Influence of Confined Acoustic Phonons on the Shubnikov – de Haas Magnetoresistance Oscillations in a Doped Semiconductor Superlattice

Pham Ngoc Thang, Le Thai Hung, Nguyen Quang Bau

***Abstract*—**The influence of confined acoustic phonons on the Shubnikov – de Haas magnetoresistance oscillations in a doped semiconductor superlattice (DSSL), subjected in a magnetic field, DC electric field, and a laser radiation, has been theoretically studied based on quantum kinetic equation method. The analytical expression for the magnetoresistance in a DSSL has been obtained as a function of external fields, DSSL parameters, and especially the quantum number *m* characterizing the effect of confined acoustic phonons. When *m* goes to zero, the results for bulk phonons in a DSSL could be achieved. Numerical calculations are also achieved for the *GaAs:Si/GaAs:Be* DSSL and compared with other studies. Results show that the Shubnikov – de Haas magnetoresistance oscillations amplitude decrease as the increasing of phonon confinement effect.

***Keywords*—**Shubnikov–de Haas magnetoresistance oscillations, quantum kinetic equation, confined acoustic phonons, laser radiation, doped semiconductor superlattices.

I. Introduction

THE Hall effect is the production of a voltage difference (the Hall voltage) across an electrical conductor transverse to an electric current in the conductor and a magnetic field perpendicular to the current. One of the special cases of Hall effect is called the Shubnikov–de Haas effect (SdH) which is an oscillation in the conductivity of a material that occurs at low temperatures in the presence of very intense magnetic fields. The SdH of Low Dimensional System (LDS) has been studied both in experimentally [1]-[8] and theoretically with the case of unconfined phonons [9]-[11]. In many recent researches, it shows that the confined electrons and confined phonons strongly influence strongly the nonlinear kinetic properties of LDS [12]-[14]. In this work, we study the SdH under the impact of confined acoustic phonons in the DSSL by using the quantum kinetic equation method. The article is organized as follows: in Section II, we present formulae for the calculation and compute the expression of the SdH. Numerical results and discussion for the GaAs:Si/GaAs:Be DSSL are given in Section III. And the final section shows remarks and conclusions.

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II. Analytic Expression for the SdH in DSSL under the Influence of Confined Acoustic Phonon

In order to find out the analytic expression for the SdH, the quantum kinetic equation method is used with four steps: (1) Establish the Hamiltonian of the confined electron-confined acoustic phonon system by using the wave function and its discrete energy levels of confined electron and confined phonons; (2) Establish the quantum kinetic equation for electron distribution function; (3) Solve this kinetic equation to find out the total current density; (4) Find out the analytic expression for the SdH.

We consider a simple model of the DSSL in which electron gas is confined by the superlattice potential along the z-direction and free in the (x −y) plane. A laser wave irradiates the sample in the *z* direction, the electric field of the laser wave is polarized in the *x*–*y* plane and $\vec{E}=\vec{E}_o\sin(\Omega t)$. The vector potential of the laser wave is $\vec{A}(t)=\frac{c}{\Omega}\vec{E}_0\cos(\Omega t)$. The motion of an electron is limited in each layer of the system, and its energy spectrum is quantized into discrete levels in the z-direction. Therefore, the single-particle wave function and corresponding eigen-energy in the DSSL has the same mathematical forms with those obtained for a harmonic oscillator. If the DSSL is subjected to a crossed DC electric field $\vec{E}=(0,0,E_1)$ and magnetic field $\vec{B}=(0,B,0)$. The Hamiltonian of the confined electron-confined acoustic phonon system detonated in a DSSL can be written:

$$\begin{aligned} H = & \sum_{N,n,\vec{p}_y} \varepsilon_{N,n}\left(\vec{p}_y - \frac{e}{\hbar c}\vec{A}(t)\right) a^+_{N,n,\vec{p}_y} a_{N,n,\vec{p}_y} \\ & + \sum_{m,\vec{q}_\perp} \hbar\omega_{m,\vec{q}_\perp} b^+_{m,\vec{q}_\perp} b_{m,\vec{q}_\perp} + + \sum_{\vec{q}} \varphi(\vec{q}) a^+_{N',n',\vec{p}_y+\vec{q}_y} a_{N,n,\vec{p}_y} \\ & + \sum_{\substack{N,N',n,n',\\ m,\vec{p}_y,\vec{q}_\perp}} C_{m,\vec{q}_\perp} I^m_{n,n'} J_{N,N'}(u) a^+_{N',n',\vec{p}_y+\vec{q}_y} a_{N,n,\vec{p}_y}\left(b_{m,\vec{q}_\perp} + b^+_{m,-\vec{q}_\perp}\right), \end{aligned} \quad (1)$$

Here, $a^+_{N,n,\vec{p}_y}$ and $a_{N,n,\vec{p}_y}$ ($b^+_{m,\vec{q}_\perp}$ and $b_{m,\vec{q}_\perp}$) are the creation and annihilation operators of electron (phonon), respectively; $|C_{m,\vec{q}_\perp}|^2 = \frac{\xi q}{2\rho v_B V}$ is the interactive coefficient of confined electronic – confined acoustic phonon in DSSL where v_s, ξ , ρ, and V are, respectively, the sound velocity, the acoustic deformation potential, the mass density, and the normalization volume of the specimen; $\vec{q} = (\vec{q}_\perp, \vec{q}_m)$ is the wave vector of confined phonons; $\omega_{m,\vec{q}_\perp} = \sqrt{\omega_c - v^2(q_\perp^2 + q_m^2)}$ is the frequency of confined phonons. The corresponding eigenenergy of an electron in a single potential well are given by:

$$\varepsilon_{N,n}(\vec{p}_y) = \left(N + \frac{1}{2}\right)\hbar\omega_c + \left(N + \frac{1}{2}\right)\hbar\omega_p - \hbar v_d p_y + \frac{1}{2} m_e v_d^2,$$

N=0, 1, 2, …. (2)

where m_e and e are the effective mass and the charge of a conduction electron, respectively; $\vec{p}_y$ is its wave vector in the y plane; $\omega_c = \frac{eB}{m_e}$ is the cyclotron frequency and $\omega_p = \left(\frac{4\pi e^2 n_D}{\kappa_o m_e}\right)^{\frac{1}{2}}$ is the plasma frequency characterizing for the DSSL confinement in the z-direction where κo is the electronic constant (vacuum permittivity) and n_D is the doping concentration; the frequency of confined acoustic phonons is $\omega_{m,\vec{q}\perp} = \sqrt{\omega_c - v^2(q_\perp^2 + q_m^2)}$.

The form factor of confined electrons can be written:

$$I_{n,n'}^m = \frac{1}{\sqrt{2^n n! 2^{n'} n'! \sqrt{\pi}.l}} \int_{-\infty}^{+\infty} e^{\pm i\vec{q}_\perp H_n\left(\frac{z}{l}\right)} H_n\left(\frac{z}{l}\right) H_{n'}\left(\frac{z}{l}\right) dz; \quad (3)$$

$$\left|J_{N,N'}(u)\right|^2 = \frac{N!}{N'!} e^u u^{N'-N} [L^{N'-N}(u)]^2; \; u = \frac{l^2 q_\perp^2}{2} \; l = \sqrt{\frac{\hbar}{m_e \omega_c}},$$

with $L_N^M(x)$ which is the Laguerre polynomial, l is the radius of the Landau; $\varphi(\vec{q}) = (2\pi i)^3 \left(e\vec{E}_1 + \omega_c[\vec{q}, h]\right) \frac{\partial}{\partial_q} \delta(\vec{q})$ is the potential scalar.

By using Hamiltonian (1)-(3) and above procedures, we obtain an equation for the partial current density in the single (constant) relaxation time approximation. After some manipulations, we obtain the expression for the conductivity tensor:

$$\sigma_{ip} = \frac{\tau}{1 + \omega_c^2 \tau^2} \left\{ \delta_{ik} - \omega_c \tau \varepsilon_{ijk} h_j + \omega_c^2 \tau^2 h_i h_k \right\} \times \times \left\{ a\delta_{kp} + b\left(\delta_{kp} - \omega_c \tau \varepsilon_{klp} h_l + \omega_c^2 \tau^2 h_k h_p\right) \right\} \quad (4)$$

where τ is the momentum relaxation time, δ_{ij} is the Kronecker delta, ε_{ijk} is the anti-symmetrical Levi - Civita tensor, the Latin symbols i, j, k, m, l, p stand for the components x, y, z of the Cartesian coordinates.

From (4), we find the components σ_{xx} and σ_{yx} of conductivity tensor in (5) and (6):

$$\sigma_{xx} = \frac{\tau}{1 + \omega_c^2 \tau^2} \left\{ a + b\left[1 - \omega_c^2 \tau^2\right] \right\} \quad (5)$$

$$\sigma_{xy} = -\frac{\tau}{1 + \omega_c^2 \tau^2} (a + 2b) \omega_c \tau. \quad (6)$$

with

$$b = \sum_{N',n',N,n,m} \frac{4\pi e^2}{m_e^2} \frac{\tau}{1 + \omega_c^2 \tau^2} (b_1 + b_2 + b_3 + b_4). \quad (7a)$$

$$b_1 = \gamma \frac{eB\bar{\Delta}x}{\hbar} \left(1 + 2\sum_{s=0}^{\infty} (-1)^s e^{-2\pi s\left(\frac{\Gamma}{\hbar\omega_c}\right)} \cos\left[2\pi s \bar{\varepsilon}_1\right] \right), \quad (7b)$$

$$b_2 = -\frac{\gamma\theta}{2} \left(\frac{eB\bar{\Delta}x}{\hbar}\right)^3 \left(1 + 2\sum_{s=0}^{\infty} (-1)^s e^{-2\pi s\left(\frac{\Gamma}{\hbar\omega_c}\right)} \cos\left[2\pi s \bar{\varepsilon}_1\right] \right), \quad (7c)$$

$$b_3 = -\frac{\gamma\theta}{4} \left(\frac{eB\bar{\Delta}x}{\hbar}\right)^3 \left(1 + 2\sum_{s=0}^{\infty} (-1)^s e^{-2\pi s\left(\frac{\Gamma}{\hbar\omega_c}\right)} \cos\left[2\pi s \bar{\varepsilon}_2\right] \right), \quad (7d)$$

$$b_4 = -\frac{\gamma\theta}{4} \left(\frac{eB\bar{\Delta}x}{\hbar}\right)^3 \left(1 + 2\sum_{s=0}^{\infty} (-1)^s e^{-2\pi s\left(\frac{\Gamma}{\hbar\omega_c}\right)} \cos\left[2\pi s \bar{\varepsilon}_3\right] \right). \quad (7e)$$

$$\bar{\Delta}x = \frac{eE_1}{\hbar v_d q_y}; \; \bar{\varepsilon}_1 = \frac{(n - n')\hbar\omega_p + eE_1\bar{\Delta}x}{\hbar\omega_c};$$
$$\bar{\varepsilon}_2 = \frac{(n - n')\hbar\omega_p + eE_1\bar{\Delta}x - \hbar\Omega}{\hbar\omega_c}$$
$$\bar{\varepsilon}_3 = \frac{(n - n')\hbar\omega_p + eE_1\bar{\Delta}x + \hbar\Omega}{\hbar\omega_c};$$
$$\gamma = \frac{AL_y \left|I_{n,n'}^m\right|^2 \left(\varepsilon_{N,n} - \varepsilon_F\right)}{(2\pi)^2 \beta\hbar^4 v_s v_d^2 l^2 \omega_c} \quad (7f)$$

From (5)-(7), we obtain the expression for the magnetoresistance (MR):

$$\rho_{xx} = \frac{\sigma_{yx}}{\sigma_{xx}^2 + \sigma_{yx}^2}, \quad (8)$$

Equation (8) is the analytical expression of magnetoresistance in DSSL in the presence of magnetic field, constant electric field and high frequency laser radiation. From (8), we can see that MR depends on the magnetic field B, frequency Ω, and compiler E_0 of laser radiation, temperature T of system, thickness d and specially the quantum numbers n, m characterizing the electron and phonon confinement. When m goes to zero, we obtain results as case of unconfined phonons.

III. Numerical Results and Discussions

In order to clarify the mechanism for MR in DSSL with impact of confined acoustic phonons, in this section, we will evaluate, plot, and discuss the expression of MR for the specific DSSL: GaAs: Be/GaAs: Si. Parameters used in this calculation are as follows: m* = 0.067 m_0, (m_0 is the free mass of an electron), v_s = 5370 ms^{-1}, Boltzmann constant $k_B = 1.38\times10^{-23} J.K^{-1}$, $\xi = 13.5eV$, $\rho = 5320 m.kg^{-3}$, $e = 2.07e_0$

Fig. 1 shows the nonlinear dependence of MR on the magnetic field in the case of confined acoustic phonons (solid red line and brick line) and unconfined acoustic phonon (dotted line), with $E = 10^5$V/m, $d = 20nm$, $T = 2K$, and low doping concentration $n_D = 5\times10^{22} m^{-3}$. In this figure, we find that the oscillation amplitude of MR in the case of confined acoustic phonons is much smaller than it in the case of bulk phonons. When the quantum number m characterizing confined phonons increases, the oscillation amplitude of MR reduces strongly.

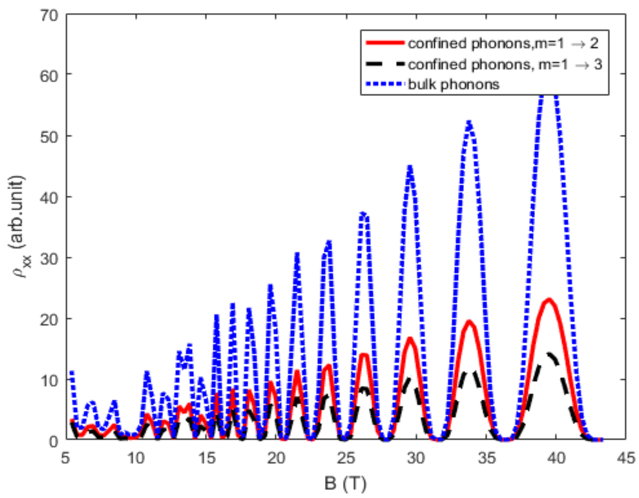

Fig. 1 Dependence of MR ρ_{xx} on magnetic field B

Fig. 2 shows that the MR is a nonlinear function of the ratio Ω/ω_c in the case of confined acoustic phonons (solid red line and brick line) and unconfined acoustic phonon (dotted line), with $B = 3T$, $E = 10^5$V/m, $d = 20nm$, $T = 2K$, and low doping concentration $n_D = 5\times10^{22} m^{-3}$. There are several resonance picks of MR in both confined phonons and bulk phonons. And we also see that when quantum number m characterizing confined phonons increases, the oscillation amplitude of MR rises strongly.

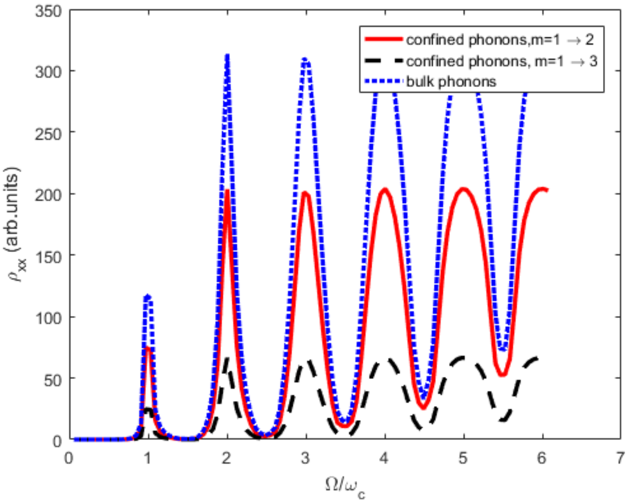

Fig. 2 Dependence of MR ρ_{xx} on Ω/ω_c

IV. Conclusion

In this paper, we have theoretically studied the Shubnikov – de Haas magnetoresistance oscillations under the influences of confined acoustic phonons by using the quantum kinetic equation method for electrons. We received the formulae of MR in the case of confined phonon, which contains a quantum number m characterizing confined phonons and easy to come back to the case of bulk phonons. We numerically calculated and graphed the MR for the specific DSSL: GaAs: Be/GaAs: Si to clarify the theoretical results. The MR depends much strongly on the quantum number m characterizing confined phonons, the magnetic field B and the ratio Ω/ω_c. There are many resonant peaks of MR in both cases. In short, the confinement of acoustic phonons in DSSL makes the MR different from case of bulk phonons.

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