Optimizing and Evaluating Performance Quality Control of the Production Process of Disposable Essentials Using Approach Vague Goal Programming

Hadi Gholizadeh, Ali Tajdin

Abstract—To have effective production planning, it is necessary to control the quality of processes. This paper aims at improving the performance of the disposable essentials process using statistical quality control and goal programming in a vague environment. That is expressed uncertainty because there is always a measurement error in the real world. Therefore, in this study, the conditions are examined in a vague environment that is a distance-based environment. The disposable essentials process in Kach Company was studied. Statistical control tools were used to characterize the existing process for four factor responses including the average of disposable glasses’ weights, heights, crater diameters, and volumes. Goal programming was then utilized to find the combination of optimal factors setting in a vague environment which is measured to apply uncertainty of the initial information when some of the parameters of the models are vague; also, the fuzzy regression model is used to predict the responses of the four described factors. Optimization results show that the process capability index values for disposable glasses’ average of weights, heights, crater diameters and volumes were improved. Such increasing the quality of the products and reducing the waste, which will reduce the cost of the finished product, and ultimately will bring customer satisfaction, and this satisfaction, will mean increased sales.

Keywords—Goal programming, quality control, vague environment, disposable glasses’ optimization, fuzzy regression.

I. INTRODUCTION

PRODUCTION begins with the decision to produce and continues until the final product is complete. To have effective production planning, it is necessary to calculate the quality control and chart control. For this purpose, the study of Kach Packaging Industries, as one of the subsidiaries of the Solico Industrial Group, is in charge of manufacturing polymer containers for packing products from Kach Dairy Company. Kach Dairy Company is one of the largest dairy and protein production plants in Iran. Due to its high production, timely and without delay, it has a significant impact on reducing its costs (Cost Stop Lines, Capital Sleep, etc.). Therefore, as part of the company's supply chain, Kach Company is required to provide the company with the most important requirements for packaging in a timely manner and with high quality. Thus, due to its constraints, the company should always provide these supplies to Kaleh Company with the highest possible output. As a result, the importance of quality in this organization has been one of the main goals.

Statistical quality control is an important approach that helps by using statistical tools to illustrate the process. Chevrolet control charts are one of the most important quality control methods used to illustrate deviations with reason. Fuzzy diagrams with the ability to formulate expert experiences and use vague and imprecise data increase the ability to control quality to improve the quality of products and services. In this research, fuzzy theory is used to empower the statistical control charts. Because of the limitation of measurement tools and uncertainty in the measurement system, such as operators, confusions, environmental conditions, etc. operators cannot provide a precise number for these characteristics, and they inevitably record them in an approximate manner. The processing ability is based on fuzzy measurements and analysis control charts. The fuzzy fashion method and the proposed method of fuzzy rules are used to create fuzzy control graphs [1]. A fuzzy method is provided for vague data in monitoring the mean and variance of fuzzy charts by [2]. One method for calculating fuzzy standard deviations is to obtain a graph for cases provided by \( x^* - s \), as well as explained by the theoretical structure of fuzzy control \( x^* - s \) which is known as local parameters [3].

Several approaches have been proposed to optimize process performance with multiple responses [4]. Therefore, multiple formalizations of goal programming (GP) models were shown for deciding the fuzzy GP (FGP) problems obtaining into account the decision maker's (DM's) priority [5]. An effective FGP technique is the weighted additive model, which considers all shapes of membership functions, with the objective to minimize the weighted deviations from the imprecise fuzzy values for all quality responses and process factors [6]. FGP has been utilized for optimizing process performance in many business applications [7]. A vague set, as well as an Intuitionistic fuzzy set, is a further generalization of a fuzzy set. We describe the Optimization of queuing theory based on vague environment [8]. In this research study, an empirical study of the fuzzy regression model was conducted to better estimate and predict. We explained fuzzy regression model for the evaluation of the functional relation between related and independent variables in the fuzzy environment, practical to diverse problems, such as prediction engineering [9]. The first engineering usage of GP was explained, due to the design and commissioning of spacecraft in the aerospace environment.
sciences via [10].

In reality, determining the combination of optimal factor settings for disposable glasses' manufacturing processes to improve multiple quality responses is a real challenge. This paper, therefore, aims at optimizing the performance of direct production process for multiple quality characteristics using statistical techniques (statistical quality control, fuzzy regression model) and weighted additive model in GP in a vague environment.

The continuation of this study is as follows. In Section II, the various concepts of the vague set theory are discussed. In Section III, vague process capability analysis and fuzzy regression models are introduced. The vague GP is introduced, and the process capability is expressed in Section IV. We conclude the subject in Section V.

II. VAGUE ENVIRONMENT

Definition 1: [8] (Fuzzy Set) A fuzzy set \( \tilde{A} = \{ u, \mu_\tilde{A}(u) \mid u \in U \} \) in a universe of discourse \( U \) is characterized by a membership function, \( \mu_\tilde{A} \), as follows:

\[
\mu_\tilde{A} : U \rightarrow [0, 1].
\]

Definition 2: [8] A vague set \( \tilde{A} \) in a universe of discourse \( U \) is characterized. It is expressed by two membership functions, which is a true membership, \( T_\tilde{A} \) , function and a false membership function, \( F_\tilde{A} \), as follows: \( T_\tilde{A} : U \rightarrow [0, 1], F_\tilde{A} : U \rightarrow [0, 1] \), and \( T_\tilde{A}(u) + F_\tilde{A}(u) \leq 1 \), where \( T_\tilde{A}(u) \) is a lower bound on the grade of membership of \( u \) derived from the evidence for \( u \), \( F_\tilde{A}(u) \) is a lower bound on the grade of membership of the negation of \( u \) derived from the evidence against \( u \). Suppose \( U = \{u_1, u_2, \ldots, u_n\} \). A vague set \( \tilde{A} \) of the universe of discourse \( U \) can be represented by \( \tilde{A} = \sum_{i=1}^{n} \frac{F_\tilde{A}(u_i)}{1-F_\tilde{A}(u_i)} u_i \), where \( 0 \leq T(u) \leq 1 \) and \( 0 \leq F(u) \leq 1 \). In this case, the degree of membership in \( u_i \) will be limited to a period of time, \( T_\tilde{A}(u_i), 1 - F_\tilde{A}(u_i) \) of \( [0, 1] \). Therefore, generalized vague collections have fuzzy sets of membership degrees \( \mu_\tilde{A}(u) \) of \( u \) in Definition 1 may be inexact as a vague set. We now depict a vague set in Fig. 1.

![Membership Functions of a Vague Set](image)

**Fig. 1 Membership Functions of a Vague Set**

Definition 3: [8]. The complement of a vague set \( \tilde{A} \) is denoted by \( \tilde{A}' \) and is defined by

\[
T_{\tilde{A}'}(u) = F_\tilde{A}(u), 1 - F_{\tilde{A}'}(u) = 1 - F_\tilde{A}(u) \quad (1)
\]

Definition 4: [8]. Let \( \tilde{A}_{\alpha T} \) be a vague set of \( U \). Then, we define \( \alpha_T \) -cuts and \( \alpha_F \) -cuts of \( \tilde{A} \) as the crisp sets of \( U \) given by

\[
\tilde{A}_{\alpha_T} = \{ u : T_\tilde{A}(u) \geq \alpha_T \} \quad \alpha_T \in [0, 1] \quad (2)
\]

\[
\tilde{A}_{\alpha_F} = \{ u : F_\tilde{A}(u) \geq \alpha_F \} \quad \alpha_F \in [0, 1] \quad (3)
\]

Definition 5: [8]. A vague set \( \tilde{A}_{F \alpha} \) of \( \mathcal{R} \) with continuous membership functions \( T_\tilde{A} \) and \( F_\tilde{A} \) is called a vague number if and only if \( \tilde{A}_{\alpha_T} \) and \( \tilde{A}_{\alpha_F} \), for all \( \alpha_T, \alpha_F \in (0, 1) \), are bounded closed intervals; i.e.

\[
\tilde{A}_{\alpha_T} = [\tilde{A}_{\alpha_T}^L, \tilde{A}_{\alpha_T}^U], \quad \tilde{A}_{\alpha_F} = [\tilde{A}_{\alpha_F}^L, \tilde{A}_{\alpha_F}^U] \quad (4)
\]

We denote the class of all vague numbers by \( \mathcal{A}(\mathcal{R}) \).

Definition 6: [8]. The vague number \( \tilde{A}_{T \alpha F} \) of \( \mathcal{R} \) is called a trapezoidal vague number, if

\[
T_\tilde{A}(u) = \begin{cases} \frac{u-a_1}{w(a_2-a_1)} & a_1 \leq u \leq a_2 \\ \frac{1}{w(a_2-a_1)} & a_2 \leq u \leq a_3 \\ \frac{a_1-u}{w(a_4-a_3)} & a_3 \leq u \leq a_4 \\ 0 & 	ext{otherwise} \end{cases} \quad (5)
\]

\[
1 - F_\tilde{A}(u) = \begin{cases} \frac{u-a_1}{w(a_2-a_1)} & a_1 \leq u \leq a_2 \\ \frac{1}{w(a_2-a_1)} & a_2 \leq u \leq a_3 \\ \frac{a_1-u}{w(a_4-a_3)} & a_3 \leq u \leq a_4 \\ 0 & 	ext{otherwise} \end{cases} \quad (6)
\]

where \( w \in [1, \infty) \). We denote such a vague number by \( \tilde{A} = (a_1, a_2, a_3, a_4, w) \).

A. Control Charts

A control chart is one of the primary monitoring techniques of Statistical Process Control (SPC). Control charts plot the points where statistics (such as an average, range, ratio, etc.) measure qualitative and quantitative characteristics of samples that have been obtained at different times of the process. The chart has a central line in the mean values (CL) as well as the upper and lower levels of control (sometimes called the "natural part of the process"), which represents the threshold "unlikely" of the resulting process and they are drawn in three standard errors from the central line (UCL and LCL, respectively). At initial factor, settings have been explained by the \( x - s \) charts for averages disposable glasses' weight, height, crater diameter, and volume [11].

\[
\text{CL}_R = \bar{x}, \text{UCL}_R = \bar{x} + A_2 \bar{S}, \text{LCL}_R = \bar{x} - A_2 \bar{S} \quad (7)
\]

B. Vague Control Charts

This paper develops vague as a control chart which is one of the primary (SPC) monitoring tools. Control charts plot the averages of measurements of quality characteristic in samples taken from the process versus the sample vague number. The chart has a center line (\( \text{CL} \)) as well as upper and
lower control limits ($UCL$ and $LCL$, respectively). The control chart for the four factors of weight, height, crater diameter and volume is shown in Fig. 2.
III. VAGUE PROCESS CAPABILITY ANALYSIS

Capability analysis is used to assess whether a process is statistically capable to meet a set of customer desired product specifications. In practice, the process standard deviation, \( \sigma \), is unknown and is frequently estimated by:

\[
\begin{align*}
\bar{x} & = \frac{1}{n} \sum_{i=1}^{n} x_i \\
\bar{S} & = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (s_i - \bar{s})^2}
\end{align*}
\]
The final average values are calculated for the four responses and recorded in Table III. Let $Y_1$, $Y_2$, $Y_3$ and $Y_4$ denote the measured averages of weight, height, crater diameter and volume, respectively. To optimize process performance, the weighted additive model GP in a vague environment was utilized. The optimization procedure is described as follows: We formulate the mathematical relationship between each quality response and process factors.

\[ Y = (12.241,13.829,20.359)\oplus(0.130,-0.019,0.008)\ominus X_1 \oplus(0.137,0.079,0.144)\ominus X_2 \oplus(-0.189,0.184,0.038)\ominus X_3 \]

The regression model for $Y_2$ is expressed as:

\[ Y_2 = (124.828,124.050,130.935)\oplus(-0.055,0.078,0.015)\ominus X_1 \oplus(0.359,0.074,-0.016)\ominus X_2 \oplus(-0.165,0.180,0.005)\ominus X_3 \]

The regression model for $Y_3$ is expressed as:

\[ Y_3 = (111.480,113.557,116.167)\oplus(0.019,0.084,0.113)\ominus X_1 \oplus(0.149,0.121,0.108)\ominus X_2 \oplus(0.064,-0.047,-0.179)\ominus X_3 \]

Finally, the regression model for $Y_4$ is expressed as:

\[ Y_4 = (825.565,833.238,838.725)\oplus(0.768,0.540,0.170)\ominus X_1 \oplus(0.659,0.749,0.968)\ominus X_2 \oplus(-0.132,-0.241,-0.109)\ominus X_3 \]

### Table II

<table>
<thead>
<tr>
<th>Total Experiment</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>$X_3$</th>
<th>$Y_1$</th>
<th>$Y_2$</th>
<th>$Y_3$</th>
<th>$Y_4$</th>
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<td>[10,15,16]</td>
<td>[15,17,19]</td>
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<td>[114,115,116]</td>
<td>[840,5,845,850]</td>
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<td>[11,12,13]</td>
<td>[11,13,15]</td>
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<td>[113,115,117]</td>
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<td>[12,14,16]</td>
<td>[8,9,10]</td>
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<td>[128,3,130]</td>
<td>[114,116,118]</td>
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<td>[12,14,16]</td>
<td>[14,13,15]</td>
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<td>[124,128,132]</td>
<td>[115,5,116,117,5]</td>
<td>[850,853,860]</td>
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<tr>
<td>5</td>
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<td>[11,12,13]</td>
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<td>[16,18,20]</td>
<td>[119,125,130]</td>
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<td>[114,8,116,117,3]</td>
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<td>[25,26,30]</td>
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<td>[115,118,121]</td>
<td>[840,5,850,860]</td>
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<tr>
<td>13</td>
<td>[15,18,21]</td>
<td>[14,16,18]</td>
<td>[10,15,16]</td>
<td>[16,18,20]</td>
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<td>[113,8,116,122,7]</td>
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<td>14</td>
<td>[15,18,21]</td>
<td>[14,16,18]</td>
<td>[11,13,15]</td>
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<td>[115,117,119]</td>
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<td>[15,18,21]</td>
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<td>[116,2,118,122,0,2]</td>
<td>[839,7,849,2,859,7]</td>
</tr>
</tbody>
</table>

### Table III

<table>
<thead>
<tr>
<th>Utility DM in the initial settings</th>
<th>Quality characteristic</th>
<th>Mean</th>
<th>deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight(g)</td>
<td>18</td>
<td>[17,5,18,2]</td>
<td>0.97</td>
</tr>
<tr>
<td>Height (mm)</td>
<td>130</td>
<td>[129,131]</td>
<td>0.86</td>
</tr>
<tr>
<td>crater diameter (mm)</td>
<td>116</td>
<td>[115,8,116,2]</td>
<td>0.65</td>
</tr>
<tr>
<td>Volume(cc)</td>
<td>850</td>
<td>[840,860]</td>
<td>0.76</td>
</tr>
</tbody>
</table>

IV. VAGUE GP

FGP can be used to plan programs for solving multiple decision problems in an irregular ambiguous environment. In this method, instead of achieving objective fuzzy value goals,
achieving membership goals for goals is to maximize or minimize uncertainty in a search process. In this study, the objectives of fusion-based distance-based programming are defined as a "false and true membership function" supported by "vague collections" [11].

We assign weights to the deviations according to their relative significance DM. Then write the complete model as:

Minimize \( Z_{af} = 0.10(\omega_{\alpha_f}^{LU}_{af} - \omega_{\alpha_f}^{LU}_{af} + 0.15(\omega_{\alpha_f}^{LU}_{af} - \omega_{\alpha_f}^{LU}_{af}) + 0.25(\omega_{\alpha_f}^{LU}_{af} - \omega_{\alpha_f}^{LU}_{af}) + 0.35(\omega_{\alpha_f}^{LU}_{af} - \omega_{\alpha_f}^{LU}_{af}) + 0.10(\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}) + \omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}) \)

\[
\begin{align*}
\frac{\delta_{\omega_f}^{LU}_{af}}{\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}} + \frac{\delta_{\omega_f}^{LU}_{af}}{\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}} &= 1 \frac{\omega_{\alpha_f}^{LU}_{af}}{\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}} \geq 26 \frac{\omega_{\alpha_f}^{LU}_{af}}{\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}} \leq 26.6 \frac{\omega_{\alpha_f}^{LU}_{af}}{\omega_{\alpha_f}^{LU}_{af} + \omega_{\alpha_f}^{LU}_{af}} \leq 1.2
\end{align*}
\]

The obtained optimal process conditions were found to be:

<table>
<thead>
<tr>
<th>TABLE III</th>
<th>THE OBTAINED OPTIMAL PROCESS CONDITIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a_f )</td>
<td>( w )</td>
</tr>
<tr>
<td>0.15</td>
<td>2</td>
</tr>
<tr>
<td>0.45</td>
<td>2</td>
</tr>
<tr>
<td>( a_f )</td>
<td>( w )</td>
</tr>
<tr>
<td>0.05</td>
<td>5</td>
</tr>
<tr>
<td>0.95</td>
<td>1</td>
</tr>
</tbody>
</table>

The expected values for the Weight (g), Height (mm), Crater diameter (mm), and Volume (cc) are calculated as 18 g, 130 mm, 116 mm and 850 cc, respectively.

The control chart for the four factors of weight, height, crater diameter and volume is shown in Fig. 3.

A Discussion

Organizations are able to make the best decisions in a variety of ways and have the best performance in order to adopt the process' effectiveness and the ability and efficiency of the production process. Considering the calculations and the control charts, we analyzed the sensitivity of the system function by considering the vague conditions of the distancing based uncertainty. The \( CPK \) values for the averages of disposables glasses' weight, height, crater diameter, and volume are 1.38, 1.89, 1.80 and 2.28, respectively. The corresponding improvement relations were found to be 69.5%, 45.5%, 36% and 33.3%, respectively. Thus, results will affect in significant recovery in quality and thereby they increase productivity. By increasing the amount of \( a_f \), \( a_f \) and \( w \), the amount of \( CPK \) decreases, as a result, the amount of dispersion increases and the process acts out of the center, which increases the amount of waste and we are at Level2σ, at this level, customers are satisfied with the organization, but they are putting a lot of pressure on the organization, such as low-profit margins, high operating costs, and a lot waste. Regarding the average deviation, we find that this deviation are due to machine error and is not related to sampling error and requires 100% inspection. Regarding waste recycling, the cost of rebuilding and rework increases. Therefore, in general, when we increase the amount of \( a_f \), \( a_f \) and \( w \), the processing ability becomes less and the process will be less accurate, but it will work properly.

The main reason for these problems can be attributed to the
accuracy of the device due to its exhaustion or the operator’s skill changes (losing skill).

\[ \alpha_r = 0.15 \]

\[ \alpha_f = 0.95 \]
(b-1)

Xbar-S Chart of vague height

Sample Mean

Sample StDev

(b-2)

Xbar-S Chart of vague height_3

Sample Mean

Sample StDev
Xbar-S Chart of vague crater diameter

\( \alpha_l = 0.15 \)

Sample Mean

\( \bar{X} = 116.88 \)

UCL = 123.45

LCL = 110.30

Sample

(c-1)

Xbar-S Chart of vague crater diameter_3

\( \alpha_r = 0.95 \)

Sample Mean

\( \bar{X} = 115.75 \)

UCL = 119.040

LCL = 112.460

Sample

(c-2)
Fig. 3 The $\bar{x} - s$ charts for average disposable glasses at vague factor settings
V. CONCLUSION
Given that the fuzzy quality control charts have greater flexibility than classical control charts; their use reduces the time needed to detect abnormalities in the process being investigated. This paper explains improving the performance of the disposable essentials process using statistical quality control and GP in an unclear environment. Therefore, in this study, the conditions are examined in a vague environment that is an environment based on distance. The statistical quality control tools were used for the four factors referred to the disposable glasses production process. Optimization results showed that the process capability index values for disposable glasses’ averages of weight, height, crater diameter and volume relative to the classical model were improved accordingly. Such improvements resulted in significant savings in production costs and increase quality. The $C_{p_k}$ values for the averages of disposable glasses' weight, height, crater diameter, and volume were improved, respectively. In this way, the results will have a significant effect on improving quality and thus increasing productivity.

The value goal of the membership function is almost equal in both conditions, which shows that the distance between these two membership functions is relatively little and they found a heuristic fuzzy membership function.

REFERENCES