Simulation of Piezoelectric Laminated Smart Structure under Strong Electric Field
Shun-Qi Zhang, Shu-Yang Zhang, Min Chen

Abstract—Applying strong electric field on piezoelectric actuators, on one hand very significant electroelastic material nonlinear effects will occur, on the other hand piezo plates and shells may undergo large displacements and rotations. In order to give a precise prediction of piezolaminated smart structures under large electric field, this paper develops a finite element (FE) model accounting for both electroelastic material nonlinearity and geometric nonlinearity with large rotations based on the first order shear deformation (FSOD) hypothesis. The proposed FE model is applied to analyze a piezolaminated semicircular shell structure.

Keywords—Smart structures, piezolaminates, material nonlinearity, geometric nonlinearity, strong electric field.

I. INTRODUCTION

SMART structures are those integrated with smart materials like piezoelectric, shape memory alloys, etc., which have a controlled loop with feedback. They have great potential in the applications of vibration control, acoustic control and health monitoring. In order to obtain high actuation forces, strong electric driving field is one of the economical ways without any modification of structural construction. However, applying strong electric fields on piezo actuators, on one hand electroelastic material nonlinear phenomenon will be introduced to the system, on the other hand thin-walled structures will undergo large displacements and rotations. In such a case, numerical models should include both electroelastic material nonlinearity and geometric nonlinearity.

Regarding the modeling and simulation of piezoelectric smart structures, there are plenty of studies available in the literature developed linear numerical models, see e.g. [1], [2] among many others. Due to the simplified assumptions, linear models are only valid for structures subjected to weak electric fields and undergoing small displacements. The numerical models for piezoelectric structures under strong electric field must take into account the electroelastic material nonlinearity. Nelson [3], Tiersten [4] first proposed nonlinear electroelastic equations for piezoelectric materials. Later, the irreversible piezoelectric nonlinearities were investigated experimentally by Li et al. [5], Masys et al. [6], and studied numerically by Landis [7], Ma et al. [8]. Furthermore, Wang et al. [9], Yao et al. [10] developed analytical models for cantilevered piezoelectric bimorph and unimorph beams under strong electric field. Kusculuoglu & Royston [11], Kapuria & Yasin developed numerical model for dynamic analysis and active vibration control of piezoelectric smart structures under strong electric fields.

The above mentioned references considered electroelastic material nonlinearity with geometrically linear. Therefore, piezoelectric materials can be subjected to strong electric fields, with the structures undergoing small displacements and rotations. Concerning piezoelectric structures undergoing large displacements and rotations, but without taking into account the electroelastic material nonlinearity, a lot of papers can be found, e.g. von Kármán type nonlinear model [12], [13], moderate rotation nonlinear model [14], and large rotation nonlinear model [15], [16]. However, applying strong electric fields usually yields large displacements and rotations, which cannot be existing separately. In such a case, Yao et al. [17] developed a finite element model with von Kármán type nonlinearity and electroelastic material nonlinearity. Von Kármán nonlinearity is the simplest one including very week geometric nonlinearity, which is only valid for structures undergoing moderately large displacements and small rotations.

In order to simulate piezoelectric smart structures under strong electric fields with large displacements and rotations, the paper develops a numerical model with both geometric and material nonlinearities for piezoelectric integrated smart structures.

II. MATHEMATICAL MODELS

A. Electroelastic Material Nonlinearity

Considering electroelastic material nonlinearity, the constitutive equations of piezoelectric materials are expressed as [4]

\[ \sigma_p = c_{pq} \varepsilon_q - e_{mp} E_m - \frac{1}{2} b_{mpn} E_m E_n, \]  

\[ D_m = e_{mq} \varepsilon_q + g_{mn} E_n + \frac{1}{2} h_{mkn} E_k E_n, \]

where \( p \) and \( q \) represent the numbers 1, 2, 3, 4, 5, and 6, while \( n \), \( m \), and \( k \) take the numbers 1, 2, 3. Assuming that the transverse normal strain is neglected for plates and shells and the electric field is applied only along the thickness direction, one yields \( p, q = 1, 2, 4, 5 \) or 6 and \( n = m = k = 3 \). Furthermore, \( \sigma_p \) and \( \varepsilon_q \) denote the components of strain and stress vector, \( E_m \) and \( D_m \) are the components of electric field and electric displacement vector, \( c_{pq} \) represent the components of elastic stiffness matrix.
The coefficients appeared above can be obtained as
\[ c_{11} = \frac{Y_1}{1 - \nu_12\nu_2}, \quad c_{12} = \frac{\nu_1\nu_2 Y_2}{1 - \nu_12\nu_2}, \]
\[ c_{22} = \frac{Y_2}{1 - \nu_12\nu_2}, \quad c_{44} = \kappa G_{23}, \]
\[ c_{55} = \kappa G_{13}, \quad c_{66} = G_{12}, \]
\[ e_{31} = d_{31}c_{11} + d_{32}c_{12}, \]
\[ e_{32} = d_{31}c_{21} + d_{32}c_{22}, \]
\[ b_{31} = \beta_{31}c_{11} + \beta_{32}c_{12}, \]
\[ b_{32} = \beta_{31}c_{21} + \beta_{32}c_{22}, \]
\[ g_{33} = e_{33} - d_{31}e_{31} - d_{32}e_{32}, \]
\[ b_{33} = \chi_{33} - d_{31}b_{31} - d_{32}b_{32}. \]

Here \((Y_1, Y_2), (\nu_12, \nu_22), (G_{12}, G_{23}, G_{13})\) and \(\kappa = 5/6\) are respectively the Young’s moduli, the Poisson’s ratios, the shear moduli, and the shear correction factor; \((d_{31}, d_{32})\) and \((e_{33})\) are the piezoelectric constants and the dielectric constants; \((\beta_{31}, \beta_{32})\) and \((\chi_{33})\) denote the nonlinear electroelastic constants and nonlinear electroelastic susceptibility constants.

### B. Geometric Nonlinearity

For plate and shell structures, using the first-order shear deformation hypothesis, one obtains the strain-displacement relations with fully geometric nonlinearity as (the details can be found in [15]-[20])

\[ \varepsilon_{\alpha\beta} = 0, \quad \varepsilon_{\alpha\beta} = \Theta^3 \varepsilon_{\alpha\beta} + \left(\Theta^3\right)^2 \varepsilon_{\alpha\beta}, \]
\[ \varepsilon_{\alpha3} = 0. \]

with the strain components
\[ 2\varepsilon_{\alpha\beta} = \psi_{\alpha\beta} + \psi_{\beta\alpha} + \frac{\psi_{\alpha\beta}}{3} + \frac{\psi_{\beta\alpha}}{3} + \frac{\psi_{\alpha\beta}}{3} + \frac{\psi_{\beta\alpha}}{3}, \]
\[ 2\varepsilon_{\alpha3} = \psi_{\alpha3} - b_{33}^1 \psi_{\alpha3} + \psi_{\alpha3} - b_{33}^1 \psi_{\alpha3} + \frac{\psi_{\alpha3}}{3} + \frac{\psi_{\alpha3}}{3} + \frac{\psi_{\alpha3}}{3} + \frac{\psi_{\alpha3}}{3}. \]

Here \(\varepsilon_{\alpha\beta}\) and \(\varepsilon_{\alpha3}\) represent respectively the in-plane strain components and the stress components. Dropping all terms with underlines yields linear plate/shell theory. Based on the linear theory, containing additionally the squares and products of transverse displacement gradients, one obtains von Kármán type nonlinear plate/shell theory. Including the fully geometrically nonlinear strain-displacement relations and considering finite shell director rotations, one leads to large rotation nonlinear theory.

In the FOSD hypothesis, 5 degrees of freedom at each node are usually considered. The fully geometrically nonlinear strain-displacement relations contain 6 parameters. In order to represent finite rotation of shell director, the 5 degrees of freedom are expressed by 6 parameters as (see [20])

\[ 0 \quad \begin{bmatrix} \varepsilon_{1} = u, \\ \varepsilon_{2} = v, \\ \varepsilon_{3} = w, \\ \varepsilon_{4} = \sin(\varphi_{1})\cos(\varphi_{2}), \\ \varepsilon_{5} = \sin(\varphi_{2}), \\ \varepsilon_{6} = \cos(\varphi_{1})\cos(\varphi_{2}) - 1. \end{bmatrix} \]

C. Finite Element Model

Using the principle of virtual work yields nonlinear static equilibrium equations and sensor equations with consideration of both geometric and material nonlinearities as

\[ K_{uu} \Delta q + K_{\phi\phi} \Delta \Phi = F_{uu} - F_{u}, \]
\[ K_{\phi\phi} \Delta q + K_{\Phi\Phi} \Delta \Phi = G_{\phi\phi} - G_{\phi}. \]

Here \(K_{uu,} \ K_{\phi\phi}, \ K_{\Phi\Phi}\) represent the stiffness matrix, the piezoelectric coupled stiffness matrix, the coupled capacity matrix and the piezoelectric capacity matrix, respectively; \(F_{uu}, \ F_{u}, \ G_{\phi\phi}\) and \(G_{\phi}\) are, respectively, the external force vector, the in-balance force vector, the external charge vector and the in-balance charge vector; \(\Phi\) and \(\Phi\) denote, respectively, the actuation voltage vector and the sensor voltage vector.

For clear description, several abbreviations are introduced. The finite element model considers geometric and material linear effects based on the FOSD hypothesis is abbreviated as LINSWE, in which WE denotes weak electric field. Employing the electroelastic material nonlinearity for strong driving electric field (SE), the FE models considering geometric linear and von Kármán nonlinear theory are respectively shorted as LINSSE and RVKSSE. Furthermore, the model including large rotation geometric nonlinear theory with material nonlinear relations is denoted by LRT5SE.

### III. Numerical Examples

In this section, a semicircular cylindrical shell is considered as the numerical example, where two piezoelectric layers are bonded at the inner and outer surfaces, as shown in Fig. 1. The host structure is made of graphite/epoxy (T300/976) stacked as 4 substrate layers \([45^\circ/ -45^\circ]\), with equal thickness of 0.254 mm. The piezoelectric layers made of 3203HD has the thickness of 0.254 mm. The material properties of 3203HD and T300/976 are given in Table I.

The semicircular cylindrical shell is subjected to a driving voltage of 254 V on both piezoelectric layers. Calculating
IV. CONCLUSION

To simulate piezoelectric smart structures under strong electric fields with large displacements and rotations, the paper has developed a nonlinear FE model for piezolaminated smart structures based on the FOSD hypothesis, in which the electroelastic material nonlinearity and large rotation geometric nonlinearity are taken into account.

The proposed model is applied to analyze a piezolaminated semicircular shell structure. The results show that large discrepancy occurs between the linear and nonlinear predictions. Moreover, in the analysis of the semicircular shell, the RVK5SE curve is very close to LIN5SE, but both of them are different from LRT56SE curve, which indicates that von Kármán type nonlinearity is too weak to predict behavior precisely.

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REFERENCES


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