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Considerations for Effectively Using Probability of Failure as a Means of Slope Design Appraisal for Homogeneous and Heterogeneous Rock Masses

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***Abstract*—**Probability of failure (PF) often appears alongside factor of safety (FS) in design acceptance criteria for rock slope, underground excavation and open pit mine designs. However, the design acceptance criteria generally provide no guidance relating to how PF should be calculated for homogeneous and heterogeneous rock masses, or what qualifies a 'reasonable' PF assessment for a given slope design. Observational and kinematic methods were widely used in the 1990s until advances in computing permitted the routine use of numerical modelling. In the 2000s and early 2010s, PF in numerical models was generally calculated using the point estimate method. More recently, some limit equilibrium analysis software offer statistical parameter inputs along with Monte-Carlo or Latin-Hypercube sampling methods to automatically calculate PF. Factors including rock type and density, weathering and alteration, intact rock strength, rock mass quality and shear strength, the location and orientation of geologic structure, shear strength of geologic structure and groundwater pore pressure influence the stability of rock slopes. Significant engineering and geological judgment, interpretation and data interpolation is usually applied in determining these factors and amalgamating them into a geotechnical model which can then be analysed. Most factors are estimated 'approximately' or with allowances for some variability rather than 'exactly'. When it comes to numerical modelling, some of these factors are then treated deterministically (i.e. as exact values), while others have probabilistic inputs based on the user's discretion and understanding of the problem being analysed. This paper discusses the importance of understanding the key aspects of slope design for homogeneous and heterogeneous rock masses and how they can be translated into reasonable PF assessments where the data permits. A case study from a large open pit gold mine in a complex geological setting in Western Australia is presented to illustrate how PF can be calculated using different methods and obtain markedly different results. Ultimately sound engineering judgement and logic is often required to decipher the true meaning and significance (if any) of some PF results.

***Keywords*—**Probability of failure, point estimate method, Monte-Carlo simulations, sensitivity analysis, slope stability.

I. Introduction

THE design of rock slopes, underground excavations and open pit mines now often include both **FS** and PF as key acceptance criteria. FS can very simply be described as (1):

$$FS = \frac{Capacity(C)}{Demand(D)} = \frac{ResistingForces}{DrivingForces} \qquad (1)$$

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FS is a simple and often used measure for stability in slopes or underground excavations. It is defined as a ratio of resisting versus driving forces or capacity (C) versus demand (D) in a given scenario or system. This simplicity allows for ease of communication, hence it is often used as a key acceptance criteria in slope stability or underground excavation design. Although easily communicable, it is commonly recognised that it has limitations. FS alone does not communicate the sensitivity of the modelled scenario to variability in material properties, especially when dealing with natural materials such as rock and soil.

Methods for calculating PF in rock slope design may include (in order of increasing complexity):

- Observational approaches (e.g. length of failed slope against total length of slope) [12].
- Kinematic methods such as the probability of undercutting geological features.
- Probabilistic empirical methods such as Q-slope [1].
- Probabilistic numerical models (e.g. limit equilibrium, finite element or finite difference analyses) [11].
- Network-path analyses or discrete fracture realization models.

Limit equilibrium is achieved when FS equals one. PF can then be described as (2):

$$PF = P[FS \leq 1] = P[C - D \leq 0] \qquad (2)$$

These design acceptance criteria concepts were originally 'borrowed' or 'adopted' from civil and mechanical engineering where materials such as steel and concrete (i.e. materials of known, well understand mechanical properties) are commonly used. The mechanical properties of materials in civil and mechanical engineering are well understood and variability is very small and remains relatively constant from project to project, and even from country to country. Furthermore, components in the analysis of civil and mechanical engineering projects are usually fixed or in pre-determined locations (i.e. the location of components is relatively known and not variable). These aspects allow relatively simple calculations to be conducted to determine PF using the concept in (2).

Contrary to the above, natural materials such as rock and soil in geotechnical and rock engineering projects are:

- Highly variable and never well-understood since site investigations comprising drilling, mapping and testing only sample very small portions of the material. From these 'pin-hole' views of the materials, large extrapolations are often made to characterize the ground.

- Strength characteristics of known intact materials have variability in compression, tension and shearing. Strengths can vary significantly within a project and certainly from country to country.
- The intact materials are separated by geological structures which have variable:
  - Strength and stiffness properties.
  - Location and orientation.
  - Occurrence as it is currently impossible to discretely model and understand each, individual geological structure both from a site investigation and modelling point-of-view.
- Complex influences from groundwater and other external factors such as in-situ stresses and environmental effects (e.g. rainfall, freeze-thaw, seismicity, etc.).
- Time dependence of material characteristics (e.g. strength loss due to exposure following excavation).

FS result sensitivity can be addressed by adjusting model inputs using geological logic and engineering judgement. These sensitivity runs are used to derive sensitivity to the result in light of likely property or input variability. The likelihood of each property varying can be incorporated into models by input of statistical information. The resultant output then contains information about both FS and sensitivity to change or the certainty or reliability of the FS result.

In modelling the PF, the number of runs providing results of FS less than one is divided by the total number of runs to find the probability of occurrence. The uncertainty of the result or distribution can be simply estimated by point estimate method [13]. Where variable distribution information can be used to estimate the resultant distribution and derive a PF. From the 2000s, PF in numerical models was generally calculated using the point estimate method, requiring several individual model scenarios with different input parameters. Although time consuming, the process utilized engineering geological logic to produce decipherable or understandable estimate results.

As slope stability software can now commonly offer the ability to input statistic information for each variable except the discrete location of geological structures, it is now more common to see both FS and PF used in design acceptance criteria. More recently in the 2010s, user-friendly limit equilibrium analysis software offer statistical parameter inputs along with Monte-Carlo or Latin-Hypercube sampling methods to conduct multiple runs; automatically calculating the PF.

The advancements in software allow for fast results and for statistical information to be incorporated into analysis. FS and PF can be easily generated, but the importance of understanding the relationship between each input parameter and the relationships between them needs consideration. However with this automation, engineering geologists and geotechnical engineers are not required to critically think about the model, as PF results are automatically produced.

There appears to be a current tendency to treat all inputs as independent variables, where there are often useful dependencies that can be used to reduce the range in output results, and hence, provide a better more appropriate result for PF.

This is possibly due to the way software tends to individually call for properties; for example, by rock type or rock mass domain. These modelling simplifications can ignore directionality of strength or gradational changes. They can lump data to form statistical inputs or ignore rock mass heterogeneity. That is, they ignore the variability in the occurrence, location and orientation of geological structures, as these parameters are not easily entered into the models. In some cases, directional shear strength or ubiquitous joint models are used to account for the orientation of the main or dominant geological structure such as bedding or foliation planes [2]. However, these have their own limitations, both for FS and even more so for PF.

Geological structures, in most cases, dictate the stability of rock slopes and underground excavations. Design acceptance criteria for rock slopes, underground excavations and in open pit mines should provide guidance relating to how PF should be calculated for homogeneous vs. more common heterogeneous rock masses.

This paper briefly discusses the scale considerations and failure criterion used in modelling and presents examples to illustrate how FS and in particular PF, can vary for the same ground conditions with very simple, yet different modelling or calculation techniques. The examples presented are for rock masses which are simplified and treated as being homogenous; such that, when a geological structure (and therefore heterogeneity) is added to models, the variability in the calculated PF likely increases proportionally with the number of additional variables.

II. Ground Modelling Considerations for Scale Dependency – Homogeneity vs. Heterogeneity and Isotropy vs. Anisotropy

Shear strength criteria such as Mohr-Coulomb [14] and Hoek-Brown [8], [9] assume homogenous, isotropic conditions. However, isotropy and homogeneity are extremely rare [15].

Homogeneity means being the same throughout. Most rocks are heterogeneous: made up of many different minerals which are often not easily sorted and separated, though are clearly distinct. However, when it comes to rock masses, scale dependency relative to the engineering application (e.g. slope stability, underground excavation etc.) determines whether a rock mass will exhibit homogeneous or heterogeneous behaviour. For example, a blocky rock mass may have interbedded siltstones and sandstones with relatively similar geomechanical properties that form near cubical blocks. Although the rock mass is heterogeneous, it may behave as a homogenous and isotropic rock mass.

Isotropic rocks and rock masses are expected to have equal properties irrespective of loading direction, whereas anisotropic rocks and rock masses have a directional dependence of properties. Directional dependence or anisotropy can be modelled in most slope stability software including limit equilibrium, finite element and of course, distinct element codes [2].

III. Failure Criterion and Co-Dependency of Shear Strength Input Parameters

Various criteria are used in rock mechanics to describe the shear strength behaviour of intact rock, rock masses and discontinuities. Each criterion has different input parameters, some of which are independent while others exhibit co-dependency. These should be understood and correctly accounted for to enable 'reasonable' mathematical approaches for calculating PF.

The authors acknowledge that co-dependency cannot be easily estimated or understood, nor can it be entered into current slope stability modelling software.

A. Mohr-Coulomb Failure Criterion

The Mohr-Coulomb failure criterion [14] has two co-dependent input parameters:

- Friction angle (Φ).
- Cohesion (c).

For intact rocks, rock masses and discontinuities, significant co-dependency is observed between cohesion and friction angle as confining stresses change.

B. Hoek-Brown Failure Criterion

The Hoek-Brown failure criterion [8], [9] input parameters are generally independent. In nature, it is possible to find materials with completely different strength, degree of fracturing and discontinuity condition. The input parameters can be estimated independently, and comprise:

- Unconfined compressive strength (UCS).
- Geological strength index (GSI).
- Dimensionless material constant for intact rock (m_i).
- Disturbance factor (D).

It is important to note that a degree of co-dependency is often introduced when GSI is estimated from the logging of drill core using rock mass rating, RMR_{89} [6] and (3) [10]. The reason is that UCS typically accounts for approximately 15% of the RMR_{89} classification rating. Therefore, as UCS increases, so does GSI when it is calculated from RMR_{89}.

$$GSI = RMR_{89} - 5 \tag{3}$$

C. Barton-Bandis Failure Criterion

The Barton-Bandis failure criterion [3], [4] input parameters are generally independent, although in some cases a correlation between joint wall compressive strength (JCS) for clean, unaltered joints and residual friction angles can be found. However, for parameter estimation purposes, input parameters in the Barton-Bandis failure criterion are estimated independently. The parameters include:

- Residual friction angle of a smoothed surface (Φ_r).
- Joint wall compressive strength (JCS).
- Joint roughness coefficient (JRC).
- Effective normal stress (σ_n).
- Length of block (L_n) and measured sample (L_o) [5].

IV. Case Study – Western Australian Gold Mine

A single material type from a large open pit gold mine in Western Australia has been selected for the purpose of this investigation.

The material is a weathered calcareous and argillaceous siltstone that exists in the upper parts of the pit to a maximum depth of approximately 100 m below surface. Below this, the degree of weathering gradationally reduces and rock mass quality improves with depth. Discrete geotechnical domains for the open pit are based on material type and the degree of weathering. Three-dimensional wireframes have been developed and improved for the geotechnical domain boundaries over several years.

At this particular gold mine, the mineral enrichment process has resulted in local zones of rock and rock mass alteration due to either silica or clay enrichment processes. These zones are not discretely defined within the three-dimensional wireframes. Observations during drill core logging and pit wall mapping suggest that these local zones of alteration can range from 5 m to approximately 25 m wide or long. These zones are a significant contributor to intact rock and rock mass strength (depending on the type of alteration) and overall slope stability.

The siltstone is locally anisotropic with a preference of sliding along bedding planes. However, in this pit, structural deformation, namely doming, has resulted in bedding dipping favourably away from the pit excavations, resulting in generally isotropic behaviour on a pit slope scale.

The mechanical properties of the weathered siltstone are quite well understood following extensive, drilling and pit wall mapping and laboratory testing. The strength of the weathered siltstone is modelled using the Hoek-Brown failure criterion [8], [9]. Input parameters vary locally and statistics are presented in TABLE I.

The design, UCS of the weathered siltstone, can be affected in three ways:

- Local variability depending on the type and degree of alteration. Clay enrichment reduces strength, whilst silica enrichment typically increases strength.
- With respect to the direction of loading relative to the anisotropy plane (bedding). The UCS data presented in TABLE I considers loading perpendicular to bedding.
- Sample bias with the preferential selection of more competent, intact samples of correct size for testing. Significant effort was taken to avoid sample bias.

TABLE I
Statistical Hoek-Brown Failure Criterion Input Parameters for Weathered Siltstone

Input Parameters	UCS (kPa)	GSI	m_i	D
Mean Average	15000	35	7	0
Standard Deviation	10000	10	2	-
Minimum	5000	25	5	0
Maximum	25000	45	9	0

Based on the available data for the weathered siltstone, it is generally concluded that the variability in UCS is a direct result of local variability due to clay or silica enrichment.

V. CASE STUDY – MONTE-CARLO SIMULATIONS

An approximately 70 m high slope in weathered siltstone was modelled with limit equilibrium analysis using a total of 5,000 non-circular slip surfaces (Fig. 1). Isotropic rock mass conditions are modelled with the use of the Hoek-Brown failure criterion. A total of 5,000 Monte-Carlo simulations using the statistical parameters were carried out for the lowest FS slip surface to calculate the PF. Note: 50,000 Monte-Carlo simulations were also run as a quality check and gave very similar results.

In Fig. 1 (A), the red material type represents the weathered siltstone with the Hoek-Brown failure criterion statistical input parameters from TABLE I. The lowest FS was 1.1 and the PF was 31%.

In Fig. 1 (B), the coloured squares are used to force the software to create multiple samples of the same materials within the simulation run; each with an area of 20 m^2. Each square is assigned with identical Hoek-Brown failure criterion statistical input parameters from TABLE I. The lowest FS remains as 1.1 and the same critical slip surface (failure mechanism) to Fig. 1 (A) is identified. However, the PF is significantly reduced from 31% to 13%. The critical slip surface intersects seven individual blocks with lesser likelihood of all sampling at extreme low or high end values.

In Fig. 1 (C), the coloured squares are reduced to 10 m^2. Again, each individual square is assigned with identical Hoek-Brown failure criterion statistical input parameters from TABLE I. The lowest FS remains as 1.1 and the same critical slip surface (failure mechanism) to Fig. 1 (A) and (B) is identified. However, the PF is further reduced from 13% to 3%. In this case the critical slip surface intersects 12 individual blocks.

TABLE II
LIMIT EQUILIBRIUM ANALYSIS RESULTS OF 5,000 NON-CIRCULAR SLIP SURFACES FOR LOWEST FS AND AUTOMATICALLY CALCULATED PF% USING MONTE-CARLO SIMULATIONS ON HETEROGENEOUS SLOPES SHOWN IN FIG. 1

Number of Identical Materials Intersected by Slip Surface	Deterministic FS	Mean FS	PF
1	1.1	1.1	31%
7	1.1	1.1	13%
12	1.1	1.1	3%

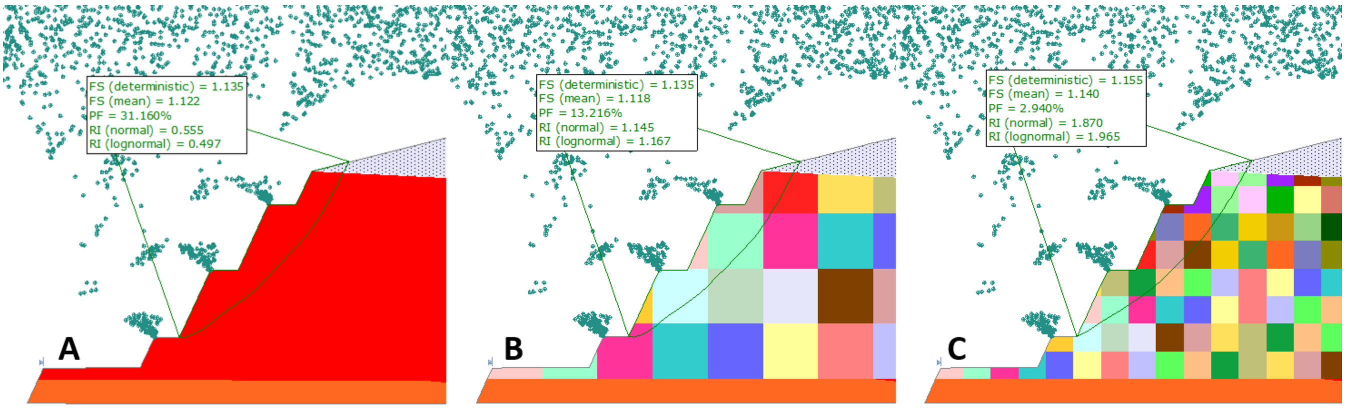

Fig. 1 Limit equilibrium analysis results for 5,000 non-circular slip surfaces – lowest FS all obtained similar surfaces and results. PF calculated for the lowest FS slip surface using 5,000 Monte-Carlo simulations of statistical properties for A: a single material; B: same material using 20 m^2 blocks, of which, seven were intersected by the lowest FS slip surface; C: same material using 10 m^2 blocks, of which, 12 were intersected by the lowest FS slip surface

Each time modelling software carry out Monte-Carlo simulations (i.e. 5000 times) to assess PF, each material's input parameters are varied based on their statistical input parameters. If more and more identical materials (blocks) are used in a model, the likelihood of each individual block having very low input parameters throughout the entire model or slip surface decreases proportionally with the number of identical materials (blocks), as shown in TABLE II.

This example illustrates how having more materials can reduce the perceived PF in a model. In most mining scenarios, where multiple rock types and more geotechnical domains are investigated simultaneously, the meaning of the PF that is calculated should be questioned. This raises the question of "which modelling method, if any, is most representative of ground behaviour, and why?"

VI. CASE STUDY – POINT ESTIMATE METHOD

The point estimate method [13] provides a direct computational procedure to obtain the moment estimates (i.e. mean and variance) for a function where each variables distribution is used to derive an output distribution. The particular shape of the probability density function is not critical to the analysis since it can be represented by the mean and two boundaries i.e. located at plus (+) and minus (-) one standard deviation from the mean average. TABLE III presents point estimate method scenarios in which Hoek-Brown input parameters UCS, GSI and m_i are treated as random variables based on the statistical input parameters from TABLE I. Note: disturbance factor (D) is considered to be deterministic with a constant value of zero.

When using point-estimate method derived input parameters, a different FS is calculated for each scenario simulation [7].

TABLE III
POINT-ESTIMATE METHOD SCENARIO HOEK-BROWN FAILURE CRITERION INPUT PARAMETERS FOR WEATHERED SILTSTONE

Point Estimate Scenario		UCS (kPa)	GSI	m_i	D
1	+++	25000	45	9	0
2	++-	25000	45	5	0
3	+--	25000	25	5	0
4	---	5000	25	5	0
5	+-+	25000	25	9	0
6	--+	5000	25	9	0
7	-++	5000	45	9	0
8	-+-	5000	45	5	0

A total of eight simulations were required for the various upper (+) and lower (-) bound combinations since three parameters were selected as random variables. The model used was Fig. 1 (A) with various input parameters or point estimate scenarios. TABLE IV presents the results for the factors of safety were obtained in the analyses in two different ways:

1. Homogenous slope with no pre-determined slip surface. Analysis uses 5,000 slip surfaces to calculate lowest FS (i.e. slip surface in each scenario is different).
2. Homogenous slope with a single, pre-determined slip surface from the lowest FS in Fig. 1 (A) (i.e. each scenario uses the same slip surface).

Based on the point estimate scenarios, mean average and standard deviation for FS were calculated. From these, PF was calculated as the probability that FS will be less than one. Therefore, from the point estimate method, PF was in the range of 43-44%. However, the mean FS remained in the order of 1.1.

Several of the point estimate scenarios (3, 4, 6 & 8) attained FS less than one. From these, it is reasonable to conclude that for this case study, when two or more input parameters are lower bound (-), failure can be anticipated. With this understanding it is possible for a geotechnical engineer or engineering geologist to review ground conditions in the field as excavations occur, and review and update modelling results using engineering geological logic and judgement.

TABLE IV
LIMIT EQUILIBRIUM ANALYSIS RESULTS FOR LOWEST FS AND MANUALLY CALCULATED PF USING POINT ESTIMATE METHOD ON HOMOGENOUS SLOPE WITH INPUT PARAMETERS FROM TABLE III

Point Estimate Scenario		Homogenous Slope 5,000 Slip Surfaces FS	Homogenous Slope Single Slip Surface from Fig. 1 (A) FS
1	+++	1.833	1.827
2	++-	1.591	1.628
3	+--	0.912	0.908
4	---	0.520	0.522
5	+-+	1.100	1.101
6	--+	0.657	0.656
7	-++	1.064	1.065
8	-+-	0.884	0.885
Mean Average		1.070	1.074
Standard Deviation		0.445	0.450
PF =P[FS<1]		44%	43%

Fig. 2 Photograph showing condition of slopes in highly weathered siltstone – effectively zero slope failures 20 years after excavation

Conversely, with the 5,000 Monte-Carlo simulations approach, this basic understanding around the controlling input parameters is less evident.

VII. CASE STUDY – OBSERVATIONAL OR EXPERIENCE APPROACH

After 20 years of mining in the weathered siltstone in the pit of interest (Fig. 2), effectively zero slope failures had occurred at similar slope design geometries to those analysed. Minor wedges (less than 10 m in height) have failed in localized areas and account for less than 1% of total bench slope crests (Fig. 3). Based on an observational approach, the FS in the slopes is above one (i.e. above equilibrium) and the PF is 0-1%.

Fig. 3 Photograph showing two ‘rare’ localized 10 m high wedge failures in lowest bench of weathered siltstone

VIII. Case Study – Results Comparison

Three different methods of assessing the PF for the same slopes were used and attained vastly different results:

- PF=3-31% from statistical inputs and Monte-Carlo simulations.
- PF=43-44% from eight individual scenarios and the point estimate method (prior to using engineering judgement).
- PF=0-1% based on the observational or experiential approach.

This highlights the importance for engineers to utilize observational inputs and relationships between parameters, rather than relying solely on software output PF results when accepting design stability analysis. It also shows how some modern software results could be manipulated to meet prescribed values of PF.

Although not discussed in the body of this paper, PF from the empirical method, Q-slope [2], and kinematic analysis (assuming infinitely continuous geological structures based on the available structure orientation data) also yielded variable results:

- PF=1-15% from Q-slope for 65° bench face angles depending on the intact material strength and stress from the slope height.
- PF=2-12% for planar and wedge sliding and toppling for bench face angles of 65° and PF=1-8% for inter-ramp slope angles of 52°.

IX. Discussion

An example has been used to illustrate how having more materials in a model can reduce the PF within a single isotropic material type or geotechnical domain.

In most rock engineering scenarios, where the stability of multiple rock types and geotechnical domains are investigated simultaneously, the meaning of the PF that is calculated in limit equilibrium analysis software should be questioned and interrogated.

When more detailed modelling approaches are carried out, perhaps involving geological faults and/or directional shear strengths (ubiquitous joint models), which require more statistically-derived input parameters, the problems associated with calculating a meaningful PF are further exacerbated.

For homogenous, isotropic rock masses where material properties vary ubiquitously throughout the rock mass, the use of statistical inputs in current slope stability packages may provide a useful and meaningful PF result. However, this scenario is incredibly rare, if not impossible to find in nature. Additionally, design acceptance criteria should provide more guidance on how to approach this, since it has been shown that various mathematical characteristics have a very large influence on the PF that is calculated.

For heterogeneous, often anisotropic rock masses with complex geological structure comprising bedding or foliation fabrics, joint sets and faults, each with different strength and stiffness properties, and moreover, variable occurrence, persistence, location and orientation, the number of variable input parameters is usually too much to: (a) precisely understand and define from site investigations (in many cases), and (b) to model in such a way to get a meaningful PF result.

The authors discourage the 'blind' application of statistical inputs for Monte-Carlo or Latin-Hypercube simulation in slope stability software without adequate consideration of input parameter applicability to shear strength criteria, statistical distributions, parameter co-dependency and the influence of geological structure, which usually dictates stability. Rather, it is suggested that a suite of 'thoughtful' and 'meaningful' sensitivity analyses are carried out in relation to the key input parameters that are expected to influence slope stability. This of course requires sound engineering geological and rock mechanics understanding.

The authors strongly encourage readers of geotechnical or rock engineering reports that present PF values, to question what assumptions and considerations were made in the calculations and whether these are reasonable for both the ground conditions and engineering project being undertaken.

X. Concluding Remarks

The engineering analysis of natural materials such as rock and soil is incredibly more complex than for human-engineered materials such as steel and concrete used in civil and mechanical engineering projects.

The adoption of design acceptance criteria for factors of safety (FS) and PF from civil and mechanical engineering disciplines should only be done with significant geological logic and engineering judgement to critically investigate plausible scenarios and reasonable material behaviour. It has been shown that PF is highly dependent on the method and modelling approach used to in the calculation. Therefore, when dealing with natural materials, the design acceptance criteria should, if possible, also provide guidance on how to calculate PF in order to provide a reasonable and realistic outcome for the problem or project.

In lieu of a solution of effectively calculating PF, the authors suggest the use of sensible and meaningful sensitivity analyses on key parameters affecting FS. This will likely unlock significant project value as opposed to a blind acceptance or drive to meet prescribed PF design acceptance criteria that may then lead to overly conservative designs.

Although the authors are unable to provide a solution to the PF design acceptance criteria problem, they encourage the engineering geological and rock engineering community to debate the issue around appropriate guidance for its use.

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