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A Ground Structure Method to Minimize the Total Installed Cost of Steel Frame Structures

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***Abstract*—**This paper presents a ground structure method to optimize the topology and discrete member sizing of steel frame structures in order to minimize total installed cost, including material, fabrication and erection components. The proposed method improves upon existing cost-based ground structure methods by incorporating constructability considerations well as satisfying both strength and serviceability constraints. The architecture for the method is a bi-level Multidisciplinary Feasible (MDF) architecture in which the discrete member sizing optimization is nested within the topology optimization process. For each structural topology generated, the sizing optimization process seek to find a set of discrete member sizes that result in the lowest total installed cost while satisfying strength (member utilization) and serviceability (node deflection and story drift) criteria. To accurately assess cost, the connection details for the structure are generated automatically using accurate site-specific cost information obtained directly from fabricators and erectors. Member continuity rules are also applied to each node in the structure to improve constructability. The proposed optimization method is benchmarked against conventional weight-based ground structure optimization methods resulting in an average cost savings of up to 30% with comparable computational efficiency.

***Keywords*—**Cost-based structural optimization, cost-based topology and sizing optimization, steel frame ground structure optimization, multidisciplinary optimization of steel structures.

I. Introduction

COMPUTATIONAL methods to optimize topology and member sizing for truss and frame structures were originally developed for aerospace applications over twenty years ago [1]. The objective of these methods was to minimize the amount of material used while satisfying structural strength and serviceability criteria. Significant subsequent research has been devoted to developing methods that can be applied steel frame building and civil structures. These methods can be divided into two major categories based on how the topological and sizing variables are represented: continuous and discrete.

Classic structural optimizations explored in the literature can be divided into the major categories of topology, sizing, connectivity and shape. The topology design space either consists of all the members of ground structure of a pre-defined density [2] in a discrete-geometry optimization problem, or all the finite elements of a discretization in a continuous-geometry problem formulation. Sizing optimization on the other hand is only implemented in discrete-geometry optimization problems (i.e. linear beam-column or truss elements), and its design space can either be characterized by continuous or discrete cross-section sizes. Connectivity optimization iterates over the degree of fixity of the nodal connections, which can be fully restrained, partially restrained or unrestrained in any degree of freedom. Finally, shape optimization is described by the variation of discrete-geometry member coordinates. Each of these design spaces can be explored simultaneously with the other two or independently, depending on the particular architecture and objective adopted. Commonly in the literature, as well as this paper, topology and sizing are optimized sequentially. Such design choice is due to the fact that changes in the topology lead to a redistribution of the forces, load path and structural response, which require member sizes to be adjusted before taking another step towards topology optimality. The methodology presented hereby also iterates through connectivity, strictly with the purpose of meeting stability requirements.

Continuous topology methods represent the structure as a single monolithic body. Often the initial structure is discretized into a set of continuum finite elements from which structural members emerge through a subtractive optimization process. Liang et al. [3] and Stromberg et al. [4] present two examples of continuous topology methods. Liang optimizes for the overall structural stiffness, keeping volume as a constraint while Stromberg compares the results of a continuum-based and finite-element-based approach, demonstrating how the latter results in topologies closer to beam-column elements. The objective of both of these methods is to minimize the minimize the weight of steel used in the structure as it is typical for engineers to estimate the cost of a steel structure early in the design process by multiplying the weight of steel by an assumed cost per unit weight [5]. However, the majority of cost of a completed steel building structure can be attributed to fabrication and erection costs which do not necessarily scale linearly with material weight [6]. Fabrication and erection costs and the constructability of the overall structure are driven primarily by the connections between linear structural members. Converting continuous finite element topologies into discrete members and connections is difficult and has limited the ability to incorporate cost and constructability considerations that are important in the design of buildings and civil structures.

Discrete topology optimization methods begin with an array of nodes that are interconnected with a dense mesh of linear members known as the ground structure. Since the optimal member topology is found from within the original ground structure, an advantage of the method is that the initial ground structure geometry can be defined so as to

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ensure feasibility of fabrication and erection of all possible solutions before the optimization process begins. The most common algorithms used for simultaneous topology and sizing optimization are genetic algorithms, as seen in Tang et al. [7], Rajan et al. [8], and Deb et al. [9]. Genetic algorithms are zero-order methods, meaning they explore the design space heuristically and without the use of gradients or a smooth objective function, and they come in different problem-specific variants. The greatest limitation of this family of algorithms is the computational time and scalability, as the design space increases very rapidly with the number of elements, complexity of the costing model, constraints, different frame types, connectivity and constructability considerations.

Achtziger et al. [10] propose an interesting formulation of discrete topology and sizing optimization, featuring a global minimum weight solution. While genetic algorithms and heuristic methods converge to local optima, this study frames the problem as a convex optimization, thus guaranteeing the global optimum. This method only applies to trusses and does not consider cost or constructability, but it poses an upper bound on optimal convergence, as the other methods described hereby are never guaranteed to yield a global optimum design.

The Ground Structure Based Topology Optimization (GRAND) method presented by Paulino et al. [11] uses a discrete-truss approximation of the full finite-element problem to dramatically improve computational efficiency compared to similar ground structure methods. The structural topologies produced by the GRAND methods are constructible. However, the GRAND method is only applicable does not account for bending forces in the members and, therefore, is only applicable to truss structures. This method aims to minimize material weight rather than the total installed cost of the structure. Asadpoure et al. [12] take a leap forward in discrete topology and sizing optimization, formulating a smooth and differentiable cost objective for truss structures. The advantages of such method is that gradient descent can be utilized to optimize the objective, which leads to very fast convergence. However, this approach does not account for actual buildability, and uses a very basic cost function that is dependent on the member weight.

Havelia [13] developed the Cost-Driven Deterministic Ground Structure Method (CDD-GSM) that utilizes a Multidisciplinary Feasibility (MDF) [14] architecture to perform capital cost-based topology and sizing optimization of 2D steel frame structures. Havelia compared minimum cost structures generated by the CDD-GSM method to minimum weight solutions and successfully demonstrated that a heavier structure can be significantly cheaper due to efficiencies in the fabrication and erection process. The CDD-GSM, however, only considers structural strength criteria and does not account for serviceability criteria that are essential for the design of building and civil structures, including limited on member deflection and inter-story drifts. Furthermore, the method did not enable members to be continuous across a joint which is commonly done for these types of structures to improve constructability and reduce cost.

II. Problem Formulation

The cost-based topology and sizing constrained optimization problem can be formalized in (1).

$$
\begin{aligned}
&minimize \quad C = \sum_{m}^{M} \mathbf{c_{capital}}(\mathbf{s}(m), t(m), \mathbf{H}(m,\mathbf{N}))\\
&\boldsymbol{\delta} = \mathbf{K^{-1}F}\\
&\mathbf{f}(m) = \mathbf{k}'(m)\mathbf{u}(m)\\
&M_t(m) = \{W, HSS_R, HSS_{SQ}\}\\
&t(m) \in \{0,1\}\\
&\mathbf{H}(m,\mathbf{N}) = \mathbf{H}(t(m), M_t(m), \mathbf{N})\\
&s(m) = s(t(m), \mathbf{f}(m), M_t(m), \boldsymbol{\delta}, \mathbf{H}(m))\\
&s.t.:\\
&\qquad d(t(\mathbf{M}), \mathbf{f}(\mathbf{M}), M_t(\mathbf{M})) <= c(t(\mathbf{M}), \mathbf{f}(\mathbf{M}), M_t(\mathbf{M}))\\
&\qquad \boldsymbol{\delta}(\mathbf{N}, \vec{\mathbf{v}}) <= \boldsymbol{\delta}_{all}(\mathbf{N}, \vec{\mathbf{v}})\\
&\qquad \xi_{Mode1} >= \xi_{control}\\
&with:\\
&\qquad \mathbf{M} = \{m_1, m_2, ...m_M\}\\
&\qquad \mathbf{N} = \{n_1, n_2, ...n_N\}\\
&\qquad \vec{\mathbf{v}} = \{\vec{\mathbf{x}}^+, \vec{\mathbf{x}}^-, \vec{\mathbf{y}}^+, \vec{\mathbf{y}}^-, \vec{\mathbf{z}}^+, \vec{\mathbf{z}}^-\}
\end{aligned}
\tag{1}
$$

TABLE I
Glossary

Variable Name	Variable Description
$\mathbf{M}$	Members
$\mathbf{N}$	Nodes
$\mathbf{C}$	Total capital cost
$\boldsymbol{\delta}$, $\mathbf{u}$	Global and local displacements
$\mathbf{K}, \mathbf{k}'$	Global and local stiffness matrices
$\mathbf{F}$, $\mathbf{f}$	Global and local forces
$\vec{\mathbf{v}}$	Coordinate directions
t	Member topology
s	Member sizing
M_t	Member section type from the AISC catalog
$\mathbf{H}$	Member fixity and sizing continuity contraints
d, c	Member demand and capacity per AISC code
ξ	Modal frequency

In this formulation, $\mathbf{c_{capital}}$ is the total capital cost associated with each member, and is described in detail in the costing section of the method. Moreover, $t(m)$ is the binary topology variable for each member of the ground structure, controlling whether a member is active or removed. Because the framework supports different section types from the AISC catalog and the sizing equations are different for each type, M_t records whether each member belongs to the wide flange, round HSS or square HSS design space. The load combinations are applied as static loads, therefore the generalized displacements $\boldsymbol{\delta}$ can be trivially obtained by inverting the global stiffness matrix $\mathbf{K}$, and the member forces $\mathbf{f}(m)$ can be retrieved from the local displacements $\mathbf{u}(m)$ and member stiffness matrix $\mathbf{k}'(m)$. Furthermore, the member size $s(t(m), \mathbf{f}(m), M_t(m), \boldsymbol{\delta}, \mathbf{H}(m))$ is a function of the topology, the member forces, the generalized displacements and the continuity constraints $\mathbf{H}(t(m), M_t(m), \mathbf{N})$. The demand/capacity ratio of each member is controlled per AISC

TABLE II
PREPROCESSING INFORMATION

Input Type	Input Description
Ground Structure	Spacing, density, structural model
Nodes	Coordinates, loaded flags, support flags
Members	Start and end nodes, fixity, section type
Loading Conditions	Design load cases and combinations, point loads
Structural properties	Material strength, frame type (OCBF or OMF)
Design Constraints	Max displacements and d/c ratio, min frequency
Constructability Constraints	Continuity in sizing and fixity, max member length
Objective Function	Weight or capital cost
Cost Data	Member, connection, fabrication and erection costs
Hyperparameters	Removal %, sizing parameters α, β

equations as a function of the topology, forces and member type. Also, the maximum drift at all nodes is constrained to be within allowable limits for each of the 6 coordinate directions identified by their unit vectors. Lastly, the frequency of the first mode is also constrained to a minimum, avoiding instabilities or singularities in the generalized stiffness.

III. METHOD

The full model architecture shown in Fig. 1 is a MDF model where the two inter-dependent sub-disciplines of topology and sizing are optimized sequentially starting from the same initial input batch at every iteration. The outer topology optimization loop ranks and removes a specified percentage of members at each iteration after the structure has been detailed and costed, and the embedded sizing loop has ensured strength and stiffness constraints are met. The individual topology, sizing and costing components highlighted in red are described in detail in the following sub-sections. The proposed methodology addresses shortcomings of the previous models by adopting constructability rules along with member-level and structure-level code compliance, guaranteeing that the converged structure is not only buildable, but also safe. Constructability is considered as an inviolable factor, and is achieved by adopting standard AISC sections, detecting member continuity and enforcing connection and member length requirements. Safety on the other hand is guaranteed by simultaneously sizing for strength and stiffness per AISC requirements. This methodology also presents advantages in problem scalability and component modularity, allowing it to be extended and tailored to specific industry projects. Scalability is achieved by an algorithm that allows for any initial geometric configuration and scales efficiently with size, whereas modularity is inherent to the particular architecture adopted, where each components of the optimization can be modified to accommodate different modeling assumptions. Lastly, this new framework has been designed to work on both moment-frame structures and braced-frame structures, the latter of which are a novelty in topology optimization and are being handled with heuristic techniques for overall structural stability.

A. Preprocessing

The pre-processing step in Table II involves assembling all the project-specific required information regarding the geometry and boundary constraints, the loading conditions, the purpose and location of the structure, the cost information

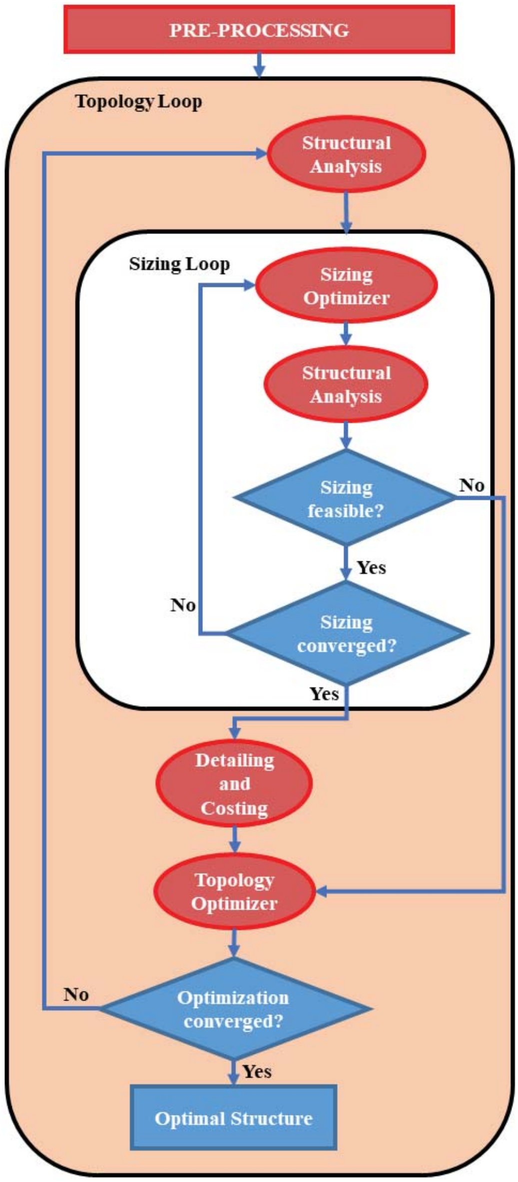

Fig. 1 Optimization flowchart. Red: key algorithm component. Blue: logic operator

and the constructability requirements. The very first step is to implement a ground structure to model the design space, which could be of different densities at the designer's discretion and without loss of generality. A fully-connected sparse ground structure, where each node is connected to every other node through a single element, is compared to a sparse ground structure in Fig. 2. The meaning of "sparse" indicates that the design space is packed using only a limited number of nodes, usually at a constant distance equal to a pre-defined span. The initial ground structure, along with the loading conditions and all the geometric information that feeds into the optimization can be automatically assembled using the SAP2000 API.

Depending on the desired sizing design space, the necessary information from the AISC section catalog is readily available for HSS round, HSS square and W sections, describing the geometric and strength properties of the sections along with

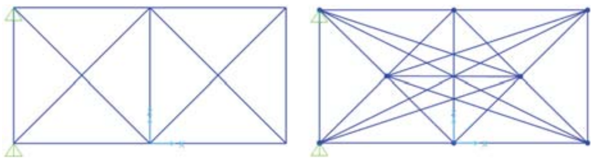

Fig. 2 Different ground structure configurations

a unit member costs obtained from the selected mills. The final pre-processing step is to specify whether the optimization will take place in 2D or 3D, identify the structure either as moment frame or braced frame and specify the maximum drift constraints for each designer-specified critical node.

B. Structural Analysis

The structural analysis components runs in two separate instances in the optimization framework described in the previous section: once after every topology update in the outer loop, and as many times as necessary to find a strength and sizing compliant structure in the inner sizing loop. The model presented here features modal and linear static analyses, used respectively to monitor the stability of the structure through the modal frequency and to apply each of the design combinations. This component represents the computational bottle-neck of the optimization, hence the overall algorithm efficiency will be expressed in terms of analyses performed. The sizing software implemented here is CSI SAP2000 v18, which has been chosen for such purpose due to its flexible API functionality, wide range of modeling capabilities, visual feedback and world-wide use in the industry. The analysis is run through the API as a ModelCenter component, reading the most current variable states hence updating sizes, topology and connection fixities, returning forces and displacements.

C. Sizing Optimization

The sizing algorithm is based on a discrete variable design space, where every member can be sized using a section from the AISC catalog. Each sizing loop Fig. 3 is passed the geometry and loading conditions from the outer optimization loop at the current iteration, and sizes the structure to meet both drift and strength constraints.

The first step of the sizing algorithm consists of *updating the continuity and hierarchy* of all the members, according to a pre-defined set of geometric rules Fig. 4:

- Collinear elements and columns are continuous for their entire span.
- At every connection, at least one element is continuous through.
- Transfer beams are always continuous.
- Braces are the hierarchical dependents of beams, and beams are the dependents of columns.

Such rules are then used by the sizing functions to ensure continuity in member sizing and beam-column joint feasibility. When two members are regarded as collinear, they are sized with the same section and the fixity at their shared joint is set to resist moment, effectively behaving as a single member. Subsequently the available section sizes are sorted

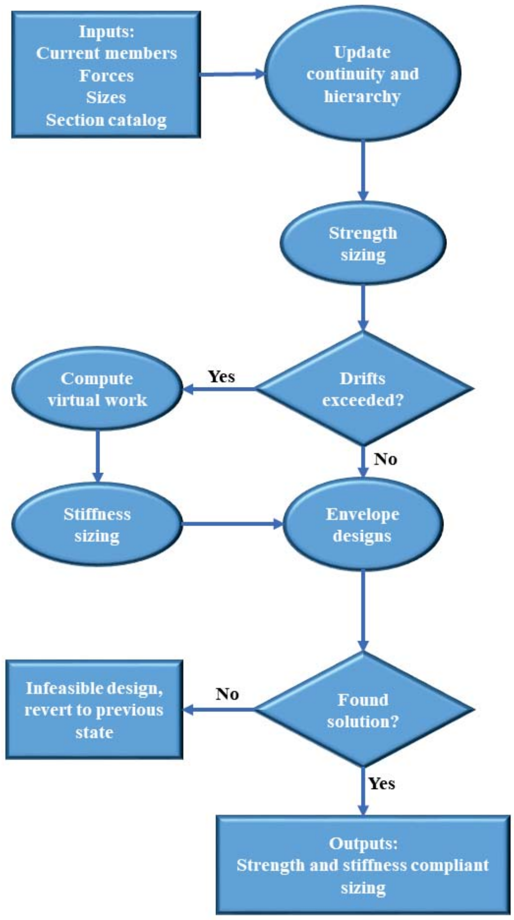

Fig. 3 Sizing optimization logic

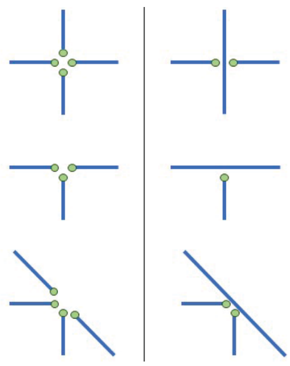

Fig. 4 Continuity rules before (left) and after (right) reassignment

using supplier cost information and *strength sizing* for each member is performed for shear and simultaneous compression and flexure (2), with an exhaustive search starting from the cheapest section to the first section satisfying the requirements.

$$\begin{aligned} &\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{if} \quad \frac{P_r}{P_c} \geq 0.2 \\ &\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \quad \text{if} \quad \frac{P_r}{P_c} < 0.2 \\ &\phi_v \frac{V_r}{V_c} \leq 1.0 \end{aligned} \tag{2}$$

While V_r, P_r, M_{rx}, M_{ry} are the forces from the critical load case acting on each member, V_c, P_c, M_{cx}, M_{cy} are the member capacities obtained from AISC sections E, F and G respectively for compression, bi-directional flexure and shear. The algorithm accounts for the entirety of the flange and web slenderness sub-cases in the code requirements for round HSS, square HSS and W sections, effectively automating all the checks that a designer would need to go through to design the element. Each member is then sized with sections from the catalog corresponding to the particular section type specified by the designer in the inputs (i.e. a member of type W in the initial ground structure will always be sized with a section of type W). The strength-sizing component will attempt to size every member from the bottom of the list, allowing to down-size with respect to previous configurations and guaranteeing the cheapest strength design at every iteration.

After finding the optimal sizes for strength, the worst-case enveloped displacements of a series of pre-defined control nodes are checked against the allowable drifts. These control nodes, defaulting as all the loaded nodes, and maximum drifts are specified by the designer on a project basis. If none of the allowable drifts are exceeded, the sizing optimization converges and the optimal strength design is passed onto the subsequent optimization components. If instead any drift is exceeded, an iterative *stiffness sizing* process based on *virtual work* is necessary to guarantee compliance [15]. This approach is used to determine which members contribute most in terms of stiffness in the direction in which drift is exceeded, allowing to rank each relative contribution and tentatively increase the section sizes accordingly. For each (load case, exceeding displacement) pair, the virtual work contribution of each of the members in the structure is computed through (3).

$$\begin{aligned} VW_i &= \int_L \left(\frac{PP'}{EA} + \frac{M(x)M'(x)}{EI} + \frac{VV'}{EA_v} + \frac{T(x)T'(x)}{GJ}\right)dx \\ VW_i &= max[VW_i, 0] \end{aligned} \quad (3)$$

where *P, M(x), V, T(x)* are the member forces corresponding to the real load case in which drifts are exceeded, and *P', M'(x), V', T'(x)* are the forces generated by a virtual case of a unit force applied at the node and in the direction of the exceeded drift. *EA, EI,* EA_v and *GJ* are the axial, flexural, shear and torsional stiffness respectively. They virtual member contributions in the formula above represent the strain energy density of each member generated by the real load case and the virtual displacement, effectively classifying the stiffness each member provides against the specified drift. Here the virtual work contribution of each member is considered no smaller than 0, since a negative contribution simply implies the member provides no stiffness at all to the displacement in question. All member virtual work contributions are normalized by the maximum and used in the heuristic formula in (4) to increase the section sizes for each member.

$$new\_size_i = Round(size_i \times (\frac{\delta_i}{\delta_{all_i}} + \beta) \times (VW_i + \alpha)) \quad (4)$$

where new_size_i and $size_i$ are indexes of the section catalog sorted by area and dept, $\frac{\delta_i}{\delta_{all_i}}$ is the ratio of actual to allowable displacement, VW_i is the normalized member virtual work contribution and α, β are hyperparameters that can be tuned for best performance. The Round operation is necessary to ensure the catalog index returned is an integer. Hence, a proposed stiffness design is obtained for each exceeded drift and the designs are conservatively enveloped to achieve a comprehensive stiffness design. Such design is then enveloped with the proposed strength design computed as previously described. However, to ensure the final proposed design is the one of minimum cost, the *stiffness-strength envelope* is performed such that the maximum of the two designs is taken for sections that have been increased to meet stiffness, and the minimum of the two designs is taken otherwise.

Upon convergence of the strength and stiffness designs, the sizing component enforces the member continuity and hierarchy rules, adjusting the final design accordingly. The sizing optimization is followed by the detailing and costing components described in the next section.

D. Costing and Detailing

The detailing component is run on the current toplogy and optimal section sizes, assembling a geometric configuration map storing information on the direction of all the members framing into each node. Such information is then passed onto the coster and optimizer. If the initial structure passed to the sizing component is unstable or a strength and stiffness-compliant design is infeasible, a flag is passed downstream to the optimizer component to handle such case. Upon convergence of the sizing component and the assembly of node-element map by the detailing logic, the total capital cost of the structure is evaluated as the sum of each member cost per (5) and is passed on to the optimizer as the objective function.

$$\begin{aligned} c_i &= c_{m_i} + c_0 + n\gamma_i c_c \\ C &= \sum_{i=1}^{M} c_i \end{aligned} \quad (5)$$

Every member incurs in a material cost c_m, a fixed connection cost c_0, and a per-member additional connection cost as a percentage of the sum of all the material costs of the member framing into the node c_c, featuring a correction factor γ based on the continuity rules described. The correction factor ensures continuous members are also costed as such, assigning a lower fraction of the total connection cost to members that are continuous through, which are hence costed as a single member. Also, multi-span continuous members uniformly share the cost incurred at all connection instances of their individual members. The three cost components $\{c_m, c_0, c_c\}$ are estimated on the basis of project location-dependent accurate fabricator and erector unit rates, and are strongly correlated to the final topology of the structure.

E. Topology Optimization

The optimizer in Fig. 5 is responsible for the member removal phase and acts as the wrapper of the optimization framework, managing the variables fed into it from the upstream components.

The first functionality of the optimizer is to determine the fitness of the members, once again, with a virtual work

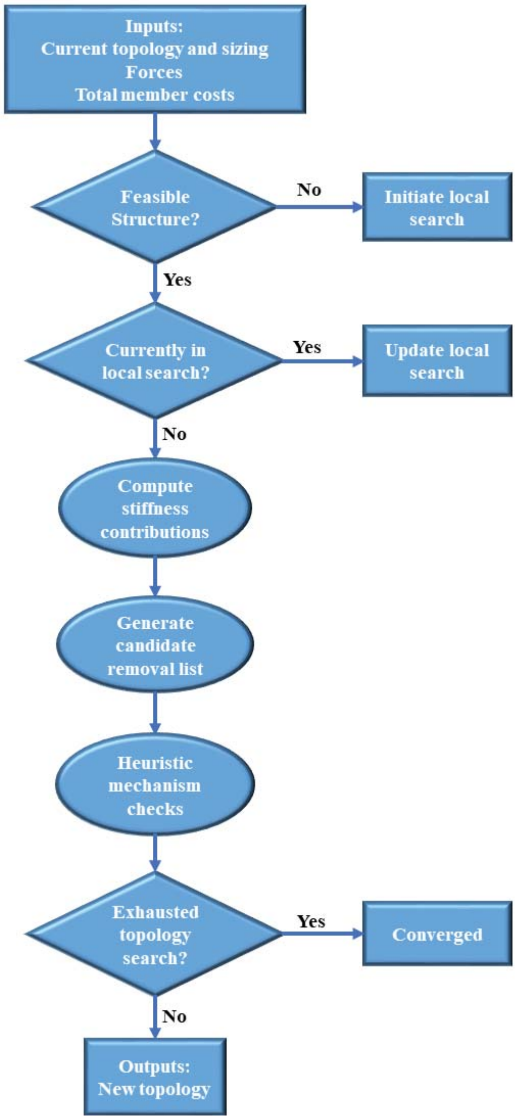

Fig. 5 Topology optimization logic

approach. Here, each member is given a score based on its overall *stiffness contribution* divided by its cost (6).

$$F_i = \sum_{j=1}^{N_{critical}} \frac{(VW_{ij} \times I_{ij})}{c_i} \quad (6)$$

After the fitness is computed, the optimizer has to *determine the candidate members for removal* based on a designer-input percentage to remove at each iteration. Such percentage can be kept as fixed or can be annealed as a function of the iteration number.

In this instance, virtual work is calculated for all the critical nodes and maximum-drift load cases, weighting the contribution of each combination for each member with an importance factor $I_{ij} = \frac{\delta_{ij}}{\delta_{all_j}}$, where δ_{ij} and δ_{all_j} are respectively the maximum and the allowable node displacements. The members are successively ranked in ascending order by their fitness value and placed on the candiate list for removal, to which all members with either virtual work contribution or forces of 0 are appended. This steps allows to short-circuit the removal process, helping the optimization to converge faster and avoid local instabilities.

The optimizer then enforces the structural continuity relations by updating each member end-fixity. At this point, the candidate list for removal goes through *heuristic checks* to help prevent instability or infeasible structures that would need to be dealt with further down the line. The heuristic check function performs the following actions on the candidate list for removal:

- Prevents loaded nodes from being removed from the structure, ensuring at least one member remains connected.
- If the structure is a braced-frame (moment frames do not incur in these stability issues), stabilizes each node recursively using a breadth-search algorithm.

The first action is achieved by removing the members among those in the candidate list with the lowest fitness. On the other hand, to stabilize the nodes connected to members in the candidate removal list, they are placed in a queue and iteratively checked one by one. A node does not pass the check if upon removal it does not meet at least one of the following requirements for stability:

- All members at the node are in the removal list.
- If the node only has one element connected to it, through-continuity is provided at the node itself or at the other end of the element.
- The elements connected to the node are not all coplanar (3D case) or collinear (2D case) with one another.
- More than one element remains connected to the node (only for 2D structures).

The requirements above are a hard constraint for the optimization, and a back-filling mechanism is triggered in case the checks above do not allow to remove the desired number of elements at a given iteration. Back-filling expands the original candidate list by doubling the desired removal percentage until such number of elements passes the heuristic checks. This functionality may also lead to an early convergence, in the case where even taking all the remaining members as candidates would not yield the desired percentage for removal. After the heuristic checks, the member removal is executed and the inputs are passed on to the next iteration.

While fairly comprehensive, the heuristic checks deployed do not succeed in preventing the three main causes of an unfeasible solution: an instability, an excessive cost increase or a broken load path. Instabilities can be of a global or localized nature, and can be efficiently detected in the frequency of vibration of the first mode. Small cost increases of 5-10% are tolerated across iterations, as these are local peaks in the objective which in most cases transition to better local minima. Broken load paths occur when a path to ground is eliminated, and can be detected in sudden changes in the total base reactions. When one of the three critical conditions manifests itself, the optimizer enters a *Local Search* mode, halting the progression of the optimization until the issue is fixed. When Local Search is triggered, all the members from the previous feasible iteration are added back. Subsequently, each member in the removal list is now removed individually and the outcome is evaluated. If removing such member yields no issue, no action is taken and the next member on the list is evaluated. If otherwise the individual removal of the member causes an infeasible solution, such member is placed on a permanent "Do-not-remove" list.

Convergence occurs when all the remaining members are on the "Do-not-remove" list, indicating that they have all been attempted to remove. Once the optimization converges, the "Do-not-remove-list" is cleared out and the optimization is run once more on the remaining members. This process helps removing members that were kept around in previous iterations for stability purposes only, but are redundant in the final configuration.

IV. Benchmark Examples

The algorithm is showcased on two examples: a cantilevered truss with a 3x3 initial ground structure, Fig. 6, and a simply supported beam with a 3x6 initial ground structure, Fig. 9. The same sets of loads, drift/strength constraints and properties are applied to the benchmark problems. The following examples serve the role of showing the topology and sizing optimization performance and outcome given different cost and continuity assumptions: each benchmark problem is optimized under the following cost and continuity models:

1) Cost-based: member cost c_m, a fixed connection cost $c_0 = \$50$, a variable connection cost c_c equal to 50% of the total member weight cost. Constructability is enforced through continuity rules (Figs. 7 and 10.
2) Weight-based: cost attributed entirely to member weight c_m, no connection cost (c_0, $c_c = 0$) and no continuity rules (Figs. 8 and 11).

In both the cost scenarios explored, the material cost c_m for each member is estimated with unit rates provided by Nucor Construction Corp. based in New York, as is a function of the member type, section, length, and steel properties. The parameters utilized in the benchmark examples are summarized below:

Available sections: AISC Catalog, wide flanges. Design method: effective length. Steel cost: \$39 per metric ton. E: 200 GPa. Fy: 345 MPa. Effective length factor: K = 1.2. Maximum vertical drift: 2 cm. Maximum lateral drift: 2 mm. Span length and interstory height: 2 m. Vertical loads: 445 kN. $\alpha = \beta = 0.1$.

The results illustrated in Table III show that the weight-based, without continuity optimal structure for the 3x3 cantilever appears much lighter than the cost-based structure with continuity rules in place. At the same time, the cost is 16.7% lower for the latter structure. In this example, not only constructability (in the form of member continuity) has converged to a solution that is more realistic to the as-built design, but it has also helped achieve a lower cost objective. In the second example presented, the 3x6 supported beam, there is no significant difference in weight between the two converged structures, however the cost-based solution with continuity achieves a cost that is 30% lower than its counter-part. Since the loads applied to both the 3x3 and 3x6 trusses are the same, it is intuitive that the optimal design of the 3x6 is cheaper and lighter than the 3x3 given that it has an additional support.

Moreover, the computational runtime, quantified by the number of structural analyses performed, is not significantly impacted by the different continuity or objective function configurations. This also shows the scalability potential of the algorithm, given that doubling the number of elements between the two benchmark examples does not require more analyses.

TABLE III
Design Comparison Summary

Example	Configuration	Cost	Weight	Analyses
3x3	Cost-based	\$8817	28.5kN	229
3x3	Weight-based	\$10287	18.4kN	199
3x6	Cost-based	\$7790	23.6kN	209
3x6	Weight-based	\$10137	24.9kN	207

V. Conclusion

The proposed topology and sizing optimization method is a step forward from classic methods in the literature, in that it aims at accurately optimizing for cost while enforcing the same constructability and code-compliance requirements that a designer would have to manually iterate through when approaching a new project. At the same time, it provides flexibility, modularity and scalability in the type, shape and dimensions of steel structural systems. With accurate material, fabrication, and erection rates, as well as specific constructability requirements on a project basis, the designer may use this optimization framework to achieve a constructible solution with minimal post-processing. This cost-based approach may generate structures that are heavier than those obtained with classic weight-based topology optimization methods, but whose cost is lower by up to 30%. The benchmark examples analyzed show how great of

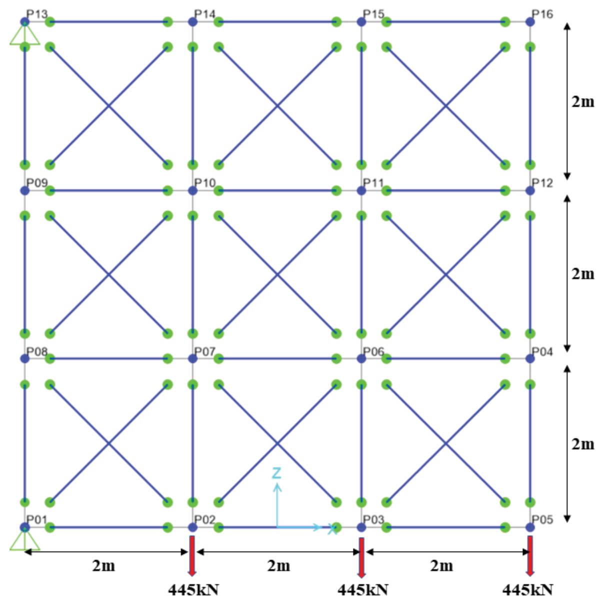

Fig. 6 3x3 cantilever ground structure

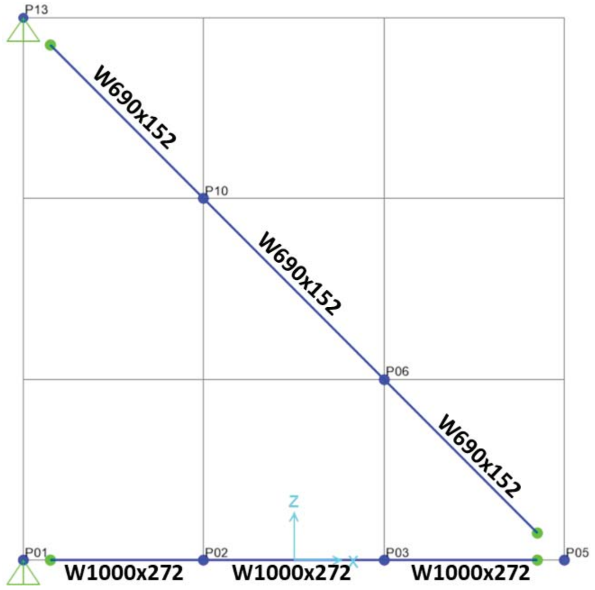

Fig. 7 3x3 Cantilever, cost-based, continuity enabled

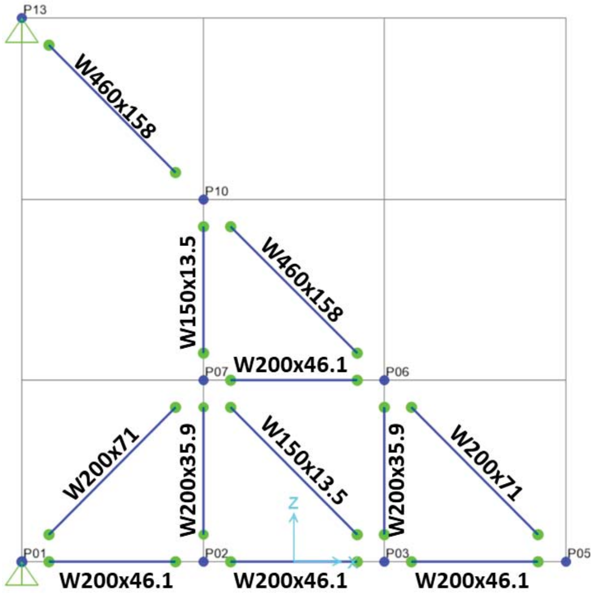

Fig. 8 3x3 Cantilever, weight-based, continuity disabled

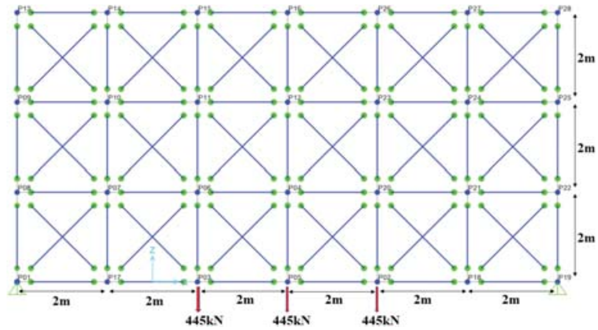

Fig. 9 3x6 beam ground structure

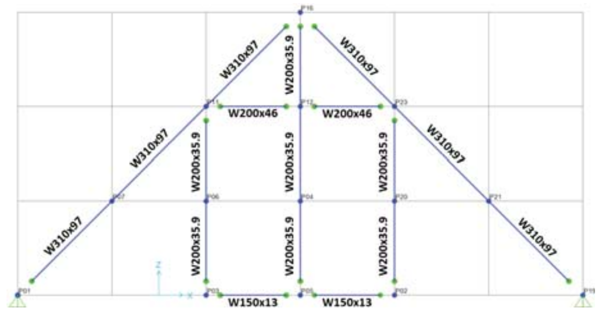

Fig. 10 3x6 Beam, cost-based, continuity enabled

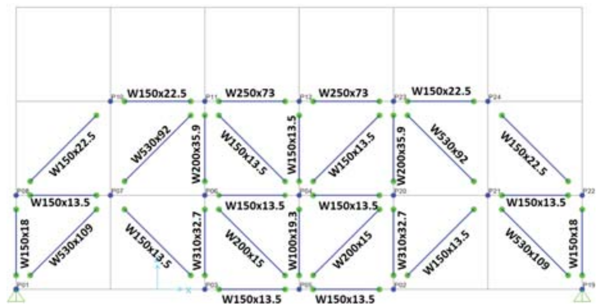

Fig. 11 3x6 Beam, weight-based, continuity disabled

an influence cost and constructability have over the optimal design, and how such parameters cannot be ignored for the optimization to be applicable in practice.

The optimization framework described in this paper is still in the early stages of development. The immediate next steps are to extend the optimization to 3D moment and braced-frame structures and to define the constructability and stability rules for this case. Larger 3D design spaces also require more complex ground structures, as well as many different real and virtual load cases. This step will be fully automated to make it easily scalable to these applications. The authors also plan to implement with a much more comprehensive costing framework that models the cost of each member and connection based on section sizes, member forces and specified connection denomination. Finally, we plan to extend the current method to include seismic design considerations and innovative building structural systems such as BRBFs (Buckling-Restrained Braced Frames) or EBFs (Eccentrically Braced Frames).

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References

[1] J. Agte, O. de Weck, J. Sobieszczanski-Sobieski, P. Arendsen, A. Morris, and M. Spieck, *MDO: assessment and direction for advancement - an opinion of one international group*, Structural and Multidisciplinary Optimization(2010) 40:17-33.

[2] Dorn W. S., et al., *Automatic Design of Optimal Structures*, Journal de Mecanique, 1964, 3(1): p. 25-52.
[3] Liang, Q. Q., Xie, Y. M., Steven, G.P., *Optimal Topology Design of Bracing Systems for Multistory Steel Frames*, J. Struct. Engrg. (2000) 126(7) pp823-829.
[4] L. L. Stromberg, A. Beghini, W. F. Baker, G. H. Paulino *Topology Optimization For Braced Frames: Combining Continuum and Beam/Column Elements*, Engineering Structures 37 (2012) 106-124.
[5] R. Baldock et al.,*Evolving Optimized Braced Steel Frameworks for Tall Buildings Using Modified Pattern Search*, Computing in Civil Engineering (2005).
[6] Paulson, B. C., *Designing to reduce construction costs*, Journal of the Construction Division, 1976. 102(4): p.587-592.
[7] W. Tang, L. Tong, Y. Gu, *Improved Genetic Algorithm for Design Optimization of Truss Structures with Sizing, Shape and Topology Variables*, Int. J. Numer. Meth. Engng 2005; 62:1737-1762.
[8] S.D. Rajan, *Sizing, Shape and Topology Design Optimization of Trusses Using Genetic Algorithm*, Journal of Structural Engineering/ October 1995/ 1480-1487.
[9] K. Deb, S. Gulati, *Design of truss-structures for minimum weight using genetic algorithms*, Finite Elements in Analysis and Design 37 (2001) 447-465.
[10] Achtziger W. and Stolpe M., *Truss topology optimization with discrete design variables Guaranteed global optimality and benchmark examples*, Struct Multidisc Optim (2007) 34, 1-20.
[11] T. Zegard, G.H. Paulino, *GRAND - Ground structure based topology optimization for arbitrary 2D domains using MATLAB*, Struct Multidisc Optim (2014) 50:861-882.
[12] A. Asadpoure, J. K. Guest, L. Valdevit, *Incorporating fabrication cost into topology optimization of discrete structures and lattices*, Struct Multidisc Optim (2015) 51:385-396.
[13] P. Havelia, *A Ground Structure Method to Optimize Topology and Sizing of Steel Frame Structures to Minimize Material, Fabrication and Erection Cost*, M.Eng. Thesis, Stanford University, 2016.
[14] S.Kodiyalam and J.Sobieski, *Multidisciplinary Design Optimization - Some Formal Methods, Framework Requirements, and Application to Vechicle Design*, Int. J. Vehicle Design, Vol.25, Nos , Special Issue, 2001.
[15] C. D. Barrar, *Structural Optimization Using the Principle of Virtual Work and an Analytical Study on Metal Buildings*, M.S. Thesis, Virginia Polytechnic Institute, 2009.