

# Modeling of a UAV Longitudinal Dynamics through System Identification Technique

Asadullah I. Qazi, Mansoor Ahsan, Zahir Ashraf, Uzair Ahmad

**Abstract**—System identification of an Unmanned Aerial Vehicle (UAV), to acquire its mathematical model, is a significant step in the process of aircraft flight automation. The need for reliable mathematical model is an established requirement for autopilot design, flight simulator development, aircraft performance appraisal, analysis of aircraft modifications, preflight testing of prototype aircraft and investigation of fatigue life and stress distribution etc. This research is aimed at system identification of a fixed wing UAV by means of specifically designed flight experiment. The purposely designed flight maneuvers were performed on the UAV and aircraft states were recorded during these flights. Acquired data were preprocessed for noise filtering and bias removal followed by parameter estimation of longitudinal dynamics transfer functions using MATLAB system identification toolbox. Black box identification based transfer function models, in response to elevator and throttle inputs, were estimated using least square error technique. The identification results show a high confidence level and goodness of fit between the estimated model and actual aircraft response.

**Keywords**—Black box modeling, fixed wing aircraft, least square error, longitudinal dynamics, system identification.

## I. INTRODUCTION

**D**URING past decades aircraft system identification had been a wide area of research. The need for aircraft modeling arises keeping in view the expensive and precarious nature of aviation industry. Utilization of a mathematical model can be appreciated in many areas like aircraft performance analysis, flight testing of modifications made in the aircraft, autopilot and simulator design. Stress and fatigue analysis can also be carried out using an aircraft model [1].

Aircraft modeling can be carried out using various techniques like analytical modeling, water toe tank testing, wind tunnel testing, and Computational Fluid Dynamics (CFD) and system identification. A large number of assumptions are made while working with these techniques however these have been improved and established over the years [2].

System identification technique is the most reliable and accurate modeling technique which involves the operation of real system under specific excitations thus eigen modes of motion of a system under test are stimulated. Statistical methods are usually applied by estimation algorithms on the recorded input/output data of the actual system for estimation of a model. The estimated model is then validated by comparing its output to the actual system output for identical

excitations. In system identification, very few assumptions are followed as compared to other modeling techniques as the actual system is operated during the process [3].

The field of aircraft system identification is satiate with various estimation algorithms in time and frequency domains, but relatively less amount of work has been done on small sized UAVs for modeling through system identification. Mathematical programming, statistics and structural mechanics were used to seek solution of system identification problem in [1]. An adaptive technique to identify a multivariable system is suggested in [2] that uses recently developed methods for optimization and feasibility of these techniques is practically analyzed by its application to the adaptive control of helicopter dynamics in [3]. Research has been carried out on the development of parameterized model of unmanned helicopter and its identification using a frequency domain technique. Model is validated by comparing results from model with the response collected during flight.

Linear time invariant model for a hover is sought by using time domain analysis techniques on the system response data for designed inputs. This model was used for controller designing [4]. System identification was done by analyzing data from steady measurements. Model was formulated using linear regression techniques which can be extended to stepwise regression for model structure determination and to data handling procedure. Second technique used was maximum likelihood estimation [5]. Optimal input design technique is presented for aircraft parameter estimation. Concept is the combination of dynamic programming method with a gradient algorithm for the optimal input synthesis [6]. Output error method was used to produce aerodynamic coefficients, stability and control derivatives. Flight training device was developed based on the parameters estimated [7]. Bayesian system identification of structural dynamic systems was performed using experimentally obtained training data [8].

Two recursive least square parameter estimation algorithms are proposed by using the data filtering technique and the auxiliary model identification idea. Input output data is filtered [9]. Two new algorithms were derived to identify mixed deterministic and stochastic systems. State sequence is determined using input output data [10]. The importance and relevance of direct continuous-time system identification and how this relates to the solution for model identification problems in practical applications has been discussed [11]. Identification and modeling of dynamics of highly maneuverable fighter aircraft is carried out through aircraft neural networks approach [12]. Online parameter

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identification algorithm is implemented to provide inflight estimate of the aircraft dynamic parameters [13].

## II. FIXED WING AIRCRAFT MODELING

A fixed wing aircraft, fundamentally being a nonlinear system, is difficult to model accurately, however the aircraft model based on nonlinear equations of motion can be linearized around a stable equilibrium point, using small perturbation theory [15]. The nonlinear aircraft model and its linearization is discussed in [14].

The linearized longitudinal model of the UAV, as deliberated in [16], can be represented in state space form as given in (1):

$$\dot{x} = Ax + Bu \quad (1)$$

where states and controls of longitudinal direction are  $x = [u \ q \ \theta \ h]^T$ ;  $u = [\delta e \ \delta t]^T$ ; and  $\theta$  = pitch angle in radians;  $q$  = pitch rate in rad/s;  $u$  = air speed in m/s;  $h$  = altitude in m;  $\delta e$  = elevator input in radians;  $\delta t$  = throttle input in percentage.

## III. PLATFORM UAV

The indigenously built aerial platform named Taurus is a conventional Remotely Controlled (RC) fixed wing UAV with four control surfaces; aileron, elevator, throttle and rudder. A snapshot of the platform is shown in Fig. 1. Significant geometrical parameters for Taurus are shown in Table I.



Fig. 1 Fixed wing UAV Taurus

## IV. SENSORS AND DATA LOGGING

The results of system identification directly depend on the quality of recorded input/output data [17]. The quality of flight data in turn depends on the number of sensors used and their degree of precision and sampling time etc. The data logger and required sensors were selected keeping in view their weight, volume, power requirement and low cost. For recording aircraft states, the employed sensor suite comprised of a digital compass (powered by Honeywell's HMC5883L-TR chip), gyroscope/accelerometers (Invensense's 6 DOF Accelerometer / Gyro MPU-6000) and barometric altitude sensor. Ublox 3DR GPS was used for sensing aircraft positions and velocities. A sampling frequency of 8 Hz was

used to log flight data by the sensor suite. Real-time wireless data telemetry kit was used for monitoring aircraft states and data logging at the ground monitoring station. Avionics was integrated on Taurus keeping in view the aircraft Centre of Gravity (CG) for weight balancing.

TABLE I  
 DIMENSIONS OF TAURUS

DIMENSIONS OF TAURUS	
<b>Fuselage</b>	
Length	36 in
Width	2.5 in
Height	8 in
<b>Wing</b>	
Area	435 in <sup>2</sup>
Span	50 in
Chord	11 in
Aspect ratio	6.7
Aileron length	7 in
Aileron chord	1 in
Dihedral	3 deg
Sweep angle	0 deg
<b>Horizontal tail</b>	
Span	7 in
Area	89 in <sup>2</sup>
Chord	2 in
Aspect ratio	1.23
Elevator chord	4 in
Elevator deflection	12.5 deg
<b>Vertical tail</b>	
Span	5 in
Area	0.67 ft <sup>2</sup>
Sweep angle	27 deg
Rudder chord	1.5 in
Rudder deflection	25 deg

## V. EXPERIMENT DESIGN AND CONDUCT

The identification experiment flight was designed to include conception of a detailed flight plan, arrangement of necessary equipment and documentation of step by step preflight and post flight checklists. In system identification, the accuracy of estimated model is directly dependent upon the design of experimental inputs [17].

These inputs are to excite aircraft modes of motion in steady trim flight condition. There are a variety of inputs used for identification but in-flight mechanics there are some most common inputs like the step, frequency sweep, doublet and 3211-multistep [18]. Doublet and 3211 multistep inputs, as shown in Fig. 2, are the most widely used inputs for aircraft system identification, and we have used the same in our experiment because of their large bandwidth.

The spectral characteristics of these inputs are shown in the Fig. 3. For designing an input signal, the step duration is calculated as a function of the natural frequency of aircraft mode of motion. In our experiment the flight inputs were designed keeping in view the historically established aircraft inputs for system identification [18].

First few missions were flown for pilot orientation about the aircraft behavior and to check if the aircraft CG is balanced. In

two days total of 63 runs were made and doublets were given at all the inputs separately keeping all other inputs constant; 37 runs were made for longitudinal direction and 26 runs were for lateral direction. Doublets of time period 2 sec were given.

Applying doublet for less than 2 sec was not possible due to pilot limitation and extending it more than 2 sec pulled aircraft into a loop. Multiple doublets were given to each control surface to get reliable data for analysis. Doublets given to each control surface were noted against time for extracting useful doublets data for further processing. Recorded flight data was extracted from data logger and preprocessed for noise filtering and bias removal in Matlab environment. Input output data related to longitudinal modes of aircraft was separated for estimation of longitudinal transfer functions.

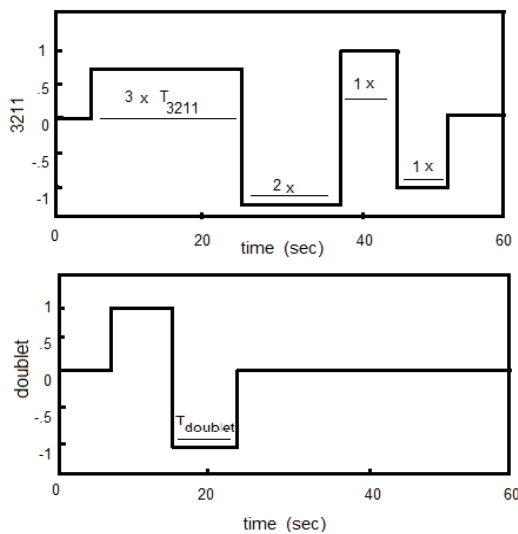


Fig. 2 Aircraft system identification inputs

Useful portion of doublet data was extracted from the complete log for estimating multiple mathematical models so that the best estimated model could be acquired.

## VI. ESTIMATION ALGORITHM

System Identification tool box of computational software MATLAB was used for estimating 3-DOF longitudinal model of the UAV, employing least square error algorithm. The working of least square error algorithm is as shown in Fig. 4; u-axis is the input axis while y-axis is the output axis. The black stars are actual data points, red line is the line that LSE algorithm is going to find in such a way that it maximally fits all the data points, by minimizing the square of distance between data points and the straight line. To find the best fit line,  $B_0$  and  $B_1$  are to be estimated as shown in Fig. 4. We define a cost function 'v' that minimizes modeling error 'e' as shown in (2):

$$v = \sum_{i=1}^r e_i^2 \quad (2)$$

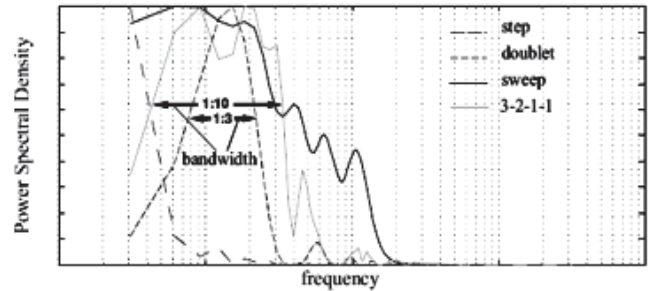


Fig. 3 Spectral characteristics of identification inputs

While modeling error is shown in (3):

$$e_i = y_i - y_i^m \quad (3)$$

where  $y_i$  is the actual output and  $y_i^m$  is the model output. This inverse parabola is partially differentiated with respect to  $B_0$  and  $B_1$  to get minimum values of each; it gives two equations which are solved simultaneously to get the values of both the parameters as shown in (4) and (5):

$$B_0 = \frac{\sum y_i \sum u_i^2 - \sum u_i \sum u_i y_i}{r \sum u_i^2 - (\sum u_i)^2} \quad (4)$$

$$B_1 = \frac{r \sum u_i y_i - \sum y_i \sum u_i}{r \sum u_i^2 - (\sum u_i)^2} \quad (5)$$

In the above equations, r is the number of data points and  $u_i$  is the actual input.

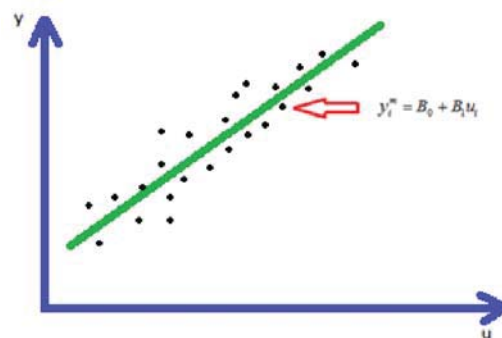


Fig. 4 Least square example

## VII. MODEL ESTIMATION AND VALIDATION

Multiple iterations were performed in System Identification toolbox to get the best estimated mathematical model by changing combination of zeros in transfer function estimation.

Doublet data was segregated into 70 to 30 ratios, 70 percent of the data points were used for estimation of the transfer function of the system that gives relation between input and each output separately. The remaining 30 percent of data was used for model validation. Estimated transfer function for elevator to pitch angle is given in (6):

$$\theta / \delta_e = \frac{1.091}{s^4 + 1.079s^3 + 3.192s^2 + 1.731s + 1.772} \quad (6)$$

Estimated transfer function for elevator to pitch rate is given in (7):

$$q / \delta_e = \frac{7.88s^3 - 11.34s^2 - 2.67s + 3.701}{s^4 + 1.031s^3 + 4.341s^2 + 3.027s + 1.624} \quad (7)$$

Estimated transfer function for air speed to elevator is given in (8):

$$u / \delta_e = \frac{-7.236s^3 + 13.9s^2 + 0.3086s - 9.785}{s^4 + 0.4466s^3 + 4.586s^2 + 0.03268s + 0.3302} \quad (8)$$

Estimated transfer function for elevator to altitude is given in (9):

$$h / \delta_e = \frac{61.68}{s^4 + 0.8699s^3 + 2.112s^2 + 1.442s + 0.2138} \quad (9)$$

Out of the total, 30% of flight data was saved for validation purpose and not utilized in the estimation process. This spare data set was used for validation of the estimated longitudinal model. Identical inputs were given to the estimated model and its outputs were recorded which were compared with the outputs in the actual flight data. The results were validated in terms of percentage of fitness of the two outputs. Percentage goodness of estimated model is shown in the validation results in Figs. 5-8.

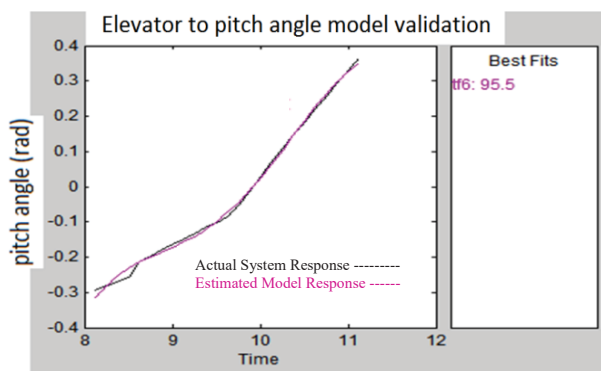


Fig. 5 Validation result for pitch angle

### VIII. CONCLUSION

In this paper, linearized longitudinal model of a conventional fixed wing UAV has been estimated and validated. Flight experiment was conducted to excite the aircraft dynamic modes of motion, in response to specifically designed input maneuvers. Appropriate type and number of sensors and data loggers were used to sense and record aircraft states and inputs. Data was preprocessed before submission to Matlab system identification toolbox. Least square error based estimation technique was used to model longitudinal transfer functions of Taurus, using the recorded flight data. Estimation

results showed high goodness of fit between the estimated model and actual system responses. The transfer function models can confidently be used for various applications discussed above, such as designing altitude and air speed controllers in an autopilot system.

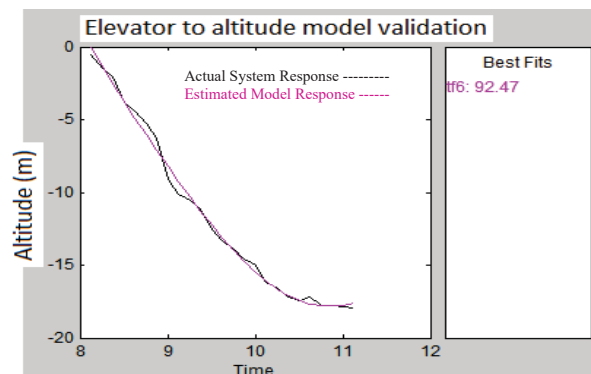


Fig. 6 Validation for altitude

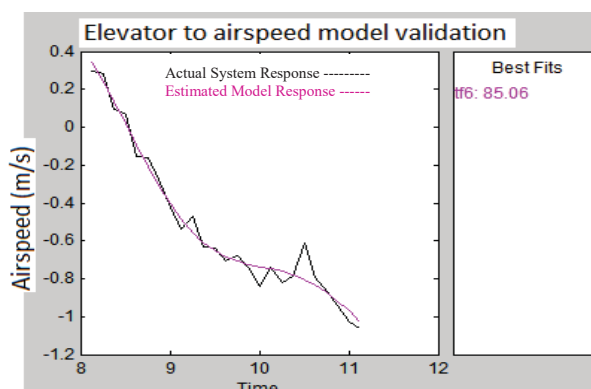


Fig. 7 Validation for air speed

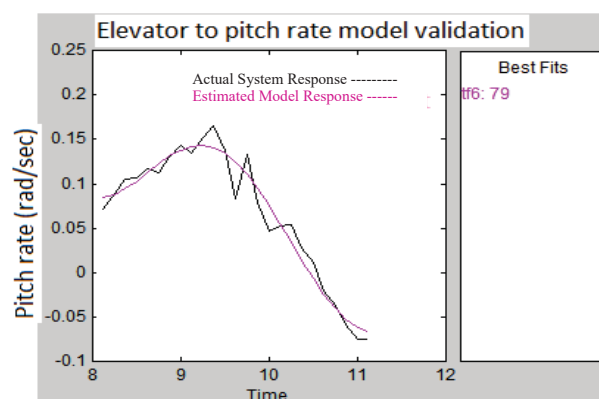


Fig. 8 Validation for pitch rate

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