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A Fuzzy Satisfactory Optimization Method Based on Stress Analysis for a Hybrid Composite Flywheel

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***Abstract*—**Considering the cost evaluation and the stress analysis, a fuzzy satisfactory optimization (FSO) method has been developed for a hybrid composite flywheel. To evaluate the cost, the cost coefficients of the flywheel components are obtained through calculating the weighted sum of the scores of the material manufacturability, the structure character, and the material price. To express the satisfactory degree of the energy, the cost, and the mass, the satisfactory functions are proposed by using the decline function and introducing a satisfactory coefficient. To imply the different significance of the objectives, the object weight coefficients are defined. Based on the stress analysis of composite material, the circumferential and radial stresses are considered into the optimization formulation. The simulations of the FSO method with different weight coefficients and storage energy density optimization (SEDO) method of a flywheel are contrasted. The analysis results show that the FSO method can satisfy different requirements of the designer and the FSO method with suitable weight coefficients can replace the SEDO method.

***Keywords*—**Flywheel energy storage, fuzzy, optimization, stress analysis.

I. INTRODUCTION

COMPARED with chemical energy storage, Flywheel Energy Storage (FES) has the advantages of no pollution, no requirement of strict environmental temperature, etc. Therefore, along with the environment protection being paid more attention, FES technology has received more and more attention within the recent years.

As the core components of a FES System (FESS), the flywheel structure is very important not only for storage capacity, but also for safety and manufacturing cost of the FESS. Since the accurate data of the cost are not available in cost optimization design, Krack et al. proposed that the cost could be normalized by the price of composite material [1], [2]. Yan designed per unit mass price of glass fiber as 1 so that other material price could be calculated by referring to the price of glass fiber [3]. However, when considering the total manufacturing process of a flywheel, it can be found that the cost of a flywheel is also affected by the structure and manufacturing process. Hence, how to evaluate the cost of a flywheel should be deeply researched.

As the structure of the flywheel decides how much kinetic energy can be stored by the whole system, many researchers used the kinetic energy as the optimization object to design a flywheel [4], [5]. However, too much mass will bring inconvenience to the assembly and transportation of the flywheel especially for the application in mobile machine. Hence, Storage energy density (SED) is used as another optimization object [6]-[8]. Along with the development of the composite material and the magnetic suspended bearing technologies, the cost of flywheel becomes an important factor affecting the application of the FESS. Hence, how to obtain big SED and low cost simultaneously is becoming more and more important. On the other hand, the safety of a high velocity flywheel is very important. In order to avoid the failure of the flywheel, Curtiss et al. researched and derived an equation which expresses the relationship between the stresses and flywheel parameters, such as the outer radii, inner radii, Poisson's ratio, maximum disk rim velocity, radial and circumferential elastic modulus, etc. [9]. Krack et al. used the equation of stresses to verify the numerical tress results, and used as a constraint for the optimization design to calculate the maximum angular velocity [1]. Ha et al. also considered the stress into the design optimization to calculate the optimum radii under the condition of the maximum angular velocity being fixed [10]. But, all the researchers did not use the stress equation for calculating the maximum angular velocity and the maximum outer radii simultaneously.

In this study, a flywheel cost evaluation method is developed. Then, to express the satisfactory degrees for different objectives reasonably, the FSO method based on fuzzy satisfactory function and the object weight coefficients is proposed. Moreover, to guarantee the safety of the high velocity flywheel, based on the analysis of the relationship between the maximum angular velocity, maximum outer radii and stresses, FSO method based on the calculation of stresses is proposed. At last, the comparison of simulation results from different optimization design methods shows the effective of the proposed method.

II. COST EVALUATION FOR FLYWHEELS

The traditional cost is calculated only according to the price of materials, it is clear that this evaluation method has great limitations. We know that although being made by same material, the flywheels with different structure have different costs. It is also obvious that some materials are easy to be machined, but some are not. Hence, we proposed that the cost of a flywheel is influenced not only by the material price, but also by the manufacturing process and the material machinability.

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To evaluate the cost more reasonably, the flywheel is divided into m components according to the material character and the flywheel structure. For every component, there are some main indicators being used to evaluate the flywheel cost, such as material manufacturability, structure character, and material price. Material manufacturability indicates that the material is easy or not to be formed and machined. Structure character means that the structure is easy or not to be manufactured. Therefore, the evaluation indicator set of the i^{th} (i=1,2,…,m) component of a flywheel is described as follows: $\psi_i=\{f_1^i, f_2^i, f_3^i\}$={material manufacturability, structure character, material price}. To evaluate the performance of each element f_j^i, the score of $x_{ij} \in [0,100]$ (i=1,2,…,m; j=1,2,3) is given to f_j^i. The higher the score for f_1^i is, the more difficulty the material manufacturing process is. On the other hand, the higher the score for f_2^i is, the more complicated the structure is. Similarly, the higher the score for f_3^i is, the more expensive the material price is.

According to the expert evaluation, the weight vector is $W=(w_1,w_2,w_3)$, $\sum_{j=1}^{3} w_j = 1$, $0<w_j<1$. Therefore, the cost coefficient of the i^{th} (i=1,2,…,m) component is:

$$C_{di} = \sum_{j=1}^{3} \frac{w_j x_{ij}}{100} \tag{1}$$

According to the cost coefficient and mass distribution, the flywheel equivalent cost can be calculated as:

$$D_r = \sum_{i=1}^{m} C_{di} \rho_i V_i(d_i, h_i) \tag{2}$$

where ρ_i is the density of the i^{th} component, d_i and h_i are the outer diameter and the height of the i^{th} component, $V_i(d_i,h_i)$ is the volume of the i^{th} component, which is the function about d_i and h_i.

III. Multi-Objective Optimization Based on FSO Method

High storage energy and low cost usually are the purpose of the flywheel optimization problem. Moreover, less mass will make the transportation more convenient and will decrease the rotational inertia which will makes the flywheel rotate more stable. Therefore, the maximum storage energy, the cost, and the mass are the optimization objects of the flywheel.

In order to express the optimization objects, $M_r(X)$, $D_r(X)$ and $E_k(X)$, respectively, are defined as the maximum kinetic energy, the equivalent cost and the mass of the flywheel. $X=(\omega, d_i, h_i)$ (i=1,2,…,m) is the decision vector, where ω is the maximum angular velocity of the flywheel, d_i and h_i are outer diameter and height of the i^{th} component.

According to the flywheel working principle, $M_r(X)$, $D_r(X)$, and $E_k(X)$ can be represented as:

$$M_r(X) = \sum_{i=1}^{m} \rho_i V_i(d_i, h_i) \tag{3}$$

$$D_r(X) = \sum_{i=1}^{m} C_{di} \rho_i V_i(d_i, h_i) \tag{4}$$

$$E_k(X) = \frac{1}{2} J \omega^2 \tag{5}$$

where J is the rotational inertia which is the function of d_i and h_i.

For different designers, their satisfactory degrees for the objective values are different. For this reason, the fuzzy satisfactory degree functions are introduced.

For multi-objective optimization problem in fuzzy environment, there are various kinds of membership functions such as linear, hyperbolic, and exponential functions [11]. To simplify the calculation and represent the desirability of designers, the triangle-like membership functions are used as the fuzzy satisfactory degree functions for the flywheel optimization objectives [12]. In order to adjust the triangle-like membership functions easily, a satisfactory coefficient of $\beta(0.5<\beta\le 1)$ is used to describe membership functions. Hence, the satisfactory degree functions of the optimization objects are represented as:

$$\mu_M(X) = \begin{cases} 1 & M_{r\min} \le M_r(X) \le (1-\beta)\ M_{r\max} + \beta M_{r\min} \\ \dfrac{M_r(X) - (1-\beta)\ M_{r\max} - \beta M_{r\min}}{\beta(M_{r\max} - M_{r\min})} & (1-\beta)\ M_{r\max} + \beta M_{r\min} < M_r(X) \le M_{r\max} \end{cases} \tag{6}$$

$$\mu_D(X) = \begin{cases} 1 & D_{r\min} \le D_r(X) \le (1-\beta)\ D_{r\max} + \beta D_{r\min} \\ \dfrac{D_r(X) - (1-\beta)\ D_{r\max} - \beta D_{r\min}}{\beta(D_{r\max} - D_{r\min})} & (1-\beta)\ D_{r\max} + \beta D_{r\min} < D_r(X) \le D_{r\max} \end{cases} \tag{7}$$

$$\mu_E(X)=\begin{cases}\dfrac{E_k(X)\text{-}E_{k\min}}{\beta(E_{k\max}\text{-}E_{k\min})} & E_{k\min}\le E_k(X)\le \beta E_{k\max}+(1-\beta)\ E_{k\min}\\ 1 & \beta E_{k\max}+(1-\beta)\ E_{k\min}<E_k(X)\le E_{k\max}\end{cases} \tag{8}$$

where $M_{r\max}$ and $M_{r\min}$, $D_{r\max}$ and $D_{r\min}$, $E_{k\max}$ and $E_{k\min}$ are upper and lower bounds of $M_r(X)$, $D_r(X)$ and $E_k(X)$, respectively. As an example, the fuzzy satisfactory degree function of flywheel mass is shown in Fig. 1.

To realize the multi-objective optimization along with the different significance of the objectives, the weight coefficients of ε ($0<\varepsilon\le1$) and γ ($0<\varepsilon+\varepsilon\gamma\le1$) are defined. ε is the coefficient of the energy significance, and γ is the coefficient of the cost significance relative to the energy. Accordingly, the multi-objective optimization formulation based on the satisfactory function is represented as follows:

$$\begin{aligned}&\text{Maximize:} && \varepsilon[\mu_E(X)+\gamma\mu_D(X)]+[1-(1+\gamma)\varepsilon]\mu_M(X)\\ &&& X_{\min}\le X\le X_{\max}\\ &\text{Subject To:} && 0<\varepsilon<1\\ &&& 0<\varepsilon(1+\gamma)\le1\end{aligned} \tag{9}$$

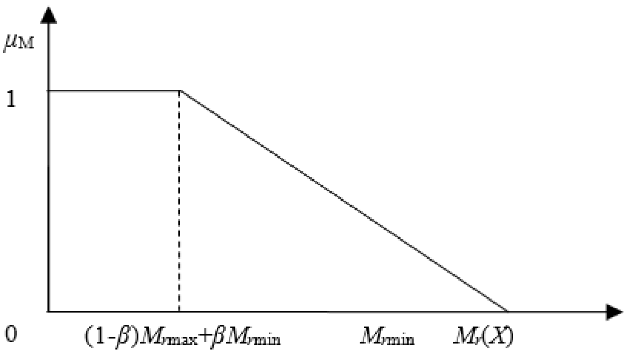

Fig. 1 Fuzzy satisfactory membership function of mass

IV. Stress Analysis for flywheel Optimization Design

Compared with metal material, the composite material has the characteristics of light weight and high strength, which is the preferred material for the high velocity flywheel.

As the safety of the flywheel is very important, the influence of the composite material stress should be fully considered. Otherwise, the flywheel failure will result in disastrous consequences.

Since the stresses on the outer ring have the greatest influence on the flywheel safety, the relationship between the stress and the material parameters and the size of this component is analyzed.

The outer ring is a kind of axisymmetric constant thickness spinning disk, so the radial stress σ_r and circumferential stress σ_θ are represented as follows [9]:

$$\begin{aligned}\sigma_\theta=&\frac{3+\nu_{\theta_r}}{9-\lambda^2}\lambda^2\rho v_e^2\Big(\frac{1+\nu_{\theta_r}/\lambda}{\lambda+\nu_{\theta_r}}\frac{R_o^{-\lambda-1}R_i^2/R_o^2-R_i^{-\lambda-1}}{R_i^{\lambda-1}R_o^{-\lambda-1}-R_o^{\lambda-1}R_i^{-\lambda-1}}r^{\lambda-1}\\&+\frac{1-\nu_{\theta_r}/\lambda}{\nu_{\theta_r}-\lambda}\frac{R_i^{\lambda-1}-R_o^{\lambda-1}R_i^2/R_o^2}{R_i^{\lambda-1}R_o^{-\lambda-1}-R_o^{\lambda-1}R_i^{-\lambda-1}}r^{-\lambda-1}-\frac{1+3\nu_{\theta_r}/\lambda^2}{3+\nu_{\theta_r}}\frac{r^2}{R_o^2}\Big)\end{aligned} \tag{10}$$

$$\begin{aligned}\sigma_r=&\frac{3+\nu_{\theta_r}}{9-\lambda^2}\rho v_e^2\Big(\frac{R_o^{-\lambda-1}R_i^2/R_o^2-R_i^{-\lambda-1}}{R_i^{\lambda-1}R_o^{-\lambda-1}-R_o^{\lambda-1}R_i^{-\lambda-1}}r^{\lambda-1}\\&+\frac{R_i^{\lambda-1}-R_o^{\lambda-1}R_i^2/R_o^2}{R_i^{\lambda-1}R_o^{-\lambda-1}-R_o^{\lambda-1}R_i^{-\lambda-1}}r^{-\lambda-1}-\frac{r^2}{R_o^2}\Big)\end{aligned} \tag{11}$$

where ρ is the material density, v_{θ_r} is the Poisson's ratio of contraction in radial direction due to circumferential θ tension, v_e is the maximum disk rim velocity, R_i and R_o are the disk inner and outer radii, $\lambda=\sqrt{E_0/E_r}$, E_0 and E_r are radial elastic moduli, and circumferential elastic modulus, r ($R_i\le r\le R_o$) is the radius of select point.

Defining $\alpha=R_i/R_o$ and $x=r/R_o$, the σ_θ and σ_r can be derived as:

$$\sigma_\theta=\rho v_e^2(c_1\lambda x^{\lambda-1}-c_2\lambda x^{-\lambda-1}-c_3x^2) \tag{12}$$

$$\sigma_r=\rho v_e^2(c_1x^{\lambda-1}+c_2x^{-\lambda-1}-c_4x^2) \tag{13}$$

where

$$c_1=\frac{3+v_{\theta_r}}{9-\lambda^2}\frac{\alpha^{3+\lambda}-1}{\alpha^{2\lambda}-1},\ c_2=\frac{3+v_{\theta_r}}{9-\lambda^2}\frac{\alpha^{2\lambda}-\alpha^{3+\lambda}}{\alpha^{2\lambda}-1},$$

$$c_3=\frac{\lambda^2+3v_{\theta_r}}{9-\lambda^2},\ c_4=\frac{3+v_{\theta_r}}{9-\lambda^2}.$$

Assume that $\sigma_{\theta in}$ and σ_{rin} are the circumferential stress coefficient and radial stress coefficient, respectively. As $v_e=\omega R_o$ and the stresses should be less than the allowable stresses, from (12) and (13), the following formulas are derived as:

$$\omega R_o\le\sqrt{[\sigma_\theta]/\rho\sigma_{\theta in}} \tag{14}$$

$$\omega R_o\le\sqrt{[\sigma_r]/\rho\sigma_{rin}} \tag{15}$$

where

$$\sigma_{\theta in} = c_1 \lambda x^{\lambda-1} - c_2 \lambda x^{-\lambda-1} - c_3 x^2$$

$$\sigma_{rin} = c_1 x^{\lambda-1} + c_2 x^{-\lambda-1} - c_4 x^2$$

The maximum of $\sigma_{\theta in}$ and σ_{rin} can be calculated for a kind of composite material. Therefore, for a fixed value of α, the multi-objective optimization formulation considering stresses constraints is represented as follows:

$$\begin{array}{ll} \text{Maximize:} & \varepsilon[\mu_E(X) + \gamma\mu_D(X)] + [1-(1+\gamma)\varepsilon]\mu_M(X) \\ & X_{\min} \le X \le X_{\max} \\ & 0 < \varepsilon < 1 \\ \text{Subject To:} & 0 < \varepsilon(1+\gamma) \le 1 \\ & \omega R_o \le \min\{\sqrt{[\sigma_\theta]/\rho\sigma_{\theta in\max}}, \sqrt{[\sigma_r]/\rho\sigma_{rin\max}}\} \\ & R_i/R_o = \alpha \end{array} \quad (16)$$

where $X_{\min}$ and $X_{\max}$ are the minimum value and maximum value of X, $\sigma_{\theta in\max}$ and $\sigma_{rin\max}$ are the maximum of σ_{rin} and $\sigma_{\theta in}$, respectively.

V. Comparison of Simulation Results

In order to analyze the presented multi-objective FSO, a composite material flywheel structure is designed which is shown in Fig. 2.

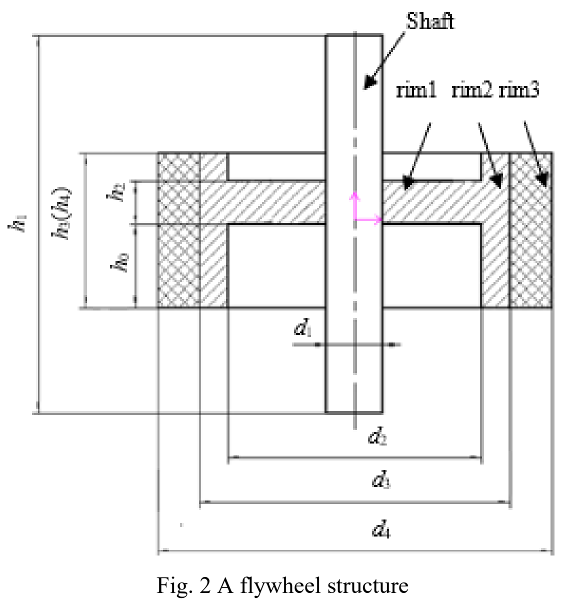

Fig. 2 A flywheel structure

According to the material and the structure, the flywheel is divided into four components: shaft, rim1, rim2, and rim3. The material of shaft is steel. The material of rim1 and rim2 is aluminium alloy. The material of rim3 is carbon fiber. Because outer ring of this flywheel is rim3, there are $R_i = d_3/2$, $R_o = d_4/2$. The variables include diameters of the four components d_1, d_2, d_3, d_4 (m) and the maximum angular velocity ω (rad/s). Setting $\alpha = 0.5$, there are $X = (\omega, d_1, d_2, d_4)$. According to the design requirement, there are $X_{\min} = (200, 0.06, 0.15, 0.6)$ and $X_{\max} = (2000, 0.08, 0.25, 0.8)$. Other structure parameters of the flywheel are $h_1 = 0.5$ m, $h_2 = 0.1$ m, $h_3 = h_4 = 0.3$ m. The densities of steel, aluminium alloy and carbon fiber are 7800 kg/m^3, 2700 kg/ m^3, and 1780 kg/ m^3, respectively.

For material manufacturability, the manufacturing processes of the steel and aluminium alloy are obviously different from the process of the carbon fiber. Because the aluminum alloy material belongs to the non-ferrous metal, the hardness is inferior to the steel, and the processing difficulty must be smaller than the steel. Due to the high requirement of the winding technology of carbon fiber flywheel, the technology maturity is not as good as the metal manufacturing process. Therefore, the material manufacturability scores of the four components are:

$$x_{11} = 30,\ x_{21} = 20,\ x_{31} = 20,\ x_{41} = 80$$

On the other hand, the structure character of the shaft is relatively complex. It not only requires strict dimensional accuracy, rigorous surface roughness, and surface hardness but also requires high shock resistance. On contrast, the structure character of rim1 and rim2 mainly reflects in the complex shape. Additionally, the demands on dimensional accuracy and surface roughness of the inner hole surface of rim1 are more rigorous than the outer circle surface of rim2. The rim3 is winded by carbon fiber which has regularly shape and lower dimensional accuracy. Hence, the score of structure character of rim3 is lowest in the scores of the four components. Therefore, the score of the structure character of the four components are:

$$x_{12} = 90,\ x_{22} = 50,\ x_{32} = 40,\ x_{42} = 10$$

Based on the market prices of steel, aluminium alloy and carbon fiber, the scores of the material price are:

$$x_{13} = 5,\ x_{23} = 12,\ x_{33} = 12,\ x_{43} = 100$$

According to the importance degree, the weight set of the evaluation indicators is: *W*= (0.3, 0.2, 0.5).

The cost coefficient of the i^{th} (i=1,2,…,m) component is calculated respectively as:

$$C_{d1} = 0.3\times\frac{30}{100} + 0.2\times\frac{90}{100} + 0.5\times\frac{5}{100} = 0.295$$

$$C_{d2} = 0.3\times\frac{20}{100} + 0.2\times\frac{50}{100} + 0.5\times\frac{12}{100} = 0.22$$

$$C_{d3} = 0.3\times\frac{20}{100} + 0.2\times\frac{40}{100} + 0.5\times\frac{12}{100} = 0.20$$

$$C_{d4} = 0.3\times\frac{80}{100} + 0.2\times\frac{10}{100} + 0.5\times\frac{100}{100} = 0.76$$

According to the mass distribution of each component, the equivalent cost of the flywheel is:

$$D_r = (2.3V_1 + 0.59V_2 + 0.54V_3 + 1.35V_4)\times 10^3$$

According to the (3), (4), (5), the $M_r(X)$, $D_r(X)$ and $E_k(X)$ can be represented as follows:

$$E_k(X) = \frac{\pi\times 10^3}{4}\omega^2(3.63d_1^4 - 0.54d_2^4 + 0.28d_3^4 + 0.53d_4^4)$$

$$D_r(X) = \frac{\pi\times 10^3}{4}(1.091d_1^2 - 0.1d_2^2 - 0.24d_3^2 + 0.4d_4^2)$$

$$M_r(X) = \frac{\pi\times 10^3}{4}(3.63d_1^2 - 0.54d_2^2 + 0.28d_3^2 + 0.54d_4^2)$$

Hence, when the β=0.8, the fuzzy satisfactory degree functions are as follows:

$$\mu_E(X) = \begin{cases} \dfrac{E_k(X)\times 10^{-6} - 3.2}{821.2} & 3.2 \le E_k(X)\times 10^{-6} \le 824.4 \\ 1 & 824.4 < E_k(X)\times 10^{-6} \le 1042 \end{cases}$$

$$\mu_D(X) = \begin{cases} 1 & 95.3 \le D_r(X) \le 113.5 \\ 1 - \dfrac{D_r(X) - 113.5}{63.1} & 113.5 < D_r(X) \le 176.6 \end{cases}$$

$$\mu_M(X) = \begin{cases} 1 & 212.8 \le M_r(X) \le 252.6 \\ 1 - \dfrac{M_r(X) - 252.6}{163.2} & 252.6 < M_r(X) \le 415.8 \end{cases}$$

when β=1, the fuzzy satisfactory degree functions are as follows:

$$\mu_E(X) = \frac{E_k(X)\times 10^{-6} - 3.2}{1039.8} \quad 3.2 \le E_k(X)\times 10^{-6} \le 1042$$

$$\mu_D(X) = 1 - \frac{D_r(X) - 95.3}{81.3} \quad 95.3 < D_r(X) \le 176.6$$

$$\mu_M(X) = 1 - \frac{M_r(X) - 212.8}{203} \quad 212.8 < M_r(X) \le 415.8$$

The parameters of the carbon fiber are v_{θ_r}=0.3, $E_0 = 203.0$ GPa, $E_r = 56$ GPa, $\rho = 1780$ kg/m^3 [13]. Hence, λ can be derived as:

$$\lambda = \sqrt{E_0/E_r} = \sqrt{203/11.2} = 4.257$$

According to (12) and (13), the values of c_1, c_2, c_3, and c_4 can be calculated as follows:

$$c_1 = \text{-}0.36,\ c_2 = \text{-}0.0014,\ c_3 = \text{-}2.085,\ c_4 = -0.362$$

Hence, using the MATLAB optimization tool box, $\sigma_{\theta inamx}$ and $\sigma_{rin\max}$ are calculated as:

$$\sigma_{\theta inamx} = 0.617,\ \sigma_{rin\max} = 0.056.$$

Since the longitudinal and transverse tensile strengths of carbon fiber are $[\sigma_\theta]$=3500MPa and $[\sigma_r]$=56MPa [11], we can obtain:

$$\sqrt{[\sigma_\theta]/(\rho\sigma_{\theta in\max})} = 1786,\ \sqrt{[\sigma_r]/(\rho\sigma_{rin\max})} = 749.5$$

Therefore, from (16) the representation of the optimization formula is rewritten as follows:

$$\begin{array}{ll} \text{Maximize:} & \varepsilon[\mu_E(X) + \gamma\mu_D(X)] + (1-\varepsilon-\varepsilon\gamma)\mu_M(X) \\ & X_{\min} \le X \le X_{\max} \\ & 0 < \varepsilon < 1 \\ \text{Subject To:} & 0 < \varepsilon(1+\gamma) \le 1 \\ & d_4\omega/2 \le 749.5 \\ & d_3/d_4 = 1/2 \end{array} \tag{17}$$

A. Comparison of FSO with Different β, ε and γ

To verify the FSO method, the optimum results of different β, ε, and γ are listed in Table I.

TABLE I
THE OPTIMUM RESULTS OF FSO WITH DIFFERENT COEFFICIENTS

Indicators	ε =0.8, γ=0.2		ε=0.3, γ=1		ε=0.3, γ=2	
	β=0.8	β=1	β=0.8	β=1	β=0.8	β=1
ω (rad/s)	1949	1899	2000	2000	2000	2000
d_1(mm)	60	60	60	60	60	60
d_2(mm)	250	250	194.4	250	250	250
D_4(mm)	769	789	648.1	600	649.9	600
E_k(MJ)	839	886	446.6	323	447	323
D_r	158	167	113.5	95.2	112.8	95.2
M_r(kg)	359	381	261.4	213	252	213
μ_E	1.0	0.80	0.54	0.31	0.54	0.31
μ_D	0.153	0.23	0.995	1	1	1
μ_M	0.21	0.27	0.946	1	0.94	1

In order to analyze the optimization effect, the values of the object which has the maximum weight among the three objects are compared. In the case of ε =0.8 and γ=0.2, the optimum value of energy of β=1 is 5.6% higher than the value of β=0.8. In the case of ε =0.3 and γ=1, the optimum value of mass of β=1 is 18.5% lower than the value of β=0.8. Moreover, in the case of ε =0.3 and γ=2, the optimum value of cost of β=1 is 15.6% lower than the value of β=0.8. All these show that when β=1, the design requirements of the satisfaction degree are more stringent. On the other hand, in Table I, when ε=0.3 and β=1, the optimum results with different γ are same. But when ε =0.3 and β=0.8, the optimum cost of γ=1 is 0.62% bigger than the cost of γ=2, and μ_D of γ=1 is 0.5% smaller than μ_D of γ=2. It is obvious that the importance of cost increases with the increase of β.

B. Comparison of FSO with and without the Stress Constraints

When β=1, the optimum results of FSO with or without the stress constraints are listed in Table II.

Considering the stress constraints, the values of $\omega d_4/2$ under different ε and γ is smaller than the allowable value of 749.5 rad·m/s, as shown in (17). On contrast, the optimum results of $\omega d_4/2$ without stress constraints are all bigger than 749.5 rad·m/s. It is obvious that the optimization with the stress constraints can guarantee the safety of the flywheel.

TABLE II
THE OPTIMUM RESULTS WITH OR WITHOUT STRESS CONSTRAINTS

Indicators	ε =0.8, γ=0.2		ε =0.6, γ=0.5	
	With	*without*	*with*	*without*
ω(rad/s)	1899	2000	2000	2000
d_1(mm)	60	60	60	60
d_2(mm)	250	208	250	250
d_4(mm)	789	800	749.4	800
E_k(MJ)	886	1039	796	1036
D_r	167	172.5	149	171
M_r(kg)	381	399	341	391
$\omega d_4/2$(rad·m/s)	749.2	800	749.4	800

C. Simulation Contrast of FSO and SEDO

In order to provide further insight on the FSO, the optimum results of FSO of β=1 and SEDO with same constraints are listed in Table III.

TABLE III
THE OPTIMUM RESULTS OF FSO AND SEDO.

Indicators	FSO		SEDO
	ε =0.8, γ=0.2	***ε =0.5, γ=0.5***	
ω(rad/s)	1899	2000	2000
d_1(mm)	60	60	60
d_2(mm)	250	250	250
d_4(mm)	789	749.5	748.3
E_k(MJ)	886	796.5	791
D_r	167	149.8	149
M_r(kg)	381	341.1	340
SED(MJ/kg)	2.325	2.335	2.327

For the optimum E_k and M_r of FSO, the SED of ε =0.8 and γ=0.2 is 0.08% smaller than that of SEDO. But, on the contrary, the SED of ε =0.5 and γ=0.5 is 0.34% bigger than that of SEDO. It can be seen that the optimum ESD can be acquired by the FSO method when the values of the weight coefficients are suitable.

VI. CONCLUSION

An equivalent cost evaluation method was proposed. To evaluate the cost more reasonably, the flywheel is divided into several components according to the material character and the flywheel structure. For every component, material manufacturability, structure character and material price were considered as the main indicators evaluating the flywheel cost.

Based on introducing the fuzzy satisfactory degree functions, the energy coefficient and the cost coefficient, the FSO method was proposed. The triangle-like membership functions are used as the fuzzy satisfactory degree functions for the flywheel optimization objectives. A satisfactory coefficient was used to adjust the triangle-like membership functions.

On the basis of the stress analysis, the relativity between the outer diameter, maximum velocity and the flywheel stresses were calculated. Using the maximum of the circumferential stress and radial stress as the constraints, the FSO method considering the stresses constraints was proposed.

To verify the effect of FSO, the FSO methods with different satisfactory coefficient and weight coefficients were simulated. The result analysis demonstrated that the FSO could realize the different design target. The contrast of FSO simulation with/without stress constraint showed that only the calculated radial stress with stress constraint was smaller than the allowance radial stress. Additionally, the contrast of SEDO and the FSO indicated that the FSO could reach a higher SED when the values of parameters are suitable.

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