# Nonstationarity Modeling of Economic and Financial Time Series

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**Abstract**—Traditional techniques for analyzing time series are based on the notion of stationarity of phenomena under study, but in reality most economic and financial series do not verify this hypothesis, which implies the implementation of specific tools for the detection of such behavior. In this paper, we study nonstationary nonseasonal time series tests in a non-exhaustive manner. We formalize the problem of nonstationary processes with numerical simulations and take stock of their statistical characteristics. The theoretical aspects of some of the most common unit root tests will be discussed. We detail the specification of the tests, showing the advantages and disadvantages of each. The empirical study focuses on the application of these tests to the exchange rate (USD/TND) and the Consumer Price Index (CPI) in Tunisia, in order to compare the Power of these tests with the characteristics of the series.

Keywords-Stationarity, unit root tests, economic time series.

#### I. INTRODUCTION

THERE are several reasons, both statistical and economic, for examining the presence of unit root in a time series. Traditional methods of processing time series have proved insufficient to predict economic phenomena. They have grown significantly with the work of Box and Jenkins [1] and the use of random processes of the ARIMA class.

Since the original work of Nelson and Plosser [2] on the non-stationary non-seasonal time series in economics, Nelson and Plosser distinguish two types of nonstationary processes: TS processes (Trend Stationary) are expressed as time function (with zero expectation and a constant variance), and DS processes (Difference Stationary) characterized by the presence of at least one unit root.

In terms of modeling, the two types behave in a radically opposite way in the long term. A TS series to reposition itself around its deterministic trend after a random shock, as in Barthelemy and Lubrano [3]; this is what we call the "mean reversion" property. A DS series does not return around its tendency to near a shock, since the shock also affects the stochastic trend of the series. The presence or absence of the "mean reversion" property has led researchers to take a close interest in the issue of unit roots.

Stock [4] proposes four motivations for testing univariate non-stationarity in economic time series:

- 1) The description of the data,
- 2) The medium and long term forecast,
- 3) A later guide to multivariate modeling, and

4) Information on the degree of persistence in a chronicle and its order of integration that can help guide the construction or test economic theories.

The objective of this paper is to present nonstationary tests of non-seasonal time series in a non-exhaustive manner. In the second section, after a definition of the notion of stationarity, we formalize the problem of nonstationary processes, namely TS and DS processes with numerical simulations, and to take stock of their statistical characteristics. The statistical and economic consequences of nonstationarity as well as the consequences of a bad stationization of a series will also be presented in this section. The theoretical aspects of some of the most common unit root tests will be discussed in the third section. The empirical study will be devoted to the fourth section, the application of these tests to the monthly exchange rate series (USD/TND) in Tunisia, in order to compare the Power of these tests with the characteristics of the series. In the last section, we present our remarks and experiences from this study.

#### II. NONSTATIONARITY APPROACHES

#### A. Nonstationary Processes

A process is stationary at the second order if the set of its moments of order one and of order two are independent of time. In contrast, a nonstationary process is a process that does not meet either of these two conditions. Thus, the origin of the nonstationarity can come from a dependence of the expected value of the time series zt with respect to time and/or a dependence of the variance or the auto-covariance with respect to the time. The fact that a process is stationary or not depends on the choice of the modeling to be adopted. As a general rule, if we consider the methodology of Box and Jenkins, if the series studied is the result of a stationary process, we then look for the best model among the stationary process class to represent it - then this model is estimated. On the other hand, if the series is the result of a nonstationary process, one must first of all seek to "stationarize" it, i.e. to find a stationary transformation of this process. Then, the parameters associated with the stationary component are modeled and estimated. The difficulty lies in the fact that there are different sources of non-stationarity and that each stationary or non-stationarity is associated with an own stationary method. We will therefore begin in this section by presenting two classes of non-stationary processes, according to the terminology of Nelson and Plosser [2]: TS (Trend Stationary) processes and DS (Difference Stationary) processes. In the next section, we present the methods of stationization for each of these process classes. But beyond the

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stakes of econometric modeling, we will see in this section that the origin of nonstationarity has very strong implications on the economic analysis of the series studied. We will see in particular that for DS processes there exists a property of shock persistence that does not exist in TS processes. Such an assumption implies, for example, that if the macroeconomic series satisfy a representation of type DS; the impact of cyclical shock may have a permanent effect on the level of the study series.

## B. TS Process

According to Nelson and Plosser [2],  $(z_t, \forall t \in Z)$  is a TS process if it can be written in the form (1):

$$z_t = f(t) + x_t \tag{1}$$

where f(t) is a function of time and  $x_t$  is a stationary stochastic process, can be an ARMA (p, q), we show that:  $E(z_t) = f(t) + x$ ; where  $E(x_t) = x$ 

The expected value  $E(z_t)$  is time dependent, which violates the first condition of the definition of a stationary process. **Example:** We consider the process  $z_t = 1, 5 + 0, 04t + x_t$ with  $x_t = 0, 3x_{t-1} + \varepsilon_t$ ;  $(t \in Z)$ ;  $\varepsilon_t$  iidn (0,1), the general evolution of this process is shown in of Fig. 1. We will cancel all the realizations of the shock at the date T = 50; then we compared the evolutions of the process and the deterministic tendency. It is verified in Fig. 1 that, from the date of stopping the shocks (T = 50), the process  $z_t$  converges towards the deterministic trend. Shocks prior to date T see their influence diminish as time passes. We verify here the non-persistence property of the shocks specific to the TS process.



#### C.DS Process

These are stochastic processes which do not satisfy the conditions of stationarity but whose difference to the order d satisfies the properties of stationarity, applying a difference filter to them  $(Bz_t = z_{t-1})$  of order d:

$$(1 - B^d) z_t = \gamma_0 + \varepsilon_t \text{ avec } \varepsilon_t \text{ is iid } (0, \sigma_{\varepsilon}^2)$$
 (2)

**Example:** If a process  $z_t$  (random walk without drift) is not stationary, we say that this process is DS, integrated of order one, noted I(1). If the process defined by the first difference:

$$z_{t} = z_{t-1} + \varepsilon_{t}$$
$$\Delta z_{t} = \varepsilon_{t}$$
$$E(\Delta z_{t}) = 0;$$
$$V(\Delta z_{t}) = E(\Delta z_{t})^{2} = E(\varepsilon_{t})^{2} = \sigma_{\varepsilon}^{2};$$
$$Pov(\Delta z_{t}, \Delta z_{t-s}) = E(\Delta z_{t} \Delta z_{t-s}) = E(\varepsilon_{t} \varepsilon_{t-s}) = 0$$

Then,  $\Delta z_t = \mathcal{E}_t$  is stationary.

С

**Definition.** According to Nelson and Plosser [2], a nonstationary process  $(z_t, \forall t \in \mathbb{Z})$  is a DS process of order d, where d denotes the integration order, if the filtered process defined by  $(1 - B)^d z_t$  is stationary. We also say that  $(z_t, \forall t \in \mathbb{Z})$  is an integrated process of order d; Noted I(d).

The DS processes relies on the presence of unit roots in the polynomial  $\Phi(B)$  associated with the autoregressive dynamics of the process.

**Remark.** One of the main properties of DS processes is hysteresis or persistence of shocks. This means that, contrary to TS processes, the random shocks  $\mathcal{E}_t$  influence the level of the I(d) variable infinity of the times.

#### III. UNIT ROOT TESTS

## A. Augmented Dikey-Fuller Test

A widely used and widespread nonstationarity test is the unit root test proposed by Dickey and Fuller in [5]. To perform this test, Dickey and Fuller in [5] propose a new statistic. This statistic no longer follows, under the null hypothesis, a classical student law, since under the null hypothesis, the process is non-stationary of the type DS.

Dikey-Fuller conducted the unit root tests taking into account the deterministic trend hypothesis. This simple Dickey-Fuller test will mostly rejects the white noise hypothesis, but most economic series are characterized by autocorrelation. To take into account the presence of autocorrelation in economic series, Dickey and Fuller [5] propose to conduct unit root tests by including one or more differentiated autoregressive terms that directly control autocorrelation in the model (and not at the estimator level). Such an approach makes it possible to whitewash the residuals and to reduce to a representation similar to that of the Dickey-Fuller Simple test. Consequently, the application of this new strategy is identical to that previously presented and we find the same asymptotic distributions, as in Hamilton [6].

The three models used to develop the Augmented Dikey-Fuller test (ADF) test are:

Model 1: 
$$\Delta z_t = \rho z_{t-1} + \sum_{j=1}^p \psi_j \Delta z_{t-j} + v_t$$
  
Model 2:  $\Delta z_t = \alpha + \rho z_{t-1} + \sum_{j=1}^p \psi_j \Delta z_{t-j} + v_t$ 
  
Model 3:  $\Delta z_t = \alpha + \beta t + \rho z_{t-1} + \sum_{j=1}^p \psi_j \Delta z_{t-j} + v_t$ 
  
(3)

The ADF test strategy consists of a first step in determining the number of delay p necessary to whitewash the residuals. The choice of the number of delays p consists in comparing different ADF models including different choices of delays, based on information criteria of AIC and SC:

$$AIC(k) = n \log(\sigma_{e_i}^2) + 2k$$

$$SC(k) = n \log(\sigma_{e_i}^2) + \log(n)$$
(4)

with *k*: number of parameters in the model and:  $(\sigma_{e_r}^2)$  is the variance of the estimated residuals. We look for the number of delay p which minimizes these two criteria.

In the second step, it is enough to apply the sequential strategy of the Dickey Fuller Simple test to the models:

We start by testing the unit root of model 3: if: H<sub>0</sub>:  $\rho=0$  is accepted, the computed statistic  $t_{\hat{\rho}=0}$  is greater than the critical threshold tabulated by Dikey-Fuller or McKinnon, we try to check if the specification of model 3, including a constant and a trend. One must construct a Fisher test of the attached hypothesis:

$$H_{0}^{3}: (\alpha, \beta, \rho) = (\alpha, 0, 0)$$
$$H_{1}^{3}: \exists \text{ at least a non zero coefficien t}$$

We test thus the nullity of the tendency, conditionally to the presence of a unitary root. The statistics for this test:

$$F_{3} = \frac{(SCR_{3}^{c} - SCR_{3})/3}{SCR_{3}/(n - (p - 1) - 3)}$$
(5)

The unit root test must be repeated from model 2 if the specification of model 3 is not adapted. We seek to verify if the specification of model 2, if: is accepted. We must construct a Fisher test of the attached hypothesis:

$$H_0^2: (\alpha, \rho) = (0, 0)$$
  
 $H_1^3: \alpha \neq 0$ 

The statistics for this test:

$$F_{2} = \frac{(SCR_{2}^{c} - SCR_{2})/2}{SCR_{2}/(n - (p - 1) - 3)}$$
(6)

If the specification of model 2 is not adapted. We seek to verify if the specification of model 1 is accepted. If the calculated statistic is greater than the critical threshold tabulated by Dikey-Fuller or McKinnon, the null hypothesis of nonstationarity is accepted. In this case, the series is a pure random walk. Otherwise, the series is stationary.

#### B. Test of Phillips and Perron

Phillips [7], and Phillips, and Perron [8] suggested using a nonparametric correction of residue autocorrelations rather than resorting to implicit parametric correction in the ADF regression.

The Phillips-Perron test statistic is a student statistic corrected for the presence of autocorrelation by taking into account an estimate of the short- and long-term variances to the presence of autocorrelation and heteroscedasticity.

The test consists of testing the unit root hypothesis in Dikey-Fuller basic models. The short-run variance estimator is given by:

$$s_{\varepsilon}^{2} = \frac{1}{n} \sum_{t=1}^{n} e_{t}^{2}$$
(7)

The long-term  $s_{\infty}^2$  is:

$$s_{\infty}^{2} = \frac{1}{n} \sum_{t=1}^{n} e_{t}^{2} + 2 \sum_{t=1}^{l} (1 - \frac{i}{l+1}) \frac{1}{n} \sum_{t=i+1}^{n} e_{t} e_{t-1} \quad (8)$$

The number 1 of delays is estimated by:

$$l \approx 4(n/100)^{2/9}$$

The corrected test statistic is:

$$t_{\rho}^{*} = \frac{s_{\varepsilon}^{2}}{s_{\infty}^{2}} n(\hat{\rho} - 1) + \frac{(s_{\infty}^{2} - s_{\varepsilon}^{2})n^{2}}{2s_{\infty}^{2}\sum_{t=1}^{n} z_{t-1}^{2}}$$
(9)

A particularly interesting feature of PP-corrected statistics is that their asymptotic distribution is identical to those derived by ADF. This implies that the test procedure can be used by referring to the tabulated asymptotic critical values of ADF even if it allows specifying much more generally the series.

The main advantage of the PP approach is that the calculation of the corrected statistics requires only: first, the OLS estimate and the computation of the associated statistics, and secondly, the estimation of correction factor based on the residual structure of this regression, using their long-run variance. It can be noted that the long-run variance takes into account all the autocorrelations of the residuals. These tests are more powerful than the ADF test because they do not require the addition of additional regressors in large numbers when the residuals have an MA (Moving Average) component. However, they suffer size distortion if the residue has a negative MA component, as in Leybourne and Newbold [9].

#### C. The Test of Kwiatkowski, Phillips, Schmidt and Shin

Kwiatkowski et al. [10] (KPSS) proposed a stationarity test that takes into consideration the possible existence of autocorrelations of the residuals of a time series. This method tests the null hypothesis of stationarity, against the alternative hypothesis of integration of order 1 of the series; it is first to consider the model:

$$z_t = \alpha + \beta t + \varepsilon_t$$

The null hypothesis will be to test the nullity of the variance of  $\varepsilon_t$ :  $H_0$ :  $\sigma_{\varepsilon}^2 = 0$ ;  $H_1$ :  $\sigma_{\varepsilon}^2 > 0$ . If  $z_t$  is with no stochastic trend, the estimated residuals will be stationary and the partial sum of these residuals will be able to be:

$$S_t = \sum_{i=1}^t e_i$$

and the long-run variance is estimated, as in the case of Phillips-Perron, test, for example, the autocorrelation phenomenon to be taken into account.

The test statistic is:

$$KPSS = \frac{\sum_{t=1}^{n} S_{t}^{2}}{n^{2} s_{\infty}^{2}}$$
(10)

The distribution of the statistics *KPSS* under the null hypothesis depends on the presence of deterministic terms in the initial regression. The decision rule is therefore: If *KPSS* < *KPSS*<sub>tab</sub>, then  $H_0$  is accepted and the series is stationary.

## IV. EMPIRICAL STUDY

We will now propose an application of the previous test strategy on the USD/TND exchange rate series (TDRATE). And the producer price index (PPI). The data base is derived from the world monetary fund, covering the period from the first month of 2008 to the sixth month of 2016 (100 observations).

A simple graphic examination, Fig. 2, clearly shows that the studied series are a priori nonstationary. The corresponding generating processes do not seem to satisfy the condition of invariance of expectation, and so does the variance. It remains to be seen whether these processes are DS or TS according to the terminology of Nelson and Plosser [2].

#### A. Exchange Rate Series USD / TND (TDRATE)

#### 1. Augmented Dikey-Fuller Test

The results of the ADF test, Table I, show that the calculated statistic is higher than the critical values for the three thresholds (1%, 5% and 10%) given by the McKinnon table. We accept the null hypothesis of unit root.

We must therefore test the nullity of the coefficient of the tendency conditionally to the presence of a unit root. For this purpose, the test is performed and the statistic is calculated,  $F_{3}$ = 3.22. The critical value at the threshold of 5% is 6.89 >

3.22, the null hypothesis of the nullity of the trend coefficient is accepted, conditional on the presence of a unit root. This means that the nonstationarity test performed with the asymptotic thresholds including a trend (model 3) must be called into question. It is therefore necessary to repeat this test from the model including only one constant (model 2).



Fig. 2 Evolution of TDRATE and IPP

TABLE I ADF Test for TDRATE Model 3						
t-Statistics Prob						
Augmented Dickey-Fuller test		-2.605443	0.2790			
Critical values	1%	-4.053392				
	-3.455842					
	10% -3.153710					

The results of the test for, Table II, ADF on the series for model 2, make us accept the unit root hypothesis. The adjusted statistic is -2.49 > at the critical values for the three thresholds (1%, 5% and 10%).

TABLE II					
ADF TE	EST FOR TDF	RATE MODEL 2			
t-Statistics Prob					
Augmented Dikey-Fuller test		-2.492317	0.1204		
Critical values	1%	-3.497727			
5%		-2890926			
	10%	-2.582514			

Now the joint test is carried out the statistic  $F_2 = 3.21$ . The critical value is equal to 4.71 at the threshold of 5%, so  $F_2 < 4.71$ ; we accept the null hypothesis of the nullity of the coefficient of the constant conditional on the presence of a unit root. The test should be repeated with model 1. The results are given in Table III.

TABLE III ADF TEST FOR TDRATE MODEL 1

		t-Statistics	Prob
Augmented Dikey-	Fuller test	0.519081	0.8262
Critical values	1%	-2.588530	
	5%	-1.944105	
	10%	-1.614596	

The adjusted statistic is 0.519 > at the critical values for the three thresholds (1%, 5% and 10%).

The residuals of the differentiated series are white noise, Fig. 3. The TDRATE series is a pure random walk without drift. Just apply the first difference filter to make it stationary.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1 2 3 4 5 6 7 8 9 10	0.113 -0.005 -0.071 -0.055 -0.068 0.112 0.098 0.153 0.136 0.056 0.056	0.113 -0.018 -0.069 -0.040 -0.059 0.124 0.067 0.132 0.124 0.124 0.053 0.123	1.2968 1.2991 1.8218 2.1432 2.6334 3.9942 5.0413 7.6282 9.6887 10.037	0.255 0.522 0.610 0.709 0.756 0.677 0.655 0.471 0.376 0.437 0.437

Fig. 3 Residuals correlogram of differentiated TDRATE

2. Phillips and Perron Tests

We carry out the PP tests, starting with the choice of truncation number from the formula [10]:

$$l \approx 4(n/100)^{2/9}$$
 (11)

Other approaches use the Akaike (AIC) and Shwarz (SC) information criteria to minimize the sum of the squares of the residuals. In this study, we chose the previous formula which gives us a truncation of l=2. The results of this test are given in Table IV for the three models.

The three models provide adjusted statistical values above the critical values for the three thresholds (1%, 5% and 10%). We accept the unit root hypothesis. Therefore, the DTRATE series is nonstationary of integration order equal to 1.

TABLE IV

PP	TEST FOR	TDRATE	
		t-Statistics	Prob
PP test Mode	13	-2.624902	0.2705
Critical values	1%	-2.588530	
	5%	-1.944105	
	10%	-1.614596	
PP test Model 2		-2.533206	0.1108
Critical values	1%	-3.497727	
	5%	-2.890926	
	10%	-2.582514	
PP test Model 1		-2.533206	0.1108
Critical values	1%	-3.497727	
	5%	-2.890926	
	10%	-2.582514	

3. Kwiatkowski, Phillips, Schmidt and Shin Tests (KPSS) The results of the KPSS test on model 2 and model 3, Table V, show that the calculated statistics are higher than the critical values for the three thresholds (1%, 5% and 10%). We therefore reject the hypothesis of stationarity. Therefore, the series DTRATE is nonstationary of order of integration 1.

TABLE V KPSS TEST FOR TDRATE					
		LM-Stat			
KPSS test Mo	1.114725				
Critical values	1%	0.216000			
	5%	0.146000			
	10%	0.119000			
KPSS Model 2		1.223452			
Critical values	1%	0.739000			
	5%	0.463000			
	10%	0.347000			

## B. Consumer Price Index (CPI)

## 1. Dikey-Fuller Test

First, the optimal delay must be sought. It should be noted that for a value of p=0, the application of the DF test causes the unit root hypothesis to be rejected. This result does not imply that the CPI series is stationary but the presence of a possible autocorrelation of the errors has distorted the statistical distributions used. That is why we applied the ADF test for different delay. From Table VI, it can be seen that the Akaike criterion leads to an optimal delay choice  $p^*=1$  for a model of type 3.

TABLE VI

	AIC AND SC FOR OPTIMAL DELAY OF CPI SERIES							
	Mo	del 3	Moo	lel 2	Model 1			
Р	AIC	SC	AIC	SC	AIC	SC		
0			4.0426	4.0950	4.0258	4.0521		
1	3.7644	3.8699	3.8398	3.9189	3.8195	3.8722		
2	3.7664	3.8991	3.8223	3.9285	3.8028	3.8824		
3	3.7883	3.9485	3.8368	3.6994	3.8174	3.9243		

The results of the AFD test, Table VII, show that the calculated statistic is higher than the critical values for the three thresholds (1%, 5% and 10%) given by the McKinnon table. We accept the null hypothesis of unit root.

TABLE VII ADF TEST FOR CPI MODEL 3				
t-Statistics Prol				
Augmented Dikey-	Augmented Dikey-Fuller test		0.1595	
Critical values	Critical values 1%			
5%		-3.456519		
	10%	-3.153989		

We must therefore test the nullity of the coefficient of the tendency conditionally to the presence of a unit root. For this purpose, the joint test is carried. The statistic F3=3.16 < 6.49, the critical value at the threshold of 5%. The null hypothesis of the nullity of the trend coefficient is assumed to be conditional on the presence of a unit root. This means that the nonstationarity test performed with the asymptotic thresholds including a trend (model 3) must be called into question. It is

therefore necessary to repeat this test from the model including only one constant.

The results of the ADF test on the series for model 2 allow us to accept the unit root hypothesis. The adjusted statistic is higher than the critical values for the three thresholds (1%, 5% and 10%), as shown in Table VIII.

TABLE VIII
ADF TEST FOR CPI MODEL 2

		t-Statistics	Prob
Augmented Dikey	-Fuller test	0.259548	0.9750
Critical values	1%	-3.498439	
	5%	-2.891234	
	10%	-2.582678	

We now carry out the joint test. The F2 statistic is equal to 3.17. The critical value is equal to 4.71 at the threshold of 5%, so is at this critical threshold, we accept the null hypothesis of the nullity of the coefficient of the constant conditionally to the presence of a unit root. The test should be repeated with model 1. The results are given in Table IX.

We accept the unit root hypothesis (critical values for the three thresholds). The CPI series is integrated of order 1 of type AR (2).

$$IPP_{t} - IPP_{t-1} = -0.421945 \Delta IPP_{t-1} + v_{t}$$

TABLE IX ADF TEST FOR CPI MODEL 1

		t-Statistics	Prob
Augmented Dikey-	Fuller test	2.533511	0.9972
Critical values	1%	-2.588772	
	5%	-1.944140	
	10%	-1.614575	

Fig. 4 shows that the residuals of the selected model are white noise.

Sample: 1 102 Included observation	is: 99					
Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1 2 3 4 5 6 7 8 9 10 11 12	0.104 -0.186 -0.082 0.046 -0.078 -0.112 0.155 -0.034 -0.141 -0.147 0.012 0.179	0.104 -0.199 -0.040 0.025 -0.116 -0.085 0.158 -0.134 -0.083 -0.133 -0.044 0.153	1.1068 4.6832 5.3792 5.6019 6.2563 7.5979 10.207 10.333 12.555 14.981 14.997 18.670	0.293 0.096 0.146 0.231 0.282 0.269 0.177 0.242 0.184 0.133 0.183 0.097

Fig. 4 Residuals correlogram of PCI

## 2. Phillips and Perron Tests

We proceed with the PP tests, starting with the choice of the number l of truncation. For l = 2, the results of this test are given in Table X for the three models.

Only for model 3, the value of the empirical statistic is lower than the critical values for the three thresholds (1%, 5% and 10%). From the results provided on model 1 and model 2, we accept the unit root hypothesis. Therefore, the CPI series is nonstationary of integration order 1.

TABLE X PP Test for CPI				
		t-Statistics	Prob	
PP test Model 3		-4.054791	0.0100	
Critical values	1%	-4.497727		
	5%	-3.455842		
	10%	-3.153710		
PP test Model 2		0.174227	0.9697	
Critical values	1%	-3.497727		
	5%	-2.890926		
	10%	-2.582514		
PP test Model 1		2.448440	0.9964	
Critical values	1%	-2.588530		
	5%	-1.944105		
	10%	-1.614596		

3. Kwiatkowski, Phillips, Schmidt and Shin Test (KPSS)

The results of this test on model 2 and model 3 for truncation l = 2, show that the calculated statistics *LM* are higher than the critical values for the three thresholds (1%, 5% and 10%). We therefore reject the hypothesis of stationarity. Therefore the CPI series is integrated of order 1 (Table XI).

TABLE XI KPSS TEST FOR CPI				
		LM-Stat		
KPSS test Mod	0.588570			
Critical values	1%	0.216000		
	5%	0.146000		
	10%	0.119000		
KPSS Model 2		3.238286		
Critical values	1%	0.739000		
	5%	0.463000		
	10%	0.347000		

#### V.CONCLUSION AND REMARKS

The object of this work was an investigation of the developments relating to the notion of nonstationarity. Such an approach necessitated reformulating the usual tests of this non-stationarity of the time series. In this context, four types of tests were presented: Dikey-Fuller (DF), Dikey-Fuller Augmented (ADF), Phillips and Perron (PP) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS).

The literature on unitary roots is very voluminous, in terms of statistical theory, as well as of empirical applications and economic theory. An attempt has been made to give as clear a line as possible on the statistical part. A study of the economic part would also be interesting to do.

We have tried to show that the unit root hypothesis led to a very particular behavior of the series and a particular model of permanence. Indeed, the empirical applications on the Tunisian context (TUNINDEX, TDRATE, and CPI), borne in this paper, proves this hypothesis.

Approaches to stationarity based on unit roots have been widely criticized in the literature. In this context, the point stressed by Campbell and Perron [11] is the almost empirical equivalence between a stationary and a stationary model in difference; the almost empirical equivalence is defined in the sense that the correlogram of the two processes can be made arbitrarily close. It suffices to consider ARMA residuals and not AR. A unit root in the AR part can then be canceled by an almost unitary root in the MA part.

Some authors such as Christiano and Eichenbaum [12] have suggested that the idea of testing the presence of a unit root should be abandoned. One can remain pragmatic by saying that these tests must be taken with caution in interpreting the results. It is illusory to attempt to contrast two economic theories on the basis of a simple unit root test. On the other hand, such a test can be a useful modeling tool.

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