Optimization of Shear Frame Structures Applying Various Forms of Wavelet Transforms

Seyed Sadegh Naseralavi, Sohrab Nemati, Ehsan Khojastehfar, Sadegh Balaghi

***Abstract*—**In the present research, various formulations of wavelet transform are applied on acceleration time history of earthquake. The mentioned transforms decompose the strong ground motion into low and high frequency parts. Since the high frequency portion of strong ground motion has a minor effect on dynamic response of structures, the structure is excited by low frequency part. Consequently, the seismic response of structure is predicted consuming one half of computational time, comparing with conventional time history analysis. Towards reducing the computational effort needed in seismic optimization of structure, seismic optimization of a shear frame structure is conducted by applying various forms of mentioned transformation through genetic algorithm.

***Keywords*—**Time history analysis, wavelet transform, optimization, earthquake.

I. Introduction

WAVELATE transform has proved many advantageous applications in dynamic analyses and optimization under the earthquake load [1], [2]. The concept of wavelet originates from the works of Fourier, a French mathematician in the last decade of the 18th century, who established a method by frequency analysis that we know as analysis Fourier today.

Next step in accentuating the present concept of wavelet was taken by Alfred Haar, a Hungarian mathematician. The Haar wavelets have the simplest form among the wavelet families, and understanding the concept of wavelets is much simpler by using them. The other researcher working on wavelets is Ingrid Daubechies. Around 1988, Daubechies used multiple analysis idea for foundation of a family of wavelets. These wavelets, which have been named as Daubechies, possess more optimum features such as compression, orthogonality, regularity, and continuity [3].

Actually, wavelet transforms break the earthquake wave into two frequencies: high and low. High frequency does not have a significant impact on the structure reaction. Hence, the structure would be affected by the low frequency part. This will reduce the earthquake points to a half, and consequently, analysis time will decrease significantly. Salajegheh and Heidari investigated structures under the earthquake load through wavelet transforms [4], [5]. They also did structure optimization under earthquake load using wavelet transforms [6], [7].

In this study, earthquake wave was filtered by Haar, db2, and db3 wavelet transforms and was used in the assessment of the optimization process. Genetic algorithm has been also utilized in the optimization task. A five-storey shear frame structure has been selected for the optimization process.

II. Wavelet Transforms

When the researchers realized that Fourier transform is not sufficient for all the requirements in the frequency analyses, wavelet transform was emerged. Wavelet transforms and its applications are widely used, so that it is known as one of the most promising techniques in the past few decades [8]. The main drawback of this method has been resolved to an extent by short-time Fourier transform (STFT), and time of each frequency is also detectable by this method. But, its efficiency would encounter with problems when dealing with unsteady waves. Unsteady waves are those that have various frequencies in different moments, of which category earthquake accelerograms would be included in [9]. In approximate dynamic analysis, it would be possible to reduce time and volume of the operations using wavelet theory and by decreasing the number of accelerogram points. Generally, wavelet transform is divided into two categories, and in this article, we use discrete wavelet transform. This transform is defined as:

$$DWT_a^{\Psi}(k,j) = \frac{1}{\sqrt{s_0^j}} \int a(t) \Psi\left(\frac{(t - k s_0^j \tau_0)}{s_0^j}\right) dt \qquad (1)$$

In (1), variables are S and τ. τ is the transform, and S is the scale and is inversely related with the frequency. Ψ(t) represents wavelet function or the mother wave. Two values S and τ are defined as Sj0 and kSj0 τ0 in discrete wavelet transforms. In discrete wavelet transforms, S and τ change rates are slow. This is not important for earthquake waves. This case is amendable using down sampling, i.e. calculations would be reduced significantly. However, inverse transform of the above equation is presented by (2) [10]:

$$a(t) = c_{\Psi} \sum \sum DWT_a^{\Psi}(k,j) \Psi_{k,j}(t) \qquad (2)$$

Based on S value (High or Low), DWT transform divides a wave into two parts:

1- For High Frequencies (low scale), approximate wave, CAi is obtained.

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2- For low frequencies (high scale), details wave CDi is obtained.

Sampling process is so that, between each two points, one point is picked as effective data, and this process is called Down Sampling. So, in the first stage, data are divided into two parts of CDi and CAi. In the next stage, operator will be applied again from CAi, and data sampling is done in the case where the number of data is halved compared to its previous state. According to Fig. 1, earthquake waves breaking process could be performed in many repeats theoretically, but this task is limited in practice because, in each stage, a number of effective data would be lost and performing the analysis with the remaining data will increase the possibility of error [11].

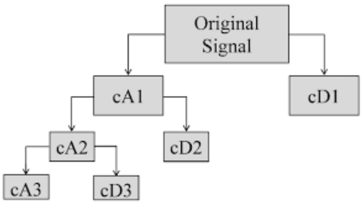

Fig. 1 Schematic illustration of wavelet transforms

In many waves, their low frequency part is the most important one. Furthermore, high frequencies show only wave details. In fact, mechanism of wave breaking is like filtering a wave which has been totally shown in Fig. 2 [12].

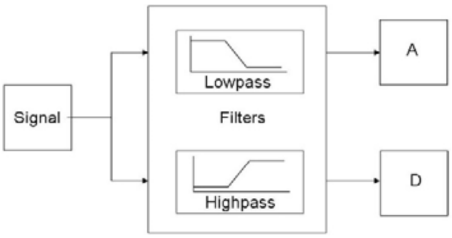

Fig. 2 Filtering wavelet

Wavelets used in this study include famous wavelets such as db2 and db3. Haar wavelet is defined as:

$$\Psi(t) = \begin{Bmatrix} 10 \le t \le 0.5 \\ -1\ 0.5 \le t \le 1 \\ 0 other \end{Bmatrix} \quad (3)$$

Daubechies wavelet is a dbn which is an asymmetrical wavelet and does not have an accurate mathematic formula. Its description has been presented below [13]:

Compared with Fourier transform, wavelet possesses very suitable localization characteristics. For example, Fourier transform is a sharp peak with many coefficients; it is because base functions of the Fourier transform are sinusoidal and cosine functions of which domain is constant across the span. But, wavelet functions are those where most energy has been centralized in a limited span and would be damped rapidly. As a consequence, appropriate selection of the mother wavelets will result in better compression than the Fourier transform. In this paper, even and odd points of the earthquake have been extracted, and each one has been introduced to the structure, separately.

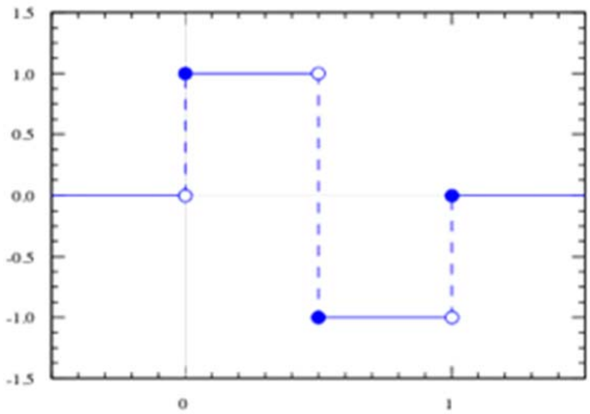

Fig. 3 Haar wavelet

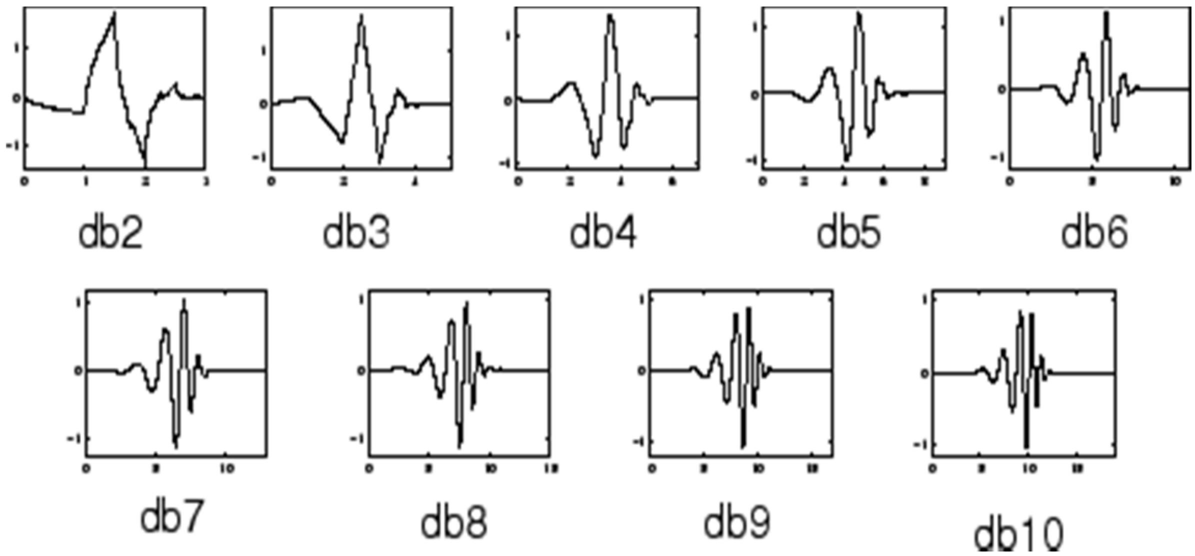

Fig. 4 Daubechies wavelet

Wavelets have many applications now in different fields such as images and signals processing, computerized pictures, information compression, and many other instances.

III. Genetic Algorithm

Genetic algorithm consists of five main steps as follows:

1) Initialization: In this part, a set of primitive solutions as initial population is randomly created in the feasible region of the search space.
2) Fitness evaluation: In this part, first the individuals of the current population are decoded and then the objective function for the individuals is calculated.
3) Selection: Herein, a set of individuals of the current population is copied and stored in the math pool for reproduction process. The individuals possessing better fitness are copied more in the math pool.
4) Reproduction: The reproduction process simulates the biological creation of a new generation. Reproduction process consists of two main operators: 1) Cross over and 2) Mutation. Cross over is simulating marriage and generation of offspring by combining two individuals (chromosomes). Mutation is an additional operator executed after cross over in some of the individuals. A certain number of genes are randomly selected to be changed. This operator prevents trapping of the solution in local minimums. In addition, the best solution of each generation is copied to the next generation in order to insure improvement of the best individual generation by generation.
5) Termination: The algorithm is stopped if the maximum number of generations is met, or if for some sequential generations the best solution does not improve.

IV. Numerical Example

For description, a five-storey sheared frame has been used for optimization through two dynamic time-story analysis and analysis by wavelet. Fig. 4 shows the structure:

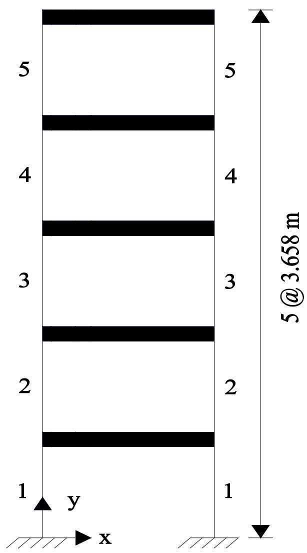

Fig. 5 A five-storey shear building

Considering the elements grouping shown in Fig. 5, box sections listed in Fig. 1 have been used for its parts.

TABLE I
Available Profiles for Shear Frames

No.	Profile
1	Box 180*180*16
2	Box 220*220*17.5
3	Box 240*240*20
4	Box 260*260*20
5	Box 280*280*20
6	Box 300*300*20
7	Box 320*320*20
8	Box 340*340*20

Used steel has the elasticity module of 2.1*10^6 N/cm^2 and a density as 7850 kg/m^3. A centralized load of 221 tones has been included for each storey. Structure damping is 0.05, and it has been analyzed under first 15 seconds of the earthquake record (Elcentro) in 0.02 s intervals (see Fig. 6).

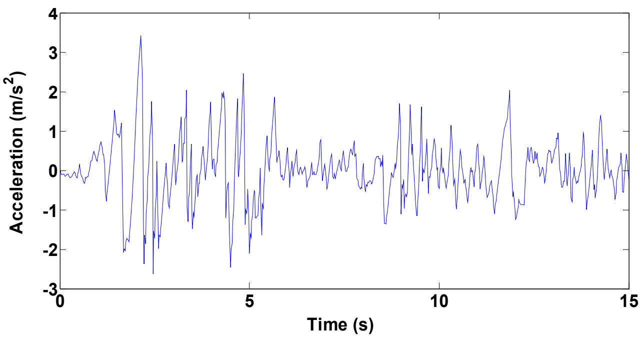

Fig. 6 Earthquake graph Elcentro

In the genetic algorithm, the primary population is 50, and the percentages of insertion and the mutation are 80 and 10, respectively.

Results of optimization under strict analysis and time-history analysis filtered by different wavelets have been presented in Table II. As can be seen in Table II, structure assessment by db3 wavelet leads to the most optimized state.

TABLE II
Optimum Design of Shear Frame Structures by Genetic Algorithm

Elements group no.	Optimum design			
	Exact	haar	Db2	Db3
1	8	8	5	8
2	5	7	5	5
3	4	3	4	4
4	2	2	2	2
5	2	2	2	2
Max. displacement (m)	0.1049	0.1048	0.1038	0.1049
Weight (kg)	5395.6	5487.5	5408.5	5395.6
Time (min)	6.00	2.30	2.32	2.26

Table III shows maximum error of optimum structure displacement through each wavelet analysis. It can be seen from Table III that db3 wavelet transform includes less errors.

TABLE III
ERROR PERCENT MAXIMUM DISPLACEMENT OPTIMIZED STRUCTURES OBTAINED BY DIFFERENT WAVELET TRANSFORMS

Wavelet	Error (%)
Haar	1.282
Db2	0.913
Db3	0.375

V. CONCLUSION

Effect of wavelet transform with different wavelets on the optimization of the structures was investigated in this paper. Results indicate that the run time has been shortened, and it can be seen that for wavelet transform, the run time is much faster than the frequent time-history analysis method which is much desired by engineers for optimization of big structures. This study also suggests that db3 wavelet transform gives better results than the classic method of Haar and db2 wavelet.

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