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Unsteady Temperature Distribution in a Finite Functionally Graded Cylinder

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***Abstract*—**In the current study, two-dimensional unsteady heat conduction in a functionally graded cylinder is studied analytically. The temperature distribution is in radial and longitudinal directions. Heat conduction coefficients are considered a power function of radius both in radial and longitudinal directions. The proposed solution can exactly satisfy the boundary conditions. Analytical unsteady temperature distribution for different parameters of functionally graded cylinder is investigated. The achieved exact solution is useful for thermal stress analysis of functionally graded cylinders. Regarding the analytical approach, this solution can be used to understand the concepts of heat conduction in functionally graded materials.

***Keywords*—**Functionally graded materials, unsteady heat conduction, cylinder, Temperature distribution.

I. Introduction

DEVELOPMENTS in designing and manufacturing of composite structures introduce them as one of the most effective materials in different areas of industry. These developments are mainly due to the unique properties of these materials such as high strength to weight ratio, corrosion resistance, formability capability, and low production costs. Composite structures are made of different layers with various properties. Regarding to the layered structure of composites, they have common defects such as delamination, especially at high temperatures. To address this problem, the idea of functionally graded material (FGM) is presented. In fact, FGMs are the new version of composite structure with continuous changes in radial direction [1].

The idea of combining two different phases to improve the properties of composite materials is presented by Bever and Duvez [2] in 1972, firstly. The FGM term was first introduced in 1980 by Japanese researchers. Since then, much researches have been done in the field of FGM thermal properties. Regarding to the complexity of heat transfer equations with variable coefficients, most studies are accomplished by numerical methods, and analytical solutions are less available. Tarn and Wang [3] investigated the one-dimensional heat conduction in a FG cylinder with varied properties in radial direction by eigen-function expansion and matrix algebra. The general boundary conditions are considered by Tarn and Wang [3]. Hosseini et al. [4] solved the one-dimensional heat transfer problem in FG cylinder at specific boundary conditions. The thermo-physical properties change as a power function of radius. Rahimi and Zamani Nejad [5] studied the thermal stress in a cylindrical rod. A cylindrical rod with functionally graded properties is considered. Jabbari et al. [6] utilized Bessel functions to solve one-dimensional heat transfer in FG cylinder with varied properties in r direction. Hosseini and Abolbashari [7] presented a new formulation for heat conduction and temperature distribution in thick hollow cylinder made of FGMs. A finite cylinder with heat conduction in radial direction is investigated. Ostrowski [8] proposed a semi-analytical method for heat conduction in FG cylinder. The material properties varied both in radial and circumferential directions. Wang [9] presented a semi-analytical method for unsteady heat transfer in FG cylinder by using the laminate theory. Heat conduction coefficient, density, and specific thermal coefficient are arbitrary functions in radial direction. An analytical solution for heat transfer in FG circular hollow cylinder with time-dependent boundary conditions was presented by Lee and Huang [10]. The properties changes as a function of the radius. Daneshjou et al. [11] presented a method for heat transfer in hollow cylinders made of FGMs. A heat source varied with time is considered in energy equation. The FG cylinder is assumed to compose of several layers in radial direction.

In this paper, a two-dimensional analytical solution for unsteady heat transfer in FG hollow cylinder is presented. Thermo-physical properties are changed in radial direction as a power function. This paper is a development of our previous paper [1] presented for steady heat conduction in FG cylinders. The results show that the proposed solution can exactly predict the temperature distribution in cylinder. It is found that the average temperature of cylinder decreases with growth of FG power index in the current specific boundary conditions.

II. Governing Equations

The energy equation for heat conduction in FGM materials could be written as:

$$\frac{1}{r}\frac{\partial}{r\partial r}\left(rk_r(r)\frac{\partial T(r,z,t)}{\partial r}\right)+\frac{\partial}{\partial z}\left(k_z(r)\frac{\partial T(r,z,t)}{\partial z}\right)=\rho c(r)\frac{\partial T(r,z,t)}{\partial t}, \quad (1)$$

where the heat conduction coefficients in r and z directions and also the density and specific heat capacity coefficient are power functions of radius:

$$k_r(r)=k_{r_0}r^p \quad (2)$$

$$k_z(r)=k_{z_0}r^p, \quad (3)$$

$$\rho c(r)=\rho c_0 r^p, \quad (4)$$

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All parameters of p, k_{r0}, k_{z0}, and ρc_0 are constants. Combining (1)-(4), the following differential equation is obtained:

$$k_{r_0}\frac{\partial^2 T(r,z,t)}{\partial r^2}+k_{r_0}(p+1)\frac{1}{r}\frac{\partial T(r,z,t)}{\partial r}+k_{z_0}\frac{\partial^2 T(r,z,t)}{\partial z^2}=\rho c_0\frac{\partial T(r,z,t)}{\partial t}, \quad (5)$$

III. Analytical Solution

In this section, an analytical solution for (5) is presented. Firstly, the modified temperature is defining as:

$$\theta(r,z,t)=T(r,z,t)-T_\infty, \quad (6)$$

where T_∞ is the environment temperature. Defining $\theta(r,z,t)=\mathcal{R}(r)\,\mathcal{Z}(z)\,\mathcal{T}(t)$, and using the separation of variables method, (3) will be separated to three independent equations:

$$\frac{\partial \mathcal{T}(t)}{\partial t}+\lambda_{nm}^2\mathcal{T}(t)=0, \quad (7)$$

$$\frac{\partial^2 \mathcal{Z}(z)}{\partial z^2}+\mu_n^2\frac{\partial \mathcal{Z}(z)}{\partial z}=0, \quad (8)$$

$$r^2\frac{\partial^2 \mathcal{R}(r)}{\partial r^2}+(p+1)r\frac{\partial \mathcal{R}(r)}{\partial r}+\frac{\eta_{nm}^2}{\alpha_0}r^2\mathcal{R}(r)=0, \quad (9)$$

where,

$$\lambda_{nm}^2=\alpha_{z0}(\eta_{nm}^2-\mu_n^2), \quad (10)$$

$$\alpha_{z0}=\frac{k_{z_0}}{\rho c_0}, \quad (11)$$

$$\alpha_0=\frac{k_{r_0}}{k_{z_0}}, \quad (12)$$

The solution of (7) is:

$$\mathcal{T}(t)=C_{1t}\,e^{-\lambda_{nm}^2 t}, \quad (13)$$

where C_{1t} is a constant. It is supposed that one side of cylinder is isolated, and the other one is in environment temperature. So, the solution of (8) could be expressed as:

$$\mathcal{Z}(z)=C_{1z}\,\varphi_n(z), \quad (14)$$

and

$$\varphi_n(z)=Sin(\mu_n z), \qquad \mu_n=\frac{(2n-1)\pi}{2L} \quad (15)$$

Equation (9) is a Bessel function and the solution is:

$$\mathcal{R}(r)=r^{\frac{-p}{2}}\left[C_{1r}J_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)+C_{1r}Y_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)\right] \quad (16)$$

It is assumed that the inside of cylinder is isolated, but the outside is under the convection boundary condition, i.e.

$$\frac{d\mathcal{R}(r_{in})}{dr}=0 \quad (17)$$

$$k_r\frac{d\mathcal{R}(r_{out})}{dr}+h\,\mathcal{R}(r_{out})=0 \quad (18)$$

where h is the convection coefficient. Applying boundary conditions ((17) and (18)) on (16) leads to the following solution:

$$\mathcal{R}(r)=C_{1r}\,\psi_{nm}(r), \quad (19)$$

and

$$\psi_{nm}(r)=r^{\frac{-p}{2}}\left[J_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)-\frac{f_{r_{in}}^1}{f_{r_{in}}^2}Y_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)\right], \quad (20)$$

η is determined by solving the following equation:

$$f_{r_{in}}^1\times f_{r_{out}}^2-f_{r_{in}}^2\times f_{r_{out}}^1=0, \quad (21)$$

and

$$f_{r_{in}}^1=\frac{-p}{2}r_{in}^{\frac{-p-2}{2}}J_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{in}\right)+r_{in}^{\frac{-p}{2}}\frac{\eta_{nm}}{\sqrt{\alpha_0}}J'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{in}\right) \quad (22)$$

$$f_{r_{in}}^2=\frac{-p}{2}r_{in}^{\frac{-p-2}{2}}Y_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{in}\right)+r_{in}^{\frac{-p}{2}}\frac{\eta_{nm}}{\sqrt{\alpha_0}}Y'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{in}\right) \quad (23)$$

$$f_{r_{out}}^1=\left[\frac{-p}{2}k_{r0}\,r_{out}^{\frac{-p-2}{2}}+h\,r_{out}^{\frac{-p}{2}}\right]J_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{out}\right)+r_{out}^{\frac{-p}{2}}\frac{\eta_{nm}}{\sqrt{\alpha_0}}J'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{out}\right) \quad (24)$$

$$f_{r_{out}}^2=\left[\frac{-p}{2}k_{r0}\,r_{out}^{\frac{-p-2}{2}}+h\,r_{out}^{\frac{-p}{2}}\right]Y_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{out}\right)+r_{out}^{\frac{-p}{2}}\frac{\eta_{nm}}{\sqrt{\alpha_0}}Y'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r_{out}\right) \quad (25)$$

where,

$$J'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)=\frac{d}{d\zeta}[J_{\frac{p}{2}}(\zeta)]_{\zeta\to\frac{\eta_{nm}}{\sqrt{\alpha_0}}r} \quad (26)$$

$$Y'_{\frac{p}{2}}\left(\frac{\eta_{nm}}{\sqrt{\alpha_0}}r\right)=\frac{d}{d\zeta}[Y_{\frac{p}{2}}(\zeta)]_{\zeta\to\frac{\eta_{nm}}{\sqrt{\alpha_0}}r} \quad (27)$$

So, the modified temperature will be:

$$\theta(r,z,t)=\sum_{n=1}^{\infty}\sum_{n=1}^{\infty}C_{nm}\,e^{-\lambda_{nm}^2 t}\,\psi_{nm}(r)\varphi_n(z), \quad (28)$$

where, $C_{nm}=C_{1t}\,C_{1r}\,C_{1z}$.

To determine C_{nm}, the initial boundary condition (constant temperature T_0) must be applied:

$$C_{nm}=\frac{\int_0^z\int_{r_{in}}^{r_{out}}\theta_0\;\psi_{nm}(r)\varphi_n(z)\,drdz}{\int_0^z\int_{r_{in}}^{r_{out}}\psi_{nm}^2(r)\varphi_n^2(z)_n\,drdz}, \quad (29)$$

where, $\theta_0=T_0-T_\infty$.

IV. Results

In this section, the achieved analytical solution is used to find temperature distribution in a hollow FGM cylinder. Fig. 1 shows the geometry of studied cylinder. Table I presents the thermo-physical properties of material and environment.

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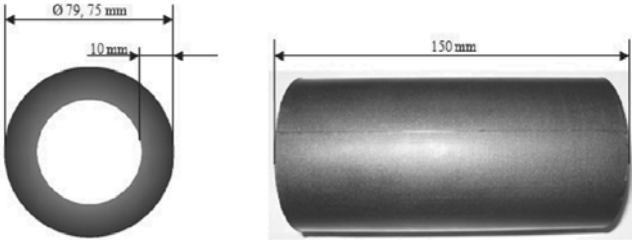


Fig. 1 Real specimen of a FGM cylinder made by centrifugal casting method [12]

TABLE I
VALUES OF THERMO-PHYSICAL PARAMETERS

Symbol	Quantity	Value
h	Convection coefficient	$20\ \frac{W}{m^2K}$
T_0	Initial temperature	$330\ K$
T_∞	Environment temperature	$310\ K$
k_{r0}	Conduction coefficient in r direction	$11.1\ \frac{W}{mK}$
k_{z0}	Conduction coefficient in z direction	$5.55\ \frac{W}{mK}$
ρ_0	Density	$935\ \frac{kg}{m^3}$
c_0	Heat capacity	$1400\ \frac{J}{Kg\ K}$

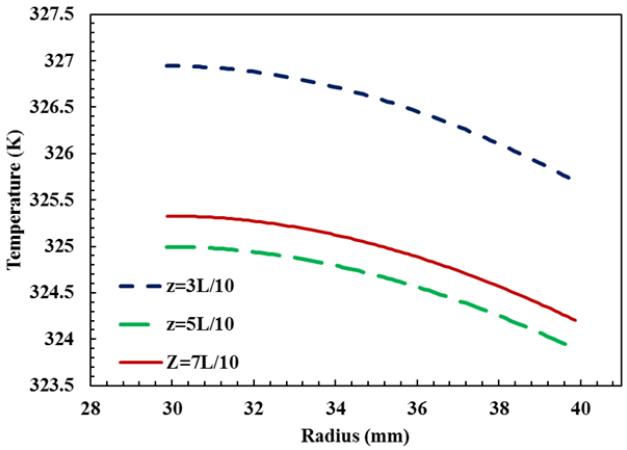


Fig. 2 Temperature variation in radial direction at different longitudinal sections (t=20s, p=0.1)

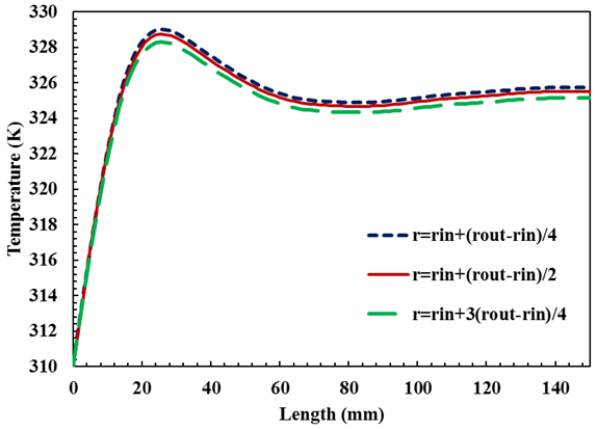


Fig. 3 Temperature variation in longitudinal direction at different radial sections (t=20s, p=0.1)

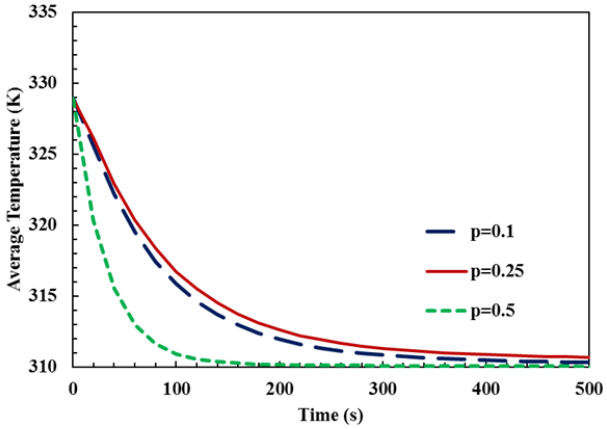


Fig. 4 Time history of average temperature for different power index of FGM

Figs. 2 and 3 show the variation of temperature in radial and longitudinal directions, respectively. These figures are depicted in different radial and longitudinal sections. As is clear from these figures, the temperature distribution is in good agreement with boundary conditions.

Fig. 4 presents the time history of average temperature of FG cylinder for different power index of FGMs. Regarding to this figure, the steady state decreases by growth of power index.

V. CONCLUSION

In this paper, an analytical solution for 2D unsteady temperature distribution in a finite FG cylinder is presented. Thermo-physical properties of FGM are power functions of radius. Thermal boundary conditions and initial condition are satisfied. The results show that the steady state occurs in former time when the power index of FG increases. The results of this analytical method can be used for validation of numerical solutions and also investigation of thermal stress in FG cylinder.

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