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Anisotropic Total Fractional Order Variation Model in Seismic Data Denoising

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***Abstract*—**In seismic data processing, attenuation of random noise is the basic step to improve quality of data for further application of seismic data in exploration and development in different gas and oil industries. The signal-to-noise ratio of the data also highly determines quality of seismic data. This factor affects the reliability as well as the accuracy of seismic signal during interpretation for different purposes in different companies. To use seismic data for further application and interpretation, we need to improve the signal-to-noise ration while attenuating random noise effectively. To improve the signal-to-noise ration and attenuating seismic random noise by preserving important features and information about seismic signals, we introduce the concept of anisotropic total fractional order denoising algorithm. The anisotropic total fractional order variation model defined in fractional order bounded variation is proposed as a regularization in seismic denoising. The split Bregman algorithm is employed to solve the minimization problem of the anisotropic total fractional order variation model and the corresponding denoising algorithm for the proposed method is derived. We test the effectiveness of theproposed method for synthetic and real seismic data sets and the denoised result is compared with F-X deconvolution and non-local means denoising algorithm.

***Keywords*—**Anisotropic total fractional order variation, fractional order bounded variation, seismic random noise attenuation, Split Bregman Algorithm.

I. Introduction

In seismic data processing the main goal of noise attenuation is to condition the seismic data so that an improved and better resolved seismic data can be obtained for further investigation for exploration and development in different gas and oil industries. One way of obtaining an improved and better resolved seismic data is through inversion methods. Variational regularization method is one of the inversion method that have been used in image processing. Partial differential equations (PDEs) based denoising methods have been discussed in different literatures [1], [2] is the most frequently used inversion method. PDE models either the nonlinear diffusion [3], or the variation of energy functional [4] have been applied for natural image processing [5]-[7]. The PDE models have been also applied for the noise attenuation of seismic data [8]-[13]. Different methods have been also proposed for the removal of noise from seismic data based on the hybrid of variational model and multi-scale geometry transforms Curvelet-based SOTV regularization, [14], shearlet and total generalized variation (TGV) [15] regularization.

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In our work, we focused on variation model which is new for seismic signal processing especially for the seismic noise attenuation. Total variation (TV) is one of the variation model which have been introduced by [4] for the noise removal from natural image. Because of its drawback in introducing the staircase artifact, TV have been not applied for seismic data processing for a long time. Now a day different improvement was employed to apply TV for seismic data processing for noise removal [10], [12], [9]. [16], [17] showed that the fractional order derivative is an alternative tool for the improvement of drawback of TV. Different methods based on fractional order derivatives have been applied for image denoising [16], [18]-[20]. Discrete optimization framework has been employed in fractional order derivative in the denoising problem as regularization [21] which is given by

$$\min\{\sum_{i,j=1}^{N}|(\nabla^{\alpha}u)|+\frac{1}{2}\sum_{j=1}^{N}2^{-2js_j}|[\lambda(f-u)_j]|, 1\le\alpha\le 2, 0\le s_j\le 1\}. \tag{1}$$

To solve (1) alternating projection algorithm have been used [22].

For this paper, we employed anisotropic total fractional order variation model defined in fractional order bounded variation to improve the signal-to-noise ratio (SNR) and remove noise from seismic data by preserving valid seismic signals.

II. Method

For this paper, the solution of anisotropic total fractional order variation model in the space of α-BV is employed to preserve both edges and smoothness of seismic data. The total order derivatives in the fractional α-order variation is defined as

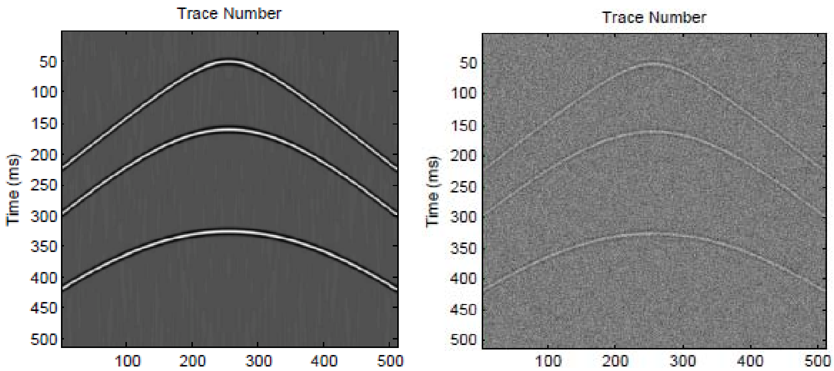
$$D^{\alpha}_{[a,x]}f(x)=\frac{1}{\Gamma(n-\alpha)}(\frac{d}{dx})^n\int_a^x\frac{f(\tau}{)}(x-\tau)^{\alpha-n+1}$$

for $0<\alpha<1$, with $\Gamma(n-\alpha)$ is a gamma function. The space available for total α-order variation (TV^{α}) using α-order derivatives is the space $BV^{\alpha}(\Omega)$ of functions of α-bounded variation on Ω. For more details and definition for total α-order variation, we refer the reader to [20]. Suppose

$$u=v+n, \tag{2}$$

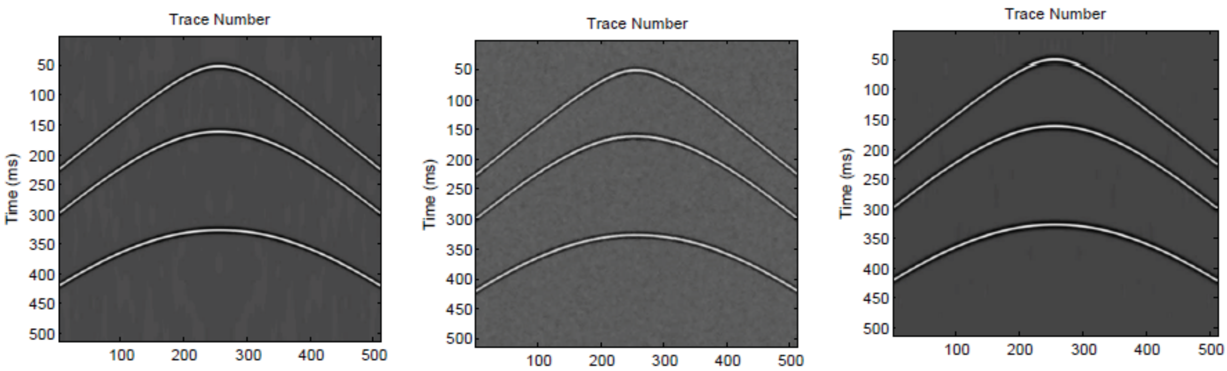
be the seismic denoising model where u is observed noisy seismic data, v is noise free seismic data, and n is band-limited Gaussian random noise with variance σ^2. Then, in the space of bounded variation (BV) we can restore the clean seismic data v by solving the following model.

$$\underset{v}{\operatorname{argmin}}\{\|v\|_{BV^{\alpha}}+\frac{\mu}{2}\|u-v\|_2^2\}, \tag{3}$$



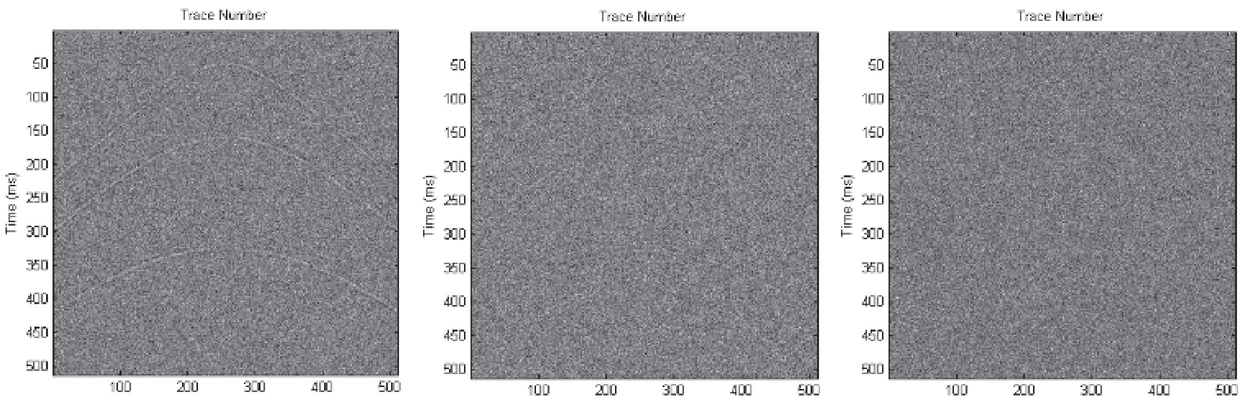
(a) (b)

Fig. 1 (a) Original data, (b) Noisy data (SNR=4.25 dB)



(a) (b) (c)

Fig. 2 (a) F-X deconvolution denoised result (SNR=13.73 dB), (b) NLM denoised result (SNR=16.18 dB), (c) Proposed method denoised result (SNR=19.58 dB)



(a) (b) (c)

Fig. 3 (a) F-X deconvolution residual, (b) NLM residual, and (c) proposed method residual

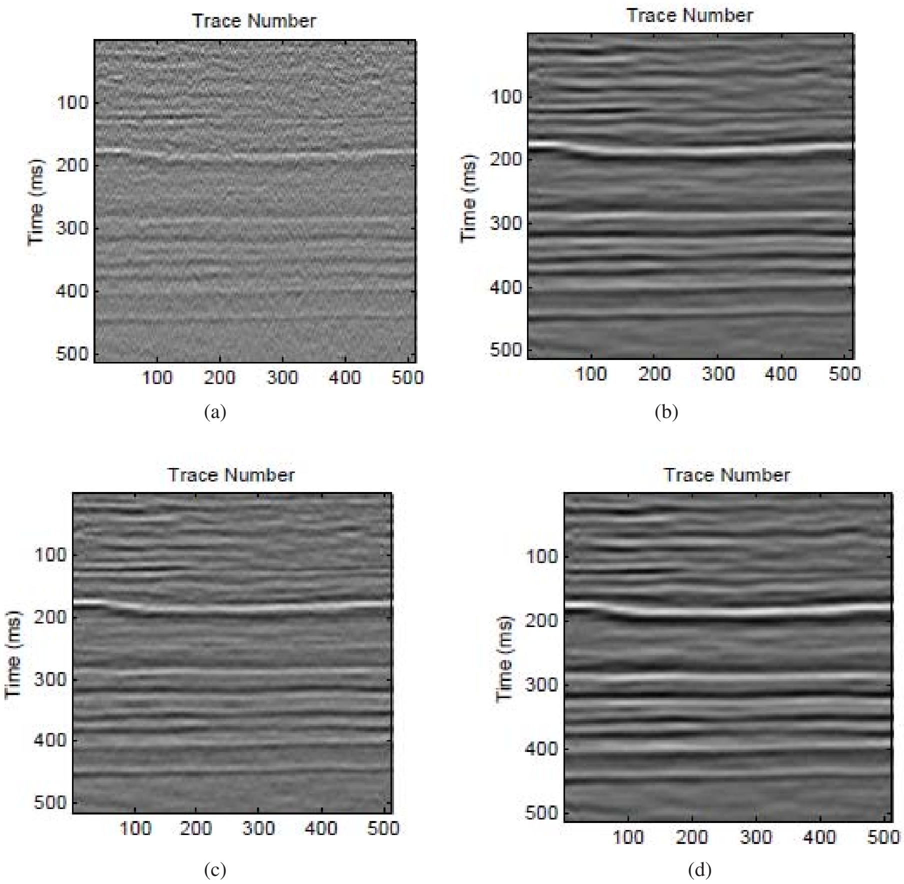


Fig. 4 (a) Original noisy real data, (b-d) The denoised result of real data by F-X deconvolution, NLM, and Proposed method respectively

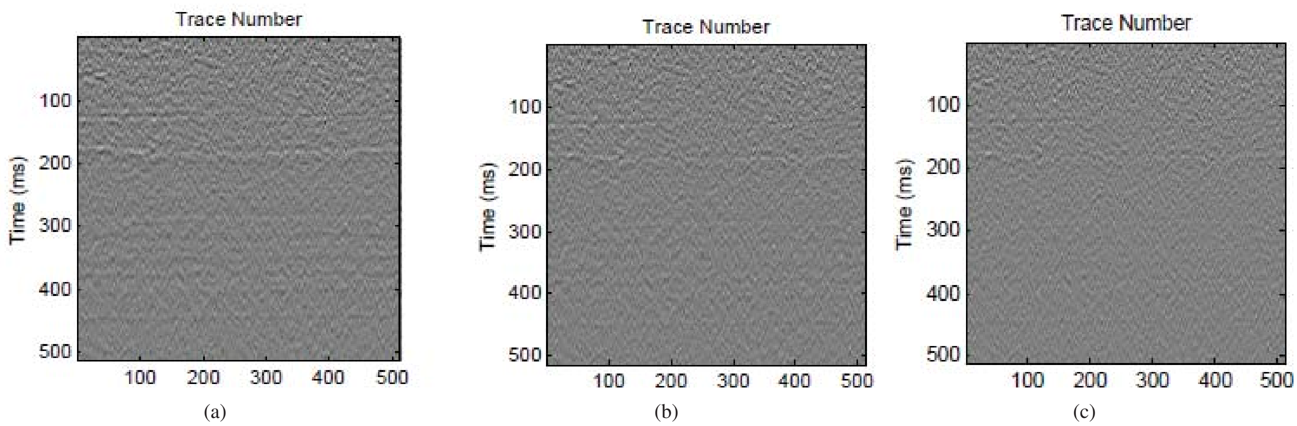


Fig. 5 (a) F-X deconvolution residual, (b) NLM residual, (c) Proposed method residual

Based on α-BV semi-norm, $\|v\|_{BV}$ defined as

$$\|v\|_{BV^\alpha} = \|v\|_{BV} + TV^\alpha(v)$$

Remark: Let $q \in N^+$ and

$$W_q^\alpha(\Omega) = \{v \in L^q(L), such that \|v\|_{W_q^\alpha} < \infty\}$$

be a function space which is embedding with

$$W_q^\alpha = (\int_\Omega |v|^q dx + \int_\Omega |\nabla^\alpha(v)|^q dx)^{\frac{1}{q}},$$

where

$$\nabla^\alpha(v) = (\frac{\partial^\alpha u}{\partial x_1}, \frac{\partial^\alpha u}{\partial x_2}, \frac{\partial^\alpha u}{\partial x_3}, ..., \frac{\partial^\alpha u}{\partial x_d})^T.$$

Since TV^α is a convex function and for $q = 1$, $v \in W_1^\alpha$, then

$$TV^\alpha = \int_\Omega |\nabla^\alpha(v)| dx.$$

Then (3) becomes,

$$\underset{v}{\operatorname{argmin}}\{\|v\|_{L^1} + \int_\Omega |\nabla^\alpha v| dx + \frac{\mu}{2}\|u - v\|_2^2\}. \tag{4}$$

For $\alpha_1 > 0$ and $\alpha_2 > 0$

$$\nabla^\alpha v = \nabla^{(\alpha_1,\alpha_2)} v = (D_x^{\alpha_1} v, D_y^{\alpha_2} v)$$

with

$$|\nabla^\alpha v| = \sqrt{(D_x^{\alpha_1} v)^2 + (D_y^{\alpha_2} v)^2}.$$

$D_x^{\alpha_1} v$ denotes the derivative of fractional order of α_1 of v with respect to x and $D_y^{\alpha_2} v$ denotes the derivative of fractional order of α_2 of v with respect to y. The model in (4) is anisotropic total fractional order variation model in $\alpha - BV$ which can be rewritten as

$$F(v) = \underset{v}{\operatorname{argmin}}\{\|v\|_{L^1} + \int_\Omega |\nabla^{(\alpha_1,\alpha_2)} v| dx + \frac{\mu}{2}\|u - v\|_2^2\}. \tag{5}$$

To solve the minimization problem in (5), we use the EulerLagrange equation. Let

$$G(v) = \underset{v}{\operatorname{argmin}}\{\|v\|_{L^1} + \int_\Omega |\nabla^{(\alpha_1,\alpha_2)} v| dx + \frac{\mu}{2}\|u - v\|_2^2\}.$$

Substitute the value of v by $v + a\gamma$ provided that $\gamma \in C^\infty(\Omega)$ and rewrite $G(v)$ as

$$\begin{aligned}\Psi(a) := G(v + a\gamma) = \underset{v}{\operatorname{argmin}}\{\|v + a\gamma\|_{L^1} +\\ \int_\Omega |\nabla^{(\alpha_1,\alpha_2)}(v + a\gamma)| dxdy + \frac{\mu}{2}\|u - (v + a\gamma)\|_2^2\}.\end{aligned} \tag{6}$$

Wee find the derivative of (6) and we get

$$\begin{aligned}\Psi^{'}(a) = \int_\Omega \frac{D_x^{\alpha_1}(v + a\gamma) D_x^{\alpha_1}\gamma + D_y^{\alpha_2}(v + a\gamma) D_y^{\alpha_2}\gamma}{\sqrt{(D_x^{\alpha_1}(v + a\gamma))^2 + (D_y^{\alpha_2}(v + a\gamma))^2}} dxdy +\\ (v + a\gamma)\gamma + \mu(v + a\gamma - u).\end{aligned} \tag{7}$$

We evaluated (7) at $a = 0$

$$Psi^{'}(0) = \int_\Omega \frac{D_x^{\alpha_1} v D_x^{\alpha_1}\gamma + D_y^{\alpha_2} v D_y^{\alpha_2}\gamma}{\sqrt{(D_x^{\alpha_1} v)^2 + (D_y^{\alpha_2} v)^2}} dxdy + v\gamma + \mu(v - u). \tag{8}$$

Equation (8) is the same with

$$\Psi^{'}(0) = \int_\Omega ((D_x^{\alpha_1})^* \frac{D_x^{\alpha_1} v}{|\nabla^{(\alpha_1,\alpha_2)} v|} + D_y^{\alpha_2} \frac{D_y^{\alpha_2} v}{|\nabla^{(\alpha_1,\alpha_2)} v|}) dxdy + v(1 + \mu) - \mu u))\gamma,$$

where $(D_x^{\alpha_1})^*$ and $(D_y^{\alpha_2})^*$ are the adjoint operators of $(D_x^{\alpha_1})$ and $(D_y^{\alpha_2})$ respectively.

To find the extreme value of the function $F(v)$ in (5), v must satisfy the EulerLagrange equation

$$\int_\Omega ((D_x^{\alpha_1})^* \frac{D_x^{\alpha_1} v}{|\nabla^{(\alpha_1,\alpha_2)} v|} + (D_y^{\alpha_2})^* \frac{D_y^{\alpha_2} v}{|\nabla^{(\alpha_1,\alpha_2)} v|}) dxdy + v(1+\mu) - \mu u) = 0$$

One method which can find the minimum of $F(v)$ is taking the smallest step size $\triangle t$ in opposite direction of Laplace of F (∇F), that means

$$v^{k+1} = v^k + \triangle t(-\nabla F).$$

We use the gradient descent method to solve the EulerLagrange equation.

$$\frac{\partial v}{\partial t} = -(D_x^{\alpha_1})^* \frac{D_x^{\alpha_1} v}{|\nabla^{(\alpha_1,\alpha_2)} v|} - (D_y^{\alpha_2})^* \frac{D_y^{\alpha_2} v}{|\nabla^{(\alpha_1,\alpha_2)} v|}) - v(1 + \mu) + \mu u$$

Let $\triangle_x^{\alpha_1}$ and $\triangle_y^{\alpha_2}$ be a fractional difference operators which are corresponding to the fractional order derivative operators $D_x^{\alpha_1}$ and $D_y^{\alpha_2}$ respectively, then we introduce the adjoint operator $(\triangle_x^{\alpha_1})^*$ and $(\triangle_y^{\alpha_2})^*$.

$$\begin{aligned}\triangle_x^{\alpha_1} v_{i,j} &= \sum_{k=0}^{K-1} W_k^{\alpha_1} v_{i-k,j}\\ \triangle_y^{\alpha_2} v_{i,j} &= \sum_{k=0}^{K-1} W_k^{\alpha_2} v_{i,j-k}\\ (\triangle_x^{\alpha_1})^* v_{i,j} &= \sum_{k=0}^{K-1-j} W_k^{\alpha_1} v_{i+k,j}\\ (\triangle_y^{\alpha_2})^* v_{i,j} &= \sum_{k=0}^{K-1-i} W_k^{\alpha_2} v_{i,j+k},\end{aligned} \tag{9}$$

with $W_k^{\alpha_1} = (-1)^k \binom{\alpha_1}{k}$ and $W_k^{\alpha_1} = (-1)^k \binom{\alpha_2}{k}$ is the coefficients of polynomial and $K \geq 3$ is the number of terms involved in α fractional order derivative. Let

$$\begin{aligned}\triangle_{xx}^{\alpha_1} v_{i,j} &= \sum_{a=0}^{K-1-j} W_k^{\alpha_1} \frac{\triangle_x^{\alpha_1} v_{i+a,j}}{\sqrt{(\nabla_x^{\alpha_1} v_{i+a,j})^2 + (\triangle_y^{\alpha_2} v_{i+a,j})^2 + \epsilon}}\\ \triangle_{yy}^{\alpha_2} v_{i,j} &= \sum_{a=0}^{K-1-j} W_k^{\alpha_2} \frac{\triangle_y^{\alpha_2} v_{i,j+a}}{\sqrt{(\nabla_x^{\alpha_1} v_{i,j+a})^2 + (\triangle_y^{\alpha_2} v_{i,j+a})^2 + \epsilon}}\end{aligned} \tag{10}$$

be the discritization of $(D_x^{\alpha_1})^* \frac{D_x^{\alpha_1} v}{|\nabla^{(\alpha_1,\alpha_2)} v|}$ and $(D_y^{\alpha_2})^* \frac{D_y^{\alpha_2} v}{|(\nabla^{\alpha_1,\alpha_2}) v|}$ respectively with a very small positive number ϵ such that $0 < \epsilon \leq 1$. Now we can solve for v^{k+1}

$$v_{i,j}^{k+1} = v_{i,j}^k - \triangle t(\triangle_{xx}^{\alpha_1} v_{i,j}^k + \triangle_{yy}^{\alpha_2} v_{i,j}^k + \mu u_{i,j} - \mu v_{i,j}^k - v_{i,j}^k)$$

III. Overview

Step 1: Input is noisy seismic data u, with $v^k = u$
Step 2: Solve (9) and (10) for $\triangle_{xx}^{\alpha_1} v_{i,j}$ and $\triangle_{yy}^{\alpha_2} v_{i,j}$
Step 3: Solve for $v_{i,j}^{k+1} = v_{i,j}^k - \triangle t(\triangle_{xx}^{\alpha_1} v_{i,j}^k + \triangle_{yy}^{\alpha_2} v_{i,j}^k + \mu u_{i,j} - \mu v_{i,j}^k - v_{i,j}^k)$ and iterate until the desired result is obtained otherwise go back to step two
Step 4: Output v^k which is denoised seismic data.

IV. Result and Discussion

In this work, we analyses and test anisotropic fractional-order derivative based total α-order variation model which is an improvement of the currently popular high order regularization models. For the choice f value of α, we employed the method which has been discussed in [19]. In signal processing, the frequency responses of fractional order differential operator are a kind of nonlinear filter and for $0 < \alpha < 1$, it can filter the high frequency component of the signal as well as it avoids its low frequency components [9] the large value of α, the proposed method removes the high frequency textures and details of the signal and for the small value of α it removes only a small amount of noises with high frequency. To effectively remove the noise with high frequency by preserving the valid seismic signal with high frequency, we chose $\alpha \in [1.2, 1.65]$. From this interval, the value of α is determined for which SNR reaches maximum value.

Finally, we test our proposed method in random noise attenuation of synthetic data with parabolic events and a real seismic data set with different events and features. The synthetic seismic data and it's noisy version is indicated in Fig. 1. Theoretically and experimentally the proposed method preferably attenuates noises, enhance the lateral continuity of seismic events, and preserve useful detail information in the horizontal as well as vertical direction while improving the signal-to-noise ratio. The performance of the proposed method compared with F-X deconvolution and non-local means (NLM) denoising algorithm in terms of SNR and resolution. As indicated in Fig. 2, our approach attains higher performance in terms of SNR and visual quality. From Fig. 3, the events and important features of the seismic signal also effectively preserved when the proposed method is applied. As we observed from Fig. 3, F-X deconvolution and NLM damages some important informations and some part of the seismic events are left in the noise section. The proposed method also applied to the real data with strong noise in Fig. 4. The proposed method shows it's performance in removing the strong and coherent noise as showen in Fig. 4 compared with F-X deconvolution and NLM denoising. In Fig. 4 F-X deconvolution over-smooths the denoised data and NLM denoisng leaves some noises in the denoised result. As we can observe from Fig. 5, F-X deconvolution and NLM denoising affact some use full features as well as informations of the data and some parts of the data left in the noise section as we can observe from Fig. 5.

V. Conclusion

In this work, we have presented an algorithm based on anisotropic total fractional order variation model in α bounded variation for random noise attenuation in seismic data. The proposed method is applied for both synthetic and real seismic data set. We compared the denoised result with F-X deconvolution and NLM and the numerical results have demonstrated that, the proposed method effectively removes random noise from seismic sections while preserving the hyperbolic events as well as weak features and improves signal-to-noise ratio.

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