

# Throughflow Effects on Thermal Convection in Variable Viscosity Ferromagnetic Liquids

G. N. Sekhar, P. G. Siddheshwar, G. Jayalatha, R. Prakash

**Abstract**—The problem of thermal convection in temperature and magnetic field sensitive Newtonian ferromagnetic liquid is studied in the presence of uniform vertical magnetic field and throughflow. Using a combination of Galerkin and shooting techniques the critical eigenvalues are obtained for stationary mode. The effect of Prandtl number ( $Pr > 1$ ) on onset is insignificant and nonlinearity of non-buoyancy magnetic parameter  $M_3$  is found to have no influence on the onset of ferroconvection. The magnetic buoyancy number,  $M_1$  and variable viscosity parameter,  $V$  have destabilizing influences on the system. The effect of throughflow Peclet number,  $Pe$  is to delay the onset of ferroconvection and this effect is independent of the direction of flow.

**Keywords**—Ferroconvection, throughflow, temperature dependent viscosity, magnetic field dependent viscosity.

## I. INTRODUCTION

**F**ERROCONVECTION in a layer of ferromagnetic liquid plays a very important role in heat transfer problems. Finlayson [1] made a detailed study of thermal convection in a ferromagnetic liquid. The various aspects on the theory of thermoconvective instability in ferromagnetic liquids has received increasing importance over the years (see [2]-[11]).

The effect of different basic temperature gradients on the onset of ferroconvection driven by combined surface tension and buoyancy forces was provided by Shivakumara et al. [12]. Abraham and Siddheshwar [13] have examined the thermal instability in a layer of a ferromagnetic liquid when the boundaries are subjected to synchronous/asynchronous imposed time-periodic boundary temperatures (ITBT) and time-periodic body force (TBF) and showed that convection influence is controlled by (ITBT) and (TBF). The detailed study of linear and non-linear analyses using the generalized energy method for the convection problem in a ferromagnetic liquid with magnetic field dependent (MFD) viscosity was made by Sunil et al. [14], [15]. The significant contribution of ferromagnetic liquids in applications is provided by Rovensweig et al. [16] and Odenbach [17].

To suppress or augment the convection, the mechanisms that have been used effectively are Coriolis force due to rotation or external magnetic/electric fields or non-uniform temperature gradient across the liquid layer. A vertical throughflow has

a significant influence on the stability of the system. The modified problem where the boundaries are permeable and there is injection of liquid through the upper plate and suction through the lower plate called the throughflow has been studied extensively by many authors (see [18]-[25]).

Now, we move on to the literature on thermal convection in liquids with variable viscosity. Siddheshwar et al. [26] studied the influence of an externally applied magnetic field on the Rayleigh-Bénard-Marangoni magnetoconvection with thermorheological effect in a Newtonian liquid for all possible boundary combinations. The detailed study on the onset of Rayleigh-Bénard, Bénard-Marangoni and Rayleigh-Bénard-Marangoni convections in a viscoelastic liquid with variable viscosity was provided by Sekhar and Jayalatha [27], [28]. Sekhar et al. [29] have studied the effects of magnetorheological and thermorheological parameters on Rayleigh-Bénard-Marangoni convection with non-uniform basic temperature gradient in ferromagnetic liquids and the influence of various parameters on the onset of convection has been analyzed. Various aspects of thermal convection in liquids with variable viscosity are studied extensively by many authors (see [30]-[32]).

In the paper, we study the effect of throughflow on the onset of thermal convection with temperature and magnetic field dependent viscosity in ferromagnetic liquids for all possible boundary combinations.

## II. MATHEMATICAL FORMULATION

The physical configuration considered here consists of infinite, horizontal and variable viscosity ferromagnetic liquid layer of thickness  $d$ . The Cartesian co-ordinate system is taken with the lower plate in the  $xy$ -plane and  $z$ -axis vertically upwards. A constant throughflow of magnitude  $w_0$  is maintained which is gravity aligned or antigravity in its direction. The uniform magnetic field  $H_i = (0, 0, H_0)$  is applied along the vertical direction. The lower and upper plates are maintained at constant temperatures  $T_0 + \Delta T$  at  $z = 0$  and  $T_0$  at  $z = d$  respectively (see Fig. 1).

### A. Governing Equations

The governing equations which represent the above physical configuration are:

Continuity equation:

$$q_{i,i} = 0, \quad (1)$$

G. N. Sekhar is with the Department of Mathematics, BMS College of Engineering, Bengaluru 560019, Karnataka, India (e-mail: drgns.maths@bmsce.ac.in).

P. G. Siddheshwar is with the Department of Mathematics, Bangalore University, Bengaluru 560056, Karnataka, India (e-mail: mathdrpgs@gmail.com).

G. Jayalatha and R. Prakash are with the Department of Mathematics, R. V. College of Engineering, R. V. Vidyanikethan Post, Bengaluru 560059, Karnataka, India (e-mail: jayalathag@rvce.edu.in, prakashr@rvce.edu.in).

TABLE I  
NOMENCLATURE

$a$	dimensionless wave number	$V$	variable parameter	viscosity
$B_i$	magnetic induction	$w_0$	constant throughflow	vertical
$C_{VH}$	specific heat volume and magnetic field		<b>Greek symbols</b>	
$d$	depth of the liquid layer	$\delta_T, \delta_H$	small constants	positive
$g_i$	gravitational acceleration (0, 0, -g)	$\alpha$	thermal expansion coefficient	
$H_i$	components of applied magnetic field	$\Delta$	difference of two values	
$H_0$	applied magnetic field	$\kappa$	thermal conductivity	
$l, m$	wave numbers	$\mu(H, T)$	variable viscosity	
$M_i$	magnetization	$\mu_0$	magnetic permeability	
$M_0$	mean value of magnetization	$\rho$	density	
$M_1$	buoyancy magnetic number	$\rho_0$	reference density	
$M_3$	non-buoyancy magnetic number	$\phi$	magnetic potential	scalar
$p$	effective pressure	$\omega$	frequency	
$Pe$	throughflow Peclet number		<b>Subscripts and Superscripts:</b>	
$Pr$	Prandtl number	$b$	basic state	
$q_i$	components of velocity (u,v,w)	$c$	critical quantity	
$R$	stationary Rayleigh number	$0$	reference value	
$t$	time	$*$	dimensionless quantity	
$T$	temperature	$'$	dimensional quantity	
$T_0$	constant temperature of the boundary			

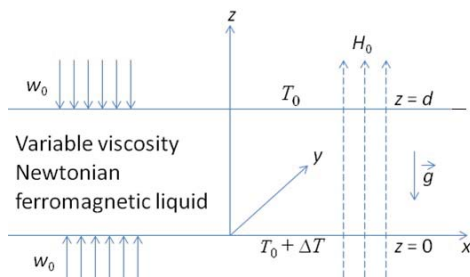


Fig. 1 Physical configuration of the problem

Momentum equation:

$$\rho_0 \left( \frac{\partial q_i}{\partial t} + q_j q_{i,j} \right) = \mu_0 (M_j H_{i,j}) + [\mu(H, T)(q_{i,j} + q_{j,i})]_{,i} - p_{,i} + \rho g_i, \quad (2)$$

Energy equation is assumed in the form:

$$\rho_0 C_{VH} \left( \frac{\partial T}{\partial t} + q_j T_{,j} \right) = \kappa T_{,jj}, \quad (3)$$

Density equation of state:

$$\rho = \rho_0 [1 - \alpha(T - T_0)], \quad (4)$$

Maxwell's equations:

$$B_{i,i} = 0 \quad \text{and} \quad \epsilon_{ijk} H_{k,j} = 0, \quad (5)$$

The magnetic induction equation given by

$$B_i = \mu_0 (M_i + H_i), \quad (6)$$

Magnetic equation of state:

$$M = M_0 + \chi_m (H - H_0) + k_l (T - T_0). \quad (7)$$

The pyromagnetic coefficient and magnetic susceptibility are given by

$$k_l = - \left( \frac{\partial M}{\partial T} \right)_{H_0, T_0}, \quad \chi_m = \left( \frac{\partial M}{\partial H} \right)_{H_0, T_0}.$$

Effective viscosity  $\mu(H, T)$  is assumed as:

$$\mu(H, T) = \frac{\mu_0}{1 + \delta_T (T - T_0) - \delta_H (H - H_0)}. \quad (8)$$

### B. Basic State Solution

The solution in the quiescent basic state is given by:

$$\left. \begin{aligned} q_{ib} &= (0, 0, w_0), \\ T_b(z) &= T_0 + \Delta T f(z), \\ \mu_b(z) &= \frac{\mu_0}{1 + \delta_T (T - T_0) - \delta_H (H - H_0)} \\ &= \frac{1}{1 + f(z)V}, \\ \rho_b(z) &= \rho_0 [1 - \alpha \Delta T f(z)], \\ M_b(z) &= M_0 - \left( \frac{k_l}{1 + \chi_m} \right) f(z), \\ H_b(z) &= H_0 + \left( \frac{k_l}{1 + \chi_m} \right) f(z). \end{aligned} \right\} \quad (9)$$

$$\text{where, } f(z) = \frac{1}{1 - e^{w_0 d / \kappa}} (e^{w_0 z / \kappa} - e^{w_0 d / \kappa})$$

$$\text{and } V = \left( \delta_T - \frac{\delta_H \kappa_l}{1 + \chi_m} \right) \Delta T.$$

On the quiescent basic state finite amplitude perturbations are super imposed in the following form:

$q_i = q_{ib} + q'_i$ ,  $p = p_b(z) + p'$ ,  $T = T_b(z) + T'$ ,  $\rho = \rho_b(z) + \rho_b$ ,  $H_i = H_{ib}(z) + H'_i$ ,  $M_i = M_{ib}(z) + M'_i$  and  $\mu = \mu_b + \mu'$ , where,  $q'_i = (u', v', w')$ ,  $p'$ ,  $T'$ ,  $H'_i = (H'_x, 0, H'_z)$ ,  $M'_i = (M'_x, 0, M'_z)$  and  $\mu'$  are the perturbed quantities. Under the Boussinesq approximation, by the classical procedure of linear stability analysis, taking  $t' = (d^2 / \kappa) t^*$ ,  $q'_i = (\kappa / d) q_i^*$ ,  $p' = (\mu \kappa / d^2) p^*$ ,  $T' = (\Delta T) T^*$  and  $\phi' = (\kappa (\Delta T) d^2 / (1 + \chi_m)) \phi^*$ , the dimensionless equations governing perturbations superposed over the quiescent basic state after dropping the primes and asterisks, can be written as

TABLE II  
 BOUNDARY COMBINATIONS AND CORRESPONDING TRIAL FUNCTIONS FOR RAYLEIGH-BÉNARD FERROCONVECTION

Case	Boundary	Boundary condition(BC)	Acronym	Trial functions
1	z = 0	w = Dw = 0 Rigid	RIFI	$w_1 = 2z^4 - 5z^3 + 3z^2$ $T_1 = z^2 - z$ $\phi_1 = \cos(\pi z)$
	z = 1	T = Dφ = 0 Isothermal w = D <sup>2</sup> w = 0 Free T = Dφ = 0 Isothermal		
2	z = 0	w = D <sup>2</sup> w = 0 Free	FIFI	$w_1 = z^4 - 2z^3 + z$ $T_1 = z^2 - z$ $\phi_1 = \cos(\pi z)$
	z = 1	T = Dφ = 0 Isothermal w = D <sup>2</sup> w = 0 Free T = Dφ = 0 Isothermal		
3	z = 0	w = Dw = 0 Rigid	RIRI	$w_1 = z^4 - 2z^3 + z^2$ $T_1 = z^2 - z$ $\phi_1 = \cos(\pi z)$
	z = 1	T = Dφ = 0 Isothermal w = Dw = 0 Rigid T = Dφ = 0 Isothermal		
4	z = 0	w = Dw = 0 Rigid	RAFI	$w_1 = 2z^4 - 5z^3 + 3z^2$ $T_1 = z^2 - 1$
	z = 1	DT = Dφ - T = 0 Adiabatic w = D <sup>2</sup> w = 0 Free T = Dφ = 0 Isothermal		
5	z = 0	w = Dw = 0 Rigid	RIFA	$w_1 = z^4 - 2z^3 + z^2$ $T_1 = z^2 - 2z$ $\phi_1 = \frac{-z^2}{2}$
	z = 1	T = Dφ = 0 Isothermal w = D <sup>2</sup> w = 0 Free DT = Dφ - T = 0 Adiabatic		
6	z = 0	w = D <sup>2</sup> w = 0 Free	FIFA	$w_1 = z^4 - 2z^3 + z$ $T_1 = z^2 - 1$ $\phi_1 = \frac{z^2}{2} - z$
	z = 1	T = Dφ = 0 Isothermal w = D <sup>2</sup> w = 0 Free DT = Dφ - T = 0 Adiabatic		
7	z = 0	w = Dw = 0 Rigid	RIRA	$w_1 = 2z^4 - 5z^3 + 3z^2$ $T_1 = z^2 - 2z$ $\phi_1 = \frac{-z^2}{2}$
	z = 1	T = Dφ = 0 Isothermal w = Dw = 0 Rigid DT = Dφ - T = 0 Adiabatic		
8	z = 0	w = D <sup>2</sup> w = 0 Free	FIRA	$w_1 = z^4 - 2z^3 + z$ $T_1 = z^2 - 2z$ $\phi_1 = \frac{-z^2}{2}$
	z = 1	T = Dφ = 0 Isothermal w = Dw = 0 Rigid DT = Dφ - T = 0 Adiabatic		
9	z = 0	w = D <sup>2</sup> w = 0 Free	FARI	$w_1 = z^4 - \frac{3}{2}z^3 + \frac{1}{2}z$ $T_1 = z^2 - 2z$
	z = 1	DT = Dφ - T = 0 Adiabatic w = Dw = 0 Rigid T = Dφ = 0 Isothermal		

$$\left(\frac{1}{Pr} \frac{\partial}{\partial t} + \frac{Pe}{Pr} \frac{\partial}{\partial z}\right) (\nabla^2 w) = g_1(z) \nabla^4 w + 2 \frac{\partial}{\partial z} (g_1(z)) \nabla^2 \left(\frac{\partial w}{\partial z}\right) + \frac{\partial^2}{\partial z^2} (g_1(z)) \left(\frac{\partial^2 w}{\partial z^2} - \nabla_1^2 w\right) \quad (10)$$

$$+ RM_1 \frac{\partial}{\partial z} (f(z)) \left(\frac{\partial}{\partial z} (\nabla_1^2 \phi) - \nabla_1^2 T\right) + R \nabla_1^2 T \frac{\partial T}{\partial t} + g_2(z) w + Pe \frac{\partial T}{\partial z} = \nabla^2 T \quad (11)$$

$$M_3 \nabla_1^2 \phi + \frac{\partial^2 \phi}{\partial z^2} - \frac{\partial T}{\partial z} = 0 \quad (12)$$

where,

$$g_1(z) = \frac{1}{1 + f(z)V},$$

$$g_2(z) = \frac{(Pe)e^{Pez}}{1 - e^{Pe}},$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla_1^2 + \frac{\partial^2}{\partial z^2}.$$

where  $(w, T, \phi)$  are the dimensionless perturbations of the velocity, the temperature, and the magnetic potential respectively. The parameters appearing in (10)-(12) are:

$$R = \frac{\alpha \rho_0 g \Delta T d^3}{\mu_0 \kappa} \quad (\text{Rayleigh number}),$$

$$Pe = \frac{w_0 d}{\frac{\kappa}{\rho_0 \kappa}} \quad (\text{throughflow Peclet number}),$$

$$Pr = \frac{\mu_0}{\rho_0 \kappa} \quad (\text{Prandtl number}),$$

$$M_1 = \frac{\mu_0 k_l^2 \Delta T}{\rho_0 g \alpha (1 + \chi_m) d} \quad (\text{buoyancy magnetic number}),$$

$$M_3 = \frac{(1 + M_0/H_0)}{(1 + \chi_m)} \quad (\text{non-buoyancy magnetic number}),$$

$$V = \left(\delta_T - \frac{\delta_H \kappa_l}{1 + \chi_m}\right) \Delta T \quad (\text{variable viscosity parameter}).$$

### C. Linear Stability Analysis

The perturbations for stationary convection are assumed to be periodic waves, employing the normal mode solution for (10)-(12) in the form:

$$w(x, y, z, t) = w(z) e^{i(lx + my)},$$

$$T(x, y, z, t) = T(z) e^{i(lx + my)}, \quad (13)$$

$$\phi(x, y, z, t) = \phi(z) e^{i(lx + my)}.$$

we get the equations governing  $w(z)$ ,  $T(z)$  and  $\phi(z)$ , the amplitudes of perturbations of velocity, temperature and magnetic potential respectively. In (13)  $l$  and  $m$  are horizontal components of wave numbers in the  $x$  and  $y$  directions with  $a^2 = l^2 + m^2$ . Using  $D = \frac{d}{dz}$  and incorporating (13) in (10)-(12), we get the following governing equations:

$$(g_1(z)(D^2 - a^2)^2) w + D^2 g_1(z) (D^2 w + a^2 w) + RM_1 a^2 D f(z) (T - D\phi) - Ra^2 T + (D^3 - a^2 D) \left(2Dg_1(z) - \frac{Pe}{Pr}\right) w = 0, \quad (14)$$

$$(D^2 - a^2) T - g_2(z) w - Pe DT = 0, \quad (15)$$

$$(D^2 - M_3 a^2) \phi - DT = 0. \quad (16)$$

### D. Galerkin Method

The critical value of Rayleigh number as well as the wave number are computed using a combination of the Galerkin technique and shooting technique. We first explain the procedure of single-term Galerkin method for determining the expression for Rayleigh number explicitly. Towards this end we select  $w(z)$ ,  $T(z)$  and  $\phi(z)$  in the form:

$$w(z) = Aw_1(z),$$

$$T(z) = BT_1(z), \quad (17)$$

$$\phi(z) = C\phi_1(z).$$

where  $A$ ,  $B$  and  $C$  are constants,  $w_1$ ,  $\phi_1$  and  $T_1$  selected satisfy the respective boundary conditions as per Table II. The detailed derivation of boundary conditions for the nine boundary combinations are available in [26], [33]-[35].

The expression for the Rayleigh number is obtained by integrating (14)-(16) with respect to  $z$  between  $z = 0$ ,  $z = 1$  after multiplying by  $w$ ,  $T$  and  $\phi$  respectively and using (17) in the resulting equation and then using the condition for non-trivial solution of the resulting homogeneous equations in  $A$ ,  $B$  and  $C$ , we get:

$$R = \frac{N_1 (X_9 - M_3 a^2 X_{10}) (X_7 - Pe X_{81})}{(N_2 N_3 X_8 + M_1 a^2 X_5 X_8 X_{11})}. \quad (18)$$

where

$$N_1 = \left(X_1 + X_2 + X_3 - \frac{Pe}{Pr} X_{21}\right),$$

$$N_2 = (M_1 a^2 X_4 - a^2 X_6),$$

$$N_3 = (M_3 a^2 X_{10} - X_9),$$

$$X_1 = \langle g_1 w_1 (D^2 - a^2)^2 w_1 \rangle,$$

$$X_2 = 2 \langle w_1 (D^2 - a^2) D w_1 D g_1 \rangle,$$

$$X_{21} = 2 \langle w_1 (D^3 - a^2 D) w_1 \rangle,$$

$$X_3 = \langle w_1 (D^2 + a^2) w_1 D^2 g_1 \rangle,$$

$$X_4 = \langle w_1 (Df(z)) T_1 \rangle,$$

$$X_5 = \langle w_1 (Df(z)) (D\phi_1) \rangle,$$

$$X_6 = \langle w_1 T_1 \rangle, \quad X_7 = \langle T_1 (D^2 - a^2) T_1 \rangle,$$

$$X_8 = \langle g_2(z) T_1 w_1 \rangle, \quad X_{81} = \langle T_1 D T_1 \rangle,$$

$$X_9 = \langle \phi_1 D^2 \phi_1 \rangle, \quad X_{10} = \langle \phi_1^2 \rangle,$$

$$X_{11} = \langle \phi_1 D T_1 \rangle.$$

### E. Numerical Solution

The set of coupled equations (14)-(16) subject to the respective boundary conditions is solved numerically using the shooting method which is based on the Runge-Kutta-Fehlberg45 (RKF45) and Newton-Raphson methods and which gives an accurate eigenvalue. In this method the system of differential equations is transformed to a nine first order differential equations. To solve this system

we require nine initial conditions but we have only four initial conditions and we need five more initial conditions to be prescribed. It is more important to choose appropriate initial conditions which can be improved by Newton-Raphson method. The solution process is repeated until the desired degree of accuracy is obtained. The detailed study of shooting method for eigen boundary value problem in convection is available in [34], [35].

### III. RESULTS AND DISCUSSION

At the onset of thermal convection the effect of vertical throughflow in temperature and magnetic field sensitive Newtonian ferromagnetic liquid is studied. Using the combination of Galerkin and shooting technique the critical eigenvalues for stationary convection are obtained. It is observed that the principle of exchange of stabilities is valid. Figs. 2-13 are the results obtained using shooting technique. Figs. 2 and 3 are respective plots of  $R_c$  and  $a_c$  versus  $V$  for different values of  $Pe$  and  $M_1$  for FIFI boundary combination. From the graph we observe that the effect of increase in the value of  $V$  is to decrease  $R_c$  both in absence/presence of throughflow. This means that the effect of variable viscosity parameter is to destabilize the system. Also, it is evident from the graph that  $a_c$  decreases with increasing  $V$  suggesting the enlargement of the cell size. As  $M_1$  increases  $R_c$  decreases and  $a_c$  increases, meaning that  $M_1$  has a destabilizing influence on the onset of ferroconvection. The effect of nonlinearity of  $M_3$  is found to have no influence on the stability of the system [1]. Figs. 4 and 5 are respective plots of  $R_c$  and  $a_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  for FIFI boundary combination. From the graph we observe that increase in the value of  $Pe$  is to increase  $R_c$  and this implies stabilization of the system. Further,  $a_c$  increases with increase in  $Pe$ . Figs. 6-9 and Figs. 10-13 are respective plots of RIRI and RIRA boundary combinations that correspond to Figs. 2-5 of FIFI. The various parameters influence on  $R_c$  and  $a_c$  are identical in all 3 boundary combination. Similar results are observed for remaining boundary combinations and not explicitly included here for want of space. We have also observed that our results are in good agreement for the limiting case of classical liquids in the absence of  $V$  with no magnetic field and no throughflow effect (see Table III). From Table IV, we observe that the  $Pr$  has insignificant influence on the onset of ferroconvection. The parameter  $V$  may take both negative and positive values. Positive values indicate temperature dominance over magnetic field in their influence on viscosity. Negative values indicate magnetic field dominance. From Figs. 14 and 15 it is clear that  $R_c$  and  $a_c$  decrease with increasing  $V$  for both negative and positive values of  $V$ .

#### A. General Results

By comparing the results on critical values,  $R_c$  and  $a_c$ , on the Rayleigh-Bénard convection in ferromagnetic liquids with vertical throughflow for respective different boundary

combinations, we observe that the following is true:

$$M_1 = 10 : \\ R_c^{RIRI} > R_c^{RIRA} > R_c^{RIFI} > R_c^{RAFI} > R_c^{RIFA} > R_c^{FIRA} > \\ R_c^{FIFI} > R_c^{FAFI} > R_c^{FIFA}.$$

$$a_c^{RIRI} > a_c^{RIRA} > a_c^{RIFI} > a_c^{FIRA} > a_c^{FIFI} > a_c^{RAFI} > \\ a_c^{RIFA} > a_c^{FAFI} > a_c^{FIFA}.$$

$$M_1 = 20 : \\ R_c^{RIRI} > R_c^{RIRA} > R_c^{RIFI} > R_c^{RIFA} > R_c^{RAFI} > R_c^{FIRA} > \\ R_c^{FIFI} > R_c^{FAFI} > R_c^{FIFA}.$$

$$a_c^{RIRI} > a_c^{RIRA} > a_c^{RIFI} > a_c^{FIRA} > a_c^{RAFI} > a_c^{FIFI} > \\ a_c^{RIFA} > a_c^{FAFI} > a_c^{FIFA}.$$

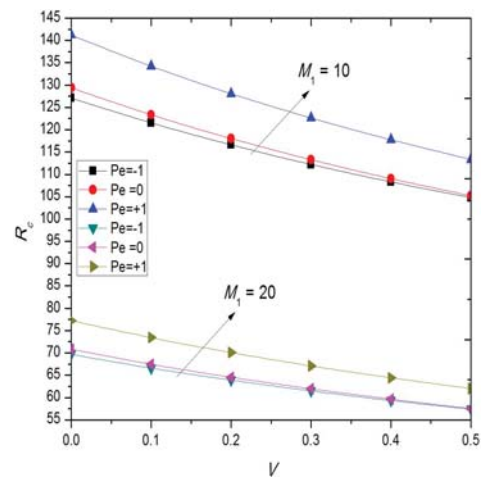


Fig. 2 Plot of  $R_c$  versus  $V$  for Different Values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the Boundary Combination FIFI

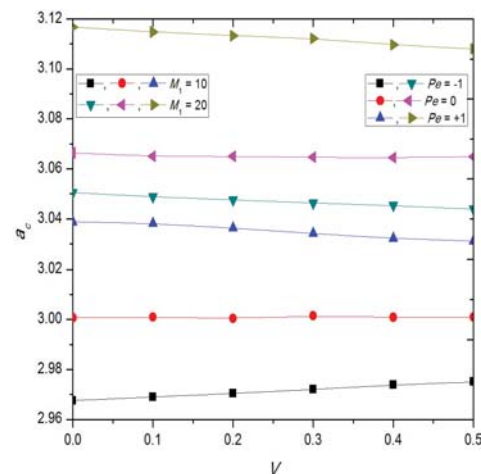


Fig. 3 Plot of  $a_c$  versus  $V$  for Different Values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the Boundary Combination FIFI



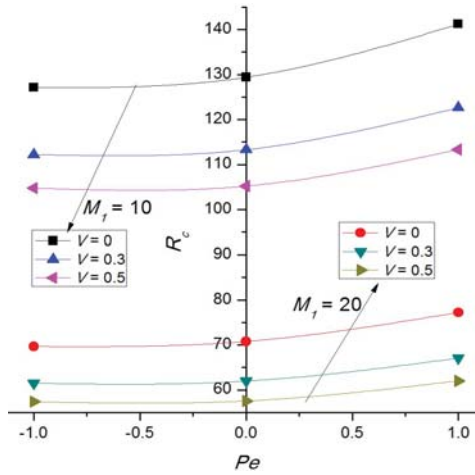


Fig. 4 Plot of  $R_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination FIFI

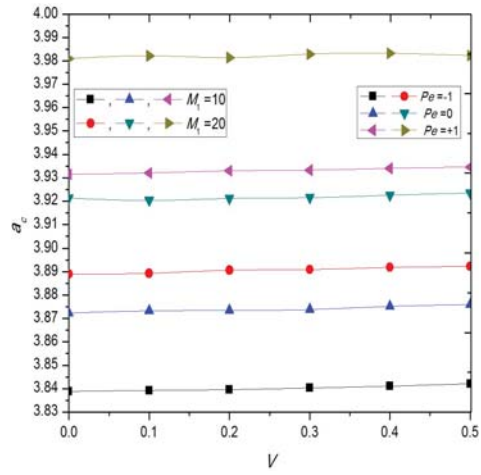


Fig. 7 Plot of  $a_c$  versus  $V$  for different values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRI

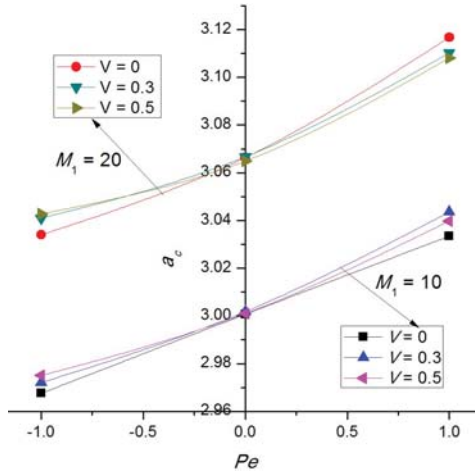


Fig. 5 Plot of  $a_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination FIFI

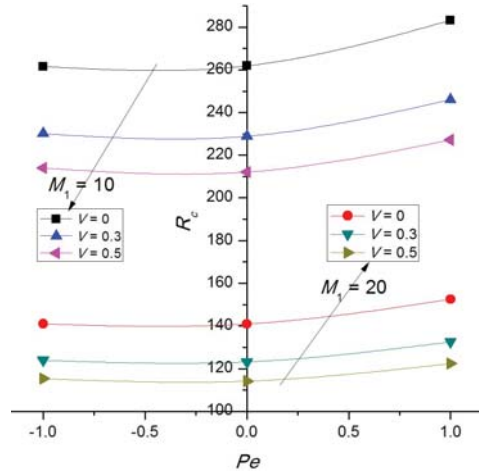


Fig. 8 Plot of  $R_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRI

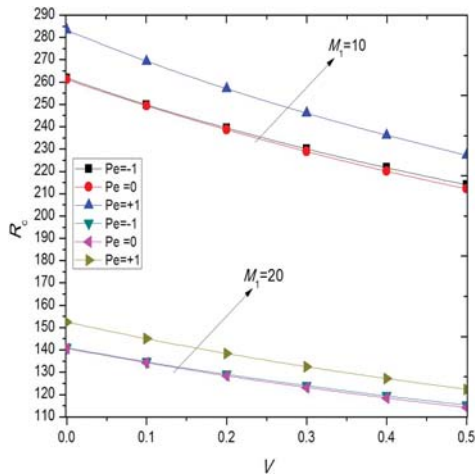


Fig. 6 Plot of  $R_c$  versus  $V$  for different values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRI

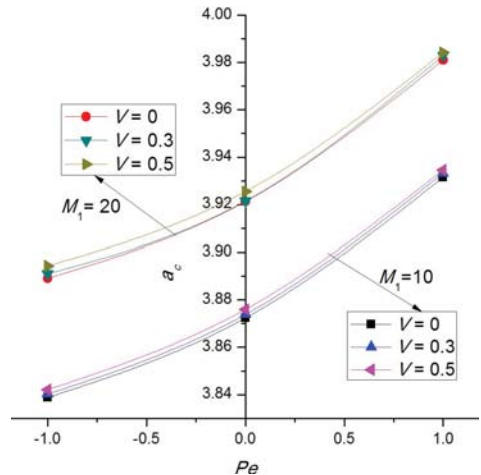


Fig. 9 Plot of  $a_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRI

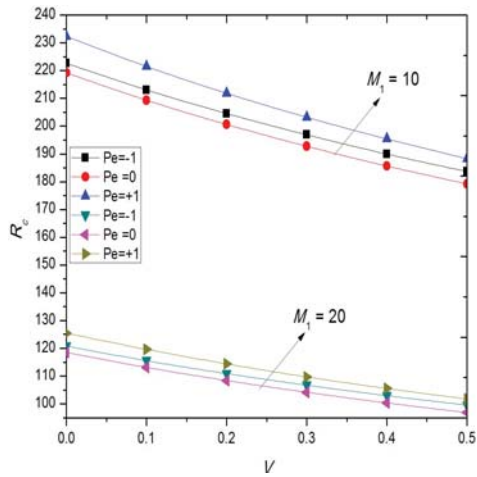


Fig. 10 Plot of  $R_c$  versus  $V$  for different values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRA

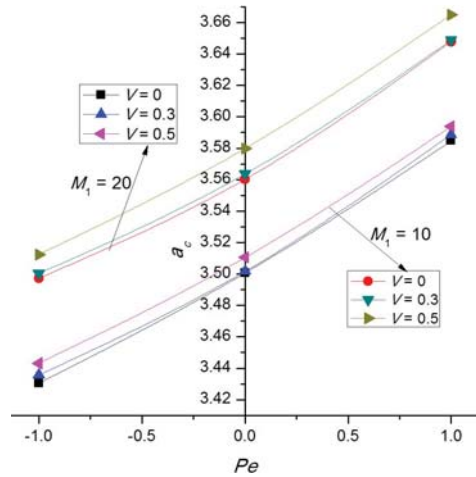


Fig. 13 Plot of  $a_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRA

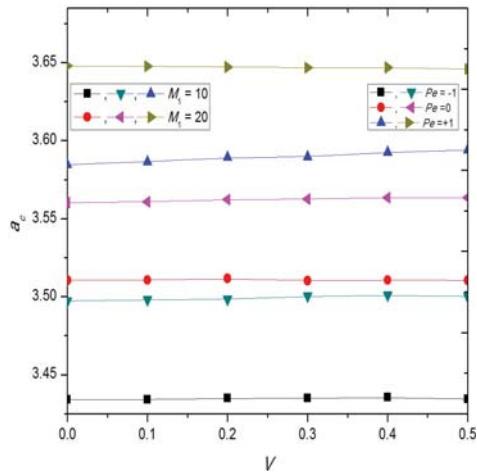


Fig. 11 Plot of  $a_c$  versus  $V$  for different values of  $Pe$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRA

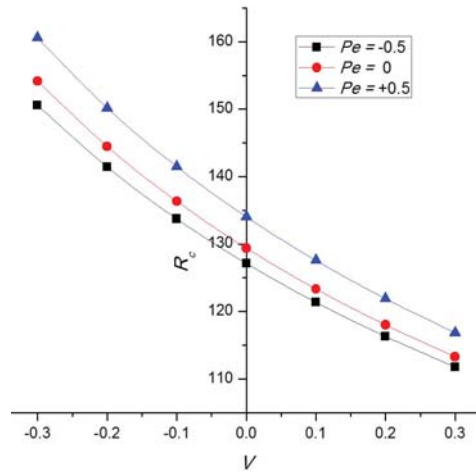


Fig. 14 Plot of  $R_c$  versus  $V$  for different values of  $Pe$  with  $M_3 = 1$ ,  $M_1 = 10$  and  $Pr = 10$  for the boundary combination FIFI

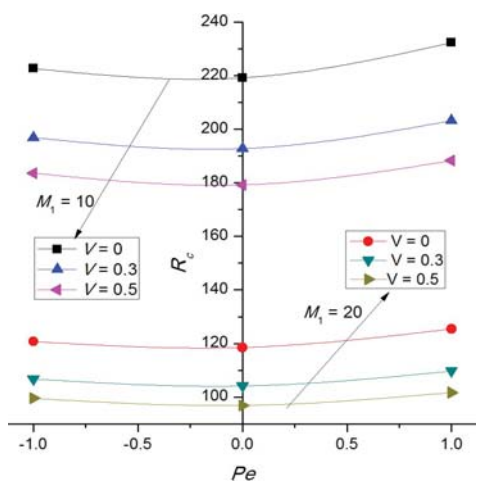


Fig. 12 Plot of  $R_c$  versus  $Pe$  for different values of  $V$  and  $M_1$  with  $M_3 = 1$  and  $Pr = 10$  for the boundary combination RIRA

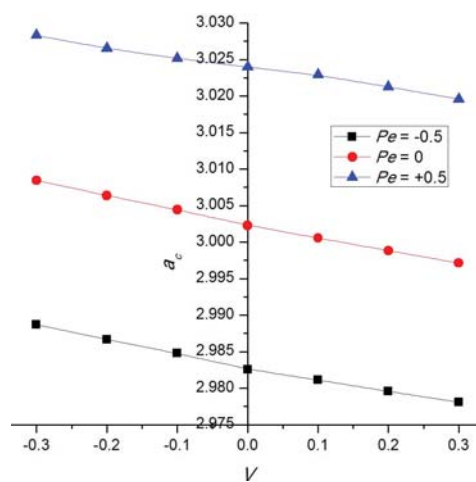


Fig. 15 Plot of  $a_c$  versus  $V$  for different values of  $Pe$  with  $M_3 = 1$ ,  $M_1 = 10$  and  $Pr = 10$  for the boundary combination FIFI

TABLE III

COMPARISON OF  $R_c$  AND  $a_c$  OBTAINED BY DIFFERENT RESEARCHERS WITH THOSE OF PRESENT STUDY FOR THE LIMITING CASE  $V = 0$  IN THE ABSENCE OF MAGNETIC FIELD AND THROUGH FLOW EFFECTS

BC	Platten and Legros [36]		Nield [18]		Present problem	
	$R_c$	$a_c$	$R_c$	$a_c$	$R_c$	$a_c$
RIFI	1100.657	2.68	1138	-	1100.65535	2.68225
RAFA	320	0	320	-	320.0000001	0.00001
RIRA	1295.781	2.55	1452	-	1295.78455	2.55185
RIFA	669.001	2.09	692	-	668.999579	2.08550
RAFI	816.748	2.21	953	-	816.747148	2.21465
FIFA	384.693	1.76	413	-	384.69325	1.7575

TABLE IV

VALUES OF  $R_c$  AND  $a_c$  FOR DIFFERENT VALUES OF  $Pr$ ,  $V$  AND  $Pe$

$M_3 = 1, M_1 = 10$  and  $Pr = 10$

V	$Pe = -1$		$Pe = 0$		$Pe = +1$	
	$R_c$	$a_c$	$R_c$	$a_c$	$R_c$	$a_c$
0	128.752599	2.9966	129.390341	3.00215	141.220796	3.05045
0.1	122.770449	2.9972	123.34098	3.00095	134.225657	3.04955
0.2	117.505829	2.99675	118.018605	3.00065	128.086344	3.04625
0.3	112.824094	2.99705	113.286531	3.00155	122.638902	3.04355

$M_3 = 1, M_1 = 10$  and  $Pr = 20$

V	$Pe = -1$		$Pe = 0$		$Pe = +1$	
	$R_c$	$a_c$	$R_c$	$a_c$	$R_c$	$a_c$
0	128.752131	2.9966	129.390341	3.00215	141.169942	3.0509
0.1	122.768935	2.9972	123.34098	3.00095	134.185338	3.0488
0.2	117.503358	2.99675	118.018605	3.00065	128.055542	3.0464
0.3	112.820691	2.9981	113.286531	3.00155	122.616846	3.0437

#### IV. CONCLUSION

The effect of  $V$  is to destabilize the system in presence of vertical through flow. The effect of increase in magnetic buoyancy number,  $M_1$  is to destabilize the system. The nonlinearity of  $M_3$  is found to have no influence on the onset of ferroconvection. The effect of increase in the value of  $Pr$  on the stability of the system is insignificant. The effect of through flow measured by the Peclet number,  $Pe$  is to delay the onset of ferroconvection and is independent of the direction of through flow.

#### ACKNOWLEDGMENT

The authors express their sincere gratitude to their respective institutions for encouragement and support.

#### REFERENCES

[1] Finlayson, B. A., 1970, *Convective instability of ferromagnetic fluids*, J. Fluid Mech., 40, 753-767.  
 [2] Das Gupta, M. and Gupta A. S., 1979, *Convective instability of a layer of ferromagnetic fluid rotating about a vertical axis*, Int J Eng Sci., 17, 271-277.  
 [3] Gotoh, and Yamada, 1982, *Thermal convection in a horizontal layer of magnetic fluids*, J. Phys. Soc. Jpn. 51, 3042-3048.  
 [4] Stiles, and Kagan, 1990, *Thermomagnetic instability of a horizontal layer of ferrofluid in a strong vertical magnetic field*, JMMM, 85, 196-198.  
 [5] Sekhar, G. N. and Rudraiah, N., 1991, *Convection in magnetic fluids with internal heat generation*, Trans. ASME: J. Heat Trans., 113, 122-127.  
 [6] Siddheshwar, P. G., 1995, *Convective instability of ferromagnetic fluids bounded by fluid-permeable, magnetic boundaries*, JMMM, 149, 148-150.  
 [7] Siddheshwar, P. G., 1999, *Rayleigh-Bénard convection in a second-order ferromagnetic fluid with second sound*, Proc. of 8th Asian Congress of fluid Mech. 6-10.  
 [8] Siddheshwar, P. G., 2002a, *Oscillatory convection in viscoelastic, ferromagnetic/dielectric liquids*, Int. J. Modern Phy B. 16, 2629-2633.

[9] Siddheshwar, P. G., 2002b, *Ferrohydrodynamic and electrodynamic instabilities in Newtonian liquids: Analogy*, East West J. Math. 16, 143-146.  
 [10] Abraham, A., 2002, *Convective instability in magnetic fluids and polarized dielectric liquids*, Ph.D. Thesis, Bangalore University, (India).  
 [11] Maruthamanikandan, S., 2005, *Instabilities in ferromagnetic, dielectric and other complex liquids*, Ph.D. Thesis, Bangalore University, (India).  
 [12] Shivakumara, I. S., Rudraiah, N., and Nanjundappa, C. E., 2002, *Effect of non-uniform basic temperature gradient on Rayleigh-Bénard-Marangoni convection in ferrofluids*, JMMM, 248, 379-395.  
 [13] Siddheshwar, P. G., and Abraham, A., 2003, *Effect of time-periodic boundary temperatures/body force on Rayleigh-Bénard convection in a ferromagnetic fluid*, Acta Mechanica, 161, 131-150.  
 [14] Sunil, Sharma, A., Sharma, D. and Kumar, P. 2008, *Effect of magnetic-field dependent viscosity on thermal convection in a ferromagnetic fluid*, Chem. Engg. Comm. 195, 571-583.  
 [15] Sunil, Sharma, P. and Mahajan, A. 2008, *A nonlinear stability analysis for thermoconvective magnetized ferrofluid with magnetic field dependent viscosity*, Int. Comm.Heat and Mass Transfer., 35, 1281-1287.  
 [16] Rosensweig, R. E., Kaiser, R. and Miskolczy, 1969, *Viscosity of magnetic fluid in a magnetic field*, J. Colloid and Interface Sci., 29, 680-686.  
 [17] Odenbach, S., 2004, *Recent progress in magnetic fluid research*, J Phys: Condens Matter., 16, R1135-R1150.  
 [18] Nield, D. A., 1987, *Throughflow effects in the Rayleigh-Bénard convective instability problem*, J. Fluid. Mech., 185, 353-360.  
 [19] Siddheshwar, P. G., 1995, *Suction-injection effects on Rayleigh-Bénard convection in a closely packed porous bed with general boundary condition on temperature*, VI Asian Cong. of Fluid Mech., 1, 590-594.  
 [20] Shivakumara, I. S., 1999, *Boundary and inertia effects on convection in porous media with throughflow*, Acta Mech., 137, 151-165.  
 [21] Shivakumara, I. S. and Suma, R., 2000, *Effects of throughflow and internal heat generation on the onset of convection in a fluid layer*, Acta Mech., 140, 207-217.  
 [22] Siddheshwar, P. G., and Pranesh, S., 2001, *Suction-injection effects on the onset of Rayleigh-Bénard-Marangoni convection in a fluid with suspended particles*, Acta Mechanica, 152, 241-252.  
 [23] Nield, D. A., and Kuznetsov, A. V., 2011, *Onset of convection in a porous medium with strong vertical throughflow*, Transp. Porous Med., 90, 883-888.  
 [24] Nanjundappa, C. E., Shivakumara, I. S. and Arunkumar R., 2014, *Ferroconvection in a porous medium with vertical throughflow*, Acta Mech., 226, 1515-1528.  
 [25] Vanishree, R. K., 2014, *Effects of throughflow and internal heat generation on thermoconvective instability in an anisotropic porous medium*, J.App. Fluid Mechanics, 7, 581-590.  
 [26] Siddheshwar, P. G., Ramachandramurthy, V. and Uma, D., 2011, *Rayleigh-Bénard and Marangoni magnetoconvection in Newtonian liquid with thermorheological effects*, Int. J. Engg. Sci., 49, 1078-1094.  
 [27] Sekhar, G. N. and Jayalatha, G., 2009, *Elastic effects on Rayleigh-Bénard-Marangoni convection in liquids with temperature-dependent viscosity*, Proc. of the ASME 2009, ISBN 978-07918-3863-1, 1-10.  
 [28] Sekhar, G. N. and Jayalatha, G., 2010, *Elastic effects on Rayleigh-Bénard convection in liquids with temperature-dependent viscosity*, Int. J. of Thermal Sci., 49, 67-79.  
 [29] Sekhar, G.N., Jayalatha, G. and Prakash, R., 2013, *Thermorheological and magnetorheological effects on Rayleigh-Bénard-Marangoni-Marangoni convection in ferromagnetic liquids with non-uniform basic temperature gradient*, Proc. ASME, Vol 7A, ISBN: 978-0-7918-5631-4, 1-10.  
 [30] Siddheshwar, P. G., 2004, *Thermorheological effect on magnetoconvection in weak electrically conducting fluids under  $1g$  and  $\mu g$* , Pramana, 62, 61-68.  
 [31] Siddheshwar, P. G., Sekhar, G.N. and Jayalatha, G., 2011, *Surface tension driven convection in viscoelastic liquids with thermorheological effect*, Int. Comm. Heat and Mass trans. 38, 468-473.  
 [32] Siddheshwar, P. G. Vanishree, R. K. and Melson, A. C., 2012, *Study of heat transport in Bénard-Darcy convection with g-jitter and thermomechanical anisotropy in variable viscosity liquids*, Trans-in Porous Media, 92, 277-288.  
 [33] Siddheshwar, P. G. and Maruthamanikandan, S., 2016, *Derivation of various possible boundary conditions for the ferroconvection problem*, (Private communication).  
 [34] Siddheshwar, P. G. and Revathi, B. R., 2009, *Shooting method for good estimates of the eigenvalue in the Rayleigh-Bénard-Marangoni convection Problem with general boundary conditions on velocity and temperature*,



Proc. ASME, Heat transfer, fluid flows and Thermal systems, 9, ISBN: 978-0-7918-4382-6, 1-10.

- [35] Sekhar, G.N., Jayalatha, G. and Prakash, R., 2017, *Thermal Convection in Variable Viscosity Ferromagnetic Liquids with Heat Source*, Int. J. Appl. Comput. Math, DOI: 10.1007/s40819-017-0313-9, ISSN: 2349-5103.
- [36] Platten, J. K. and Legros, J. C., 1984, *Convection in liquids*, Springer-Verlag, Berlin Heidelberg, New York.



**G. N. Sekhar** is a Professor in the Department of Mathematics, BMS College of Engineering. He has several noteworthy research papers to his credit and has been an important member of the research community in his college that implements several programs under TEQIP.

**P. G. Siddheshwar** is a Professor in the Department of Mathematics, Bangalore University. He has more than 100 noteworthy research papers to his credit on the topic of natural convection in clear fluids non-Newtonian fluids and nanofluids. He has visited a number of countries world wide on academic assignments.

**G. Jayalatha** is a Associate Professor in the Department of Mathematics, R. V. College of Engineering. She has several noteworthy research papers to her credit on the topic of natural convection in viscoelastic liquids and ferromagnetic liquids and she has been an important member of the research community in her college.

**R. Prakash** is a Assistant Professor in the Department of Mathematics, R. V. College of Engineering and Research scholar at BMS College of Engineering. He is in the final year of his Ph. D. Program. His research topic is *Convection in Variable Viscosity Ferromagnetic Liquids*.