leq 1$ ) is the friction coefficient and  $u$  ( $\text{m} \cdot \text{s}^{-1}$ ) is the average flow velocity. Formulas for calculating  $f$  are known in literature [11]. Therefore, there is a critical slope which determines the location of the plant. If the terrain slope is larger than the critical slope, the plant is economically advantageous to be located at the end of it. On the other hand, when the terrain slope is smaller than the critical, then the plant is economically advantageous to be located at the start of it. This critical slope means that the profit is the same independent of the location of the plant. Setting the profit at the beginning equal to the profit at the end, we can calculate the critical slope as shown below.

$$\begin{aligned} Profit_{start} = Profit_{end} \Rightarrow Inflows_{start} - Inflows_{end} = \\ Outflows_{start} - Outflows_{end} \Rightarrow 9.81 \cdot Q \cdot H_{man}^{start} \cdot n \cdot C_{kWh} \cdot t \cdot \\ \alpha - (C_{pen} \cdot L_{pen}^{start} + C_{E/M}^{start} + A) \cdot \varepsilon = 9.81 \cdot Q \cdot H_{man}^{end} \cdot n \cdot C_{kWh} \cdot t \cdot \\ \alpha - (C_{pen} \cdot L_{pen}^{end} + C_{E/M}^{end} + A) \cdot \varepsilon \Rightarrow 9.81 \cdot Q \cdot (s \cdot L_{pen}^{start} - \\ \Delta h_{start} + h) \cdot n \cdot C_{kWh} \cdot t \cdot \alpha - C_{E/M}^{start} \cdot \varepsilon - A \cdot \varepsilon = 9.81 \cdot Q \cdot \\ (s \cdot L_{pen}^{end} - \Delta h_{end} + h) \cdot n \cdot C_{kWh} \cdot t \cdot \alpha - C_{pen} \cdot L_{pen} \cdot \varepsilon - C_{E/M}^{end} \cdot \\ \varepsilon - A \cdot \varepsilon \Rightarrow 9.81 \cdot Q \cdot n \cdot C_{kWh} \cdot t \cdot \alpha \cdot s \cdot L_{pen} - 9.81 \cdot Q \cdot n \cdot C_{kWh} \cdot \\ t \cdot \alpha \cdot \Delta h - C_{pen} \cdot L_{pen} \cdot \varepsilon - C_{E/M}^{end} \cdot \varepsilon + C_{E/M}^{start} \cdot \varepsilon = 0 \end{aligned}$$

The final form of the critical slope equation is:

$$S_{cr} = \frac{[C_{pen} \cdot L_{pen} + (C_{E/M}^{end} - C_{E/M}^{start})] \cdot \varepsilon}{9.81 \cdot Q \cdot n \cdot C_{kWh} \cdot t \cdot \alpha \cdot L_{pen}} + \frac{\Delta h}{L_{pen}} \quad (8)$$

where,  $L_{pen} = L_{pen}^{end} - L_{pen}^{start}$  and  $\Delta h = \Delta h_{end} - \Delta h_{start}$ ,  $\frac{\Delta h}{L_{pen}} = \frac{f \cdot u^2}{D_{in} \cdot 2 \cdot 9.81}$

If we consider that the cost of electromechanical equipment is almost equal ( $C_{E/M}^{end} = C_{E/M}^{start}$ ), then the critical slope does not depend on the length of penstock and is:

$$S_{cr} = \frac{C_{pen} \cdot \varepsilon}{9.81 \cdot Q \cdot n \cdot C_{kWh} \cdot t \cdot \alpha} + \frac{f \cdot u^2}{D_{in} \cdot 2 \cdot 9.81} \quad (9)$$

## IV. CASE STUDY

### A. Study Area and Technical Part

A case study took place, in order to understand and confirm the new formula, at the river near the village Mesovouni in central Greece. At the beginning, the drainage basin was delimited based on the location of the weir, the run-off was estimated, and the diagram of flow duration curve was created. Then, two candidate locations were designated in order to carry out the preliminary study.

The necessary technical details were determined in order to calculate the cost of the projects and the economic parameters. Specifically, run-off was estimated using meteorological data of the area for 31 years' time series, based on Thornthwaite's model for water balance [12], snowmelt models [13] and Soil Conservation Service rainfall run-off model [14]. After that, the dimensioning of the project took place. Fig. 1 presents the two different candidate locations of the plant, while in Tables I

and II the technical parameters for each plant are shown. Location 1 is closer to the weir-intake and is about 883 m from the forebay, while location 2 is farther than location 1 and about 1413 m from the forebay. It is worth mentioning that, while the diameter of the penstock increases, there is a power increase but also cost increase. So, the optimum

diameter for each case is calculated and presented in Table II. However, a more cost effective design can be examined in which the penstock can be divided in more sections with different diameters and/or wall thickness, especially for longer penstocks [15].

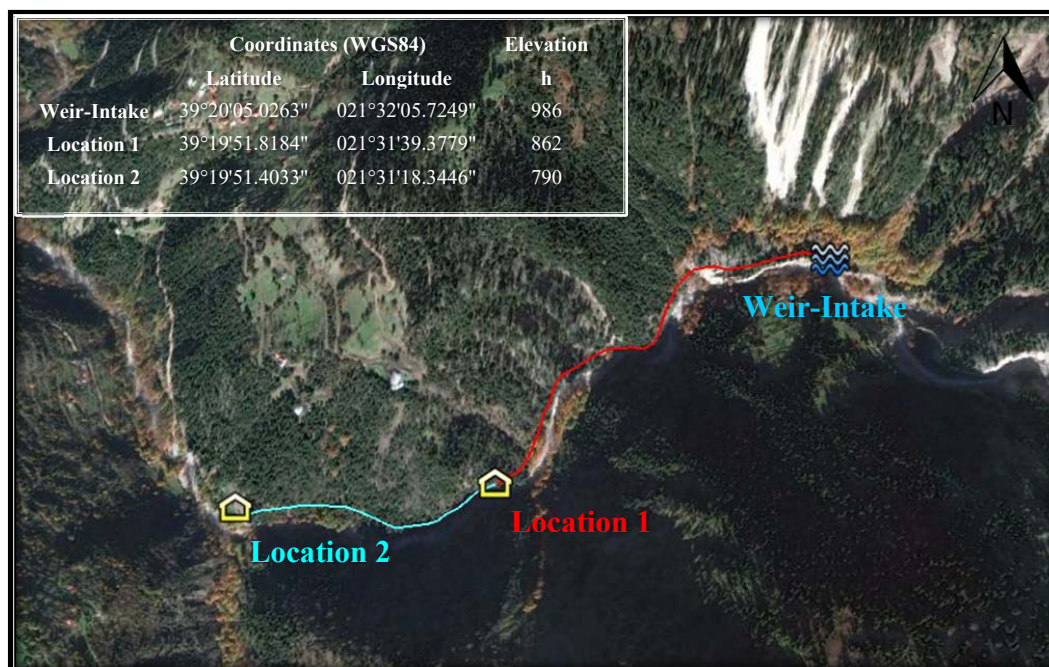


Fig. 1 Study area with two candidate locations

TABLE I  
 HYDROPOWER PLANT PARAMETERS

Plant Parameter	Location	
	1	2
Mean Annual Stream Flow (m <sup>3</sup> /s)	0.37	
Mean Annual Hydropower Flow (m <sup>3</sup> /s)	0.50	
Nominal Flow (m <sup>3</sup> /s)	0.78	
Head (m)	122.1	191.3
Nominal Power (MW)	0.799	1.252
Mean Annual Electric Production (GWh)	2.72	4.30
Utilized Precipitation (%)	52.2	
Utilized Runoff (%)	81.2	
Annual Operation Time (%)	59.8	

TABLE II  
 PENSTOCK PARAMETERS

Penstock Parameter	Location	
	1	2
Material	Steel	
Length (m)	883	1413
Outer Diameter (mm)	914.4	812.8
Wall Thickness (mm)	7	10.7
Δh (m)	1.4	4.4
Penstock Cost (€/m)	298	402
Electromechanical Equipment Cost (€)	412276	482434

### B. Techno-Economic Analysis

An economic analysis took place because the economic side

of the study is very critical and needs more investigation. Evaluation of the investment helps to choose the best location of a SHP. For this reason, a financial analysis was carried out for evaluating the investment for both candidate locations.

The necessary steps are:

1. Estimation of initial investment cost, annual fixed costs and annual inflows.
2. Estimation of financing and loan repayment.
3. Cash flow and Internal Rate of Return (IRR) calculation.

The initial investment cost is defined as the sum of fixed assets, which includes capital expenditure and asset investments, as well as working capital which represents the liquidity of the company. Working capital depicts the company's ability to cover its long term liabilities during its first months of operations. Since the product produced has no creditors and the inflows are immediate, we can assume that the working capital is negligible, so the initial cost of the investment equals the fixed assets [16], [17].

The fixed assets, which is the investment cost, consists of:

- Capital expenditure
  - Market research
  - Studies and licenses
- Asset investments
  - Civil works (weir, open channels, powerhouse building, etc.)
  - Electromechanical and auxiliary equipment (turbine,

- generator, etc.)
- o Land purchase and legal costs
- o Grid connection

Annual fixed costs are mainly the maintenance of equipment, staff salary, and taxes. In addition, the total initial cost of the investment will be covered by specific sources of funding. Specifically, we consider 35% to be covered by grants, 30% from own funds and 35% from lending. As for loan repayment, the interest rate on the loan that will be raised is 5%, while the repayment period is 5 years. The annual installment will be equal to the loan amount multiplied by the annual depreciation coefficient which is given by (5).

Production costs consist of the sum of the annual costs, excluding taxes, interest and depreciation of assets. Depreciation of assets remains constant for each year. In this case, the lifetime of the project was considered to be 30 years, therefore the depreciation coefficient was assumed to be equal to 3.3%. It is worth mentioning that the land cost is not accounted for. The annual inflows were estimated from electricity sale, based on the price of kWh (0.08785 €) and the readjustment per year due to inflation, which was considered equal to 0.5%.

Net cash flow is one of the most important components in investment evaluation techniques, since it is the final outcome of the investment. Cash flow is the difference between inflows and outflows that are related to the investment [18], [19].

The IRR is the most popular evaluation method. It is defined as the discount rate that makes cash flow zero; it is the rate at which the initial investment equals the sum of future cash flows. In the other words, IRR is the interest rate that makes Net Present Value (NPV) zero (0). The Present Value (PV) for year  $n$  is the multiplication of NCF and the Coefficient Present Value (CPV) which is given by the following relation [20]:

$$CPV = \frac{1}{(1+i)^n} \quad (10)$$

where,  $i$  is the discount rate and  $n$  the examined year.

The NPV calculated by the following equation:

$$NPV = \sum_{t=1}^n PV_t - IC \quad (11)$$

where, IC is the investment costs, which is the sum of own funds and the amount of the loan.

NPV depends on the least acceptable discount rate  $i$ , or the Weighted Average Cost of Capital (WACC), that is chosen every time. If the NPV is positive, for a given WACC, then the company should undertake the investment. In addition, in such a case, the profitability ratio is greater than one (1), and the net profit ratio is higher than zero (0). Therefore, the interest rate ( $i$ ), which makes NPV zero and is the IRR was required [18]-[20]. Table III presents the final economic analysis results.

TABLE III  
 HYDROPOWER ECONOMIC PARAMETERS

Economic Parameter	Location	
	1	2
Initial Investment Cost (€)	1081789	1475935
Annual Fixed Cost (€)	43272	59037
Investment Cost (€)	703163	959358
Average Annual Inflows (€)	256853	406706
Average Annual Production Cost (€)	57281	73997
IRR (%)	23.2	27.5

### C. Critical Slope - Results

The terrain slope between location 1 and location 2 is:

$$s = \frac{H_{man}^2 - H_{man}^1}{L_{pen}^2 - L_{pen}^1} = \frac{191.3 - 122.1}{1413 - 883} = 0.130 \text{ or } 13\%$$

According to (8), in our case the value of the critical slope is:

$$S_{cr} = \frac{[C_{pen} \cdot L_{pen} + (C_{E/M}^{end} - C_{E/M}^{start})] \cdot \varepsilon}{9.81 \cdot Q \cdot n \cdot C_{kWh} \cdot t \cdot a \cdot L_{pen}} + \frac{\Delta h}{L_{pen}} = =$$

$$\frac{[402 \cdot 530 + (482434 - 412276)] \cdot 0.173}{9.81 \cdot 0.78 \cdot 0.85 \cdot 0.08785 \cdot 5240 \cdot 0.7 \cdot 530} + \frac{3}{530} \Rightarrow S_{cr} = 0.050 \text{ or } 5\%$$

The final results of calculated terrain and critical slopes show that terrain slope is larger than critical. This fact indicates that the appropriate location for constructing the hydropower plant is location 2. Also, the techno-economic analysis shows that the location 2 is economically more advantageous because of the IRR results ( $IRR_2 > IRR_1$ ).

### V. CONCLUSION

The best location of a SHP, which is a renewable source of energy, is an important issue in engineering design in order to maximize profit. In this paper, a new formula is investigated and presented for estimating the best location of a SHP, based on terrain slope. The further the hydropower plant is located from forebay, the applied head increases, but on the other hand it causes an increase of the total cost. The named "critical slope" is given in order to determine the location of the plant by terrain slope and the new formula demonstrates the parameters that it depends on. In this formula, the term  $\Delta h/L_{pen}$  is usually very small and it can be neglected simplifying calculations. Furthermore, if we consider that there is no change on electromechanical equipment cost, then (9) can be used which is simpler and does not depend on penstock length. These equations can be used for a first estimation of a SHP location and show if a candidate location requires more detailed investigation. It has the advantage that it is simple, and no techno-economic analysis is needed, which is time and money consuming. In addition, the formulas can be applied in order to determine the location of the plant in cases where there is a change in terrain slope. The case study that took place confirms that critical slope equation is useful for such projects. In this study, two candidate locations for a SHP were investigated. The terrain slope between them was higher than the critical slope which means that the plant

should be constructed further from the forebay. The analytic economic analysis was in agreement with critical slope results because  $IRR_2 > IRR_1$ . In conclusion, the new formulas for critical slope are useful and applicable.

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