The Magnetized Quantum Breathing in Cylindrical Dusty Plasma

A. Abdikian

Abstract**—**A quantum breathing mode has been theatrically studied in quantum dusty plasma. By using linear quantum hydrodynamic model, not only the quantum dispersion relation of rotation mode but also void structure has been derived in the presence of an external magnetic field. Although the phase velocity of the magnetized quantum breathing mode is greater than that of unmagnetized quantum breathing mode, attenuation of the magnetized quantum breathing mode along radial distance seems to be slower than that of unmagnetized quantum breathing mode. Clearly, drawing the quantum breathing mode in the presence and absence of a magnetic field, we found that the magnetic field alters the distribution of dust particles and changes the radial and azimuthal velocities around the axis. Because the magnetic field rotates the dust particles and collects them, it could compensate the void structure.

Keywords**—**The linear quantum hydrodynamic model, the magnetized quantum breathing mode, the quantum dispersion relation of rotation mode, void structure.

I. Introduction

When the tiny dust particulates of carbon, silicon, etc. from nano to micron sizes enter into the plasma, dusty plasma is formed [1]. In recent years, extensive researches in theoretical [2], [3] and laboratory [4], [5] fields have been done on the dusty plasma due to the wide range of its applications in different environments. Classical plasma is associated with high temperatures and the low density of particle number [6]. The effects of quantum mechanics will appear when the plasma is extremely cooled and the density of particle number is high so that the de Broglie wavelengths of the charge carriers could be comparable to the dimensions of the system. Such system is called the quantum plasma [7]. The dynamics of quantum plasma could be investigated by using the Poisson-Schrödinger model, the Wigner-Poisson model and the Quantum Hydrodynamic Equations (QHD) [8]-[10]. Quantum effects play a crucial role in the manufacture of microelectronic devices and in the study of dense astrophysical systems and plasma lasers [11]. The reason for the importance of these studies is that these particles can sometimes be useful (for example, in producing solar cells), and sometimes harmful (as they become contaminants in reactors or as they decrease the efficiency of surface processes with the growth of micro-particles). Therefore, controlling such particles in the laboratory seems to be an important issue in improving the quality of semiconductors or computer microchips etc. [1]. The QHD model is widely used in studying quantum effects in dusty plasma. For instance, in recent years, by using this model, Shukla et al. derived the linear dispersion relation of dust quantum acoustic waves and then studied the soliton solutions in the form of very cold dust plasma [11]. In addition, many researchers have investigated wave propagation in single and double wall carbon nanotubes by using QHD model [12], [13]. On the other hand, the collective excitations of the quantum dusty plasma have been investigated [14], [15]. Excitation of the quantum breathing mode has been reported in the laboratory [14], [16]. It has been shown that the quantum breathing mode can be created in a system of particles trapped in a harmonic potential whose particles interact with each other in the form of either $1/r^{\gamma}$ ($\gamma \in \Re_{\neq 0}$) or $\log r$ [17], [18]. In this way, when this mode is propagated towards the outside, the boundary conditions that can be a circular potential barrier cause the wave to return. Hence, it can produce propagation like a uniform expansion and contraction along the radial. The propagation of this mode causes the distribution of the particle density to be in the radial direction. Because of the quantum breathing mode, a void is formed in the center of vibration. The creation of void has been reported experimentally. Firstly, during growth of carbon nanoparticles on direct electrical discharge, Arnas et al. [19] found that the void would be formed. Then, Kumar et al. [20] noticed that the void is created in the process of growth of tungsten particles. Investigating these types of modes is important in the production of experimental and laboratory instruments. These modes disturb the distribution of plasma particles and disrupt the measurement parameters. And because a void is created in the plasma center, it can facilitate the removal of micro particles (impurities) of the production process. Therefore, studying these modes is useful for both scientific researches and for plasma technology applications.

In this paper, it has been theoretically attempted to study the quantum breathing mode by using a set of QHD equations in the presence of a static and constant magnetic field. The structure of this paper is as follows: in Section II, QHD model is used for an electron fluid in the presence of a magnetic field to obtain the dispersion relation of rotation modes. Section III is devoted to the numerical analysis of the dispersion relation. The last section is the conclusion.

II. Fundamental Equation

We assume that dusty plasma includes the dust particles with mass M and negative charges $-Ze$ (Z is integer) confined in a cylindrical geometry which are subjected to a constant and axial magnetic field $\vec{\mathbf{B}}_0 = B_0\hat{\mathbf{z}}$. It is also assumed that the charge of the electron is $-e$ and the ion

A. Abdikian is with the Department of Physics, Malayer University, Malayer 65719-95863, Iran (e-mail: abdykian@malayeru.ac.ir).

charge is $+e$. We have assumed that the plasma particles obey the Fermi gas state equation in zero temperature

$$P = \frac{m_j v_{Fj}^2}{3n_{j0}^2} n_j^3,$$

where $j = e,i,d$ and $v_{Fj}^2 = 2k_B T_{Fj} / m_j$ is the Fermi speed defined in terms of Boltzmann constant k_B, T_{Fj} is Fermi temperature, m_j is mass, n_j is the number density, and n_{j0} is its equilibrium value which satisfies the charge neutrality condition $n_{i0} = n_{e0} + Zn_{d0}$.

The set of equations of quantum dusty plasma governing in cylindrical coordinates assuming the low phase velocity and low frequency are [21]

$$\frac{\partial \vec{\mathbf{u}}}{\partial t} + (\vec{\mathbf{u}} \cdot \nabla)\vec{\mathbf{u}} = \frac{Ze}{m}\left[-\nabla\Phi + \vec{\mathbf{u}} \times \vec{\mathbf{B}}\right] + \frac{\nabla P_d}{M\, n_d} + \frac{\hbar^2}{2M}\nabla\left(\frac{\nabla^2\sqrt{n_d}}{\sqrt{n_d}}\right) \tag{1}$$

$$\nabla^2\Phi = \frac{e}{\varepsilon_0}\left(Zn_d + n_e - n_i\right), \tag{2}$$

$$\frac{\partial n_d}{\partial t} + \nabla \cdot \left(n_d\, \vec{\mathbf{u}}\right) = 0, \tag{3}$$

$$0 = e\nabla\Phi - \frac{\nabla P_e}{m_e n_e} + \frac{\hbar^2}{2m_e}\nabla\left(\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right), \tag{4}$$

$$0 = -e\nabla\Phi - \frac{\nabla P_i}{m_i n_i} + \frac{\hbar^2}{2m_i}\nabla\left(\frac{\nabla^2\sqrt{n_i}}{\sqrt{n_i}}\right), \tag{5}$$

where $\vec{\mathbf{u}}$ and Φ represent the dust fluid velocity and the electrostatic potential, respectively. And also ε_0 and $\hbar$ are the dielectric costant and the Planck constant divided by 2π, respectively. The linear set of the above equations in the cylindrical coordinate with axial symmetry (we can ignore $\partial / \partial z$ in our model) can be normalized and written as follows

$$\frac{\partial u_r}{\partial t} = \frac{\partial\Phi}{\partial r} - \frac{\omega_{cd}}{\omega_{pd}}u_\theta - \delta\frac{\partial n_d}{\partial r} + \frac{\nabla P_d}{M\, n_d} + \frac{H_d^2}{2}\frac{\partial}{\partial r}\left(\nabla^2 n_d\right), \tag{6}$$

$$\frac{\partial u_\theta}{\partial t} = \frac{1}{r}\frac{\partial\Phi}{\partial r} + \frac{\omega_{cd}}{\omega_{pd}}u_r - \frac{\delta}{r}\frac{\partial n_d}{\partial\theta} + \frac{\nabla P_d}{M\, n_d} + \frac{H_d^2}{2}\frac{\partial}{\partial\theta}\left(\nabla^2 n_d\right), \tag{7}$$

$$\frac{\partial n_d}{\partial t} + \frac{1}{r}(ru_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial\theta} = 0, \tag{8}$$

$$\nabla^2\Phi = n_d + \mu_e n_e - \mu_i n_i, \tag{9}$$

$$n_e = \frac{\Phi}{\sigma}, \tag{10}$$

$$n_i = -\Phi, \tag{11}$$

The parameters in the above equations are normalized as:

$$t \to t\,\omega_{pd},\, r \to r/\lambda_D,\, \Phi \to \frac{e\Phi}{2K_B T_{Fi}},\, u_{r,\theta} \to u_{r,\theta}/C_d$$

$$n_j \to n_j / n_{0j},\, \omega_{pd} = \sqrt{\frac{Z^2 e^2 n_{d0}}{\varepsilon_0 M}},\ \lambda_D = \sqrt{\frac{2\varepsilon_0 K_B T_{Fi}}{Z\ e^2 n_{d0}}},$$

$$C_d = \sqrt{\frac{2Z\ K_B T_{Fi}}{M}},\, H_d = \sqrt{\frac{2\hbar\,\omega_{pd}^2}{M^2 C_d^4}},\quad \omega_{cd} = \frac{Z\,e\,B_0}{M},$$

$$\delta = \frac{T_{Fd}}{Z\,F_{Fi}},\, \sigma = \frac{T_{Fe}}{T_{Fi}},\ \mu_e = \frac{1}{\mu - 1},\, \mu_i = \frac{\mu}{\mu - 1},\, \mu = \frac{n_{0i}}{n_{0e}}$$

Using the Fourier transformations, the radial and angular terms can be separated

$$f = \tilde{f}(r)\exp(i\ell\theta - i\omega t),$$

Since we have cylindrical symmetry, the variables should have the same value at θ and $2\pi + \theta$, so the mode number (ℓ) must be an integer. Equations (6)-(11) give

$$u_r = \frac{i}{\omega^2 - \omega_{cd}^2}\left[\left(\omega\frac{\partial}{\partial r} + \omega_{cd}\frac{\ell}{r}\right)\Phi - \delta\left(\omega\frac{\partial}{\partial r} + \omega_{cd}\frac{\ell}{r}\right)n_d + \frac{H_d^2}{2}\left(\omega\frac{\partial}{\partial r} + \omega_{cd}\frac{\ell}{r}\right)\nabla^2 n_d\right], \tag{12}$$

$$u_\theta = \frac{i}{\omega^2 - \omega_{cd}^2}\left[\left(\omega\frac{\ell}{r} + \omega_{cd}\frac{\partial}{\partial r}\right)\Phi - \delta\left(\omega\frac{\ell}{r} + \omega_{cd}\frac{\partial}{\partial r}\right)n_d + \frac{H_d^2}{2}\left(\omega\frac{\ell}{r} + \omega_{cd}\frac{\partial}{\partial r}\right)\nabla^2 n_d\right], \tag{13}$$

And combining (12), (13) and (8)

$$n_d = \frac{i}{\omega^2 - \omega_{cd}^2}\left[\nabla^2\Phi - \delta\nabla^2 n_d + \frac{H_d^2}{2}\nabla^4 n_d\right], \tag{14}$$

The normalized Poisson's equation for the dust particles is

$$\nabla^2\Phi = n_d + \left(\frac{\mu_e}{\sigma} + \mu_i\right)\Phi, \tag{15}$$

Substituting (15) into (14)

$$n_d = \frac{i}{\omega^2-\omega_{cd}^2}\left[\frac{H_d^2}{2}\nabla^6\Phi-\left(\delta+\frac{H_d^2}{2}\left(\frac{\mu_e}{\sigma}+\mu_i\right)\right)\nabla^4\Phi + \left(1+\delta\left(\frac{\mu_e}{\sigma}+\mu_i\right)\right)\nabla^2\Phi\right], \quad (16)$$

By combining (15) and (16), we obtain

$$a\nabla^6\Phi-b\nabla^4\Phi+c\nabla^2\Phi+d\,\Phi=0, \quad (17)$$

where the expressions are chosen as $a=H_d^2/2$, $b=\left[\delta+H_d^2/2(\mu_e/\sigma+\mu_i)\right]$, $c=\left[1+\delta(\mu_e/\sigma+\mu_i)-(\omega^2-\omega_{cd}^2)\right]$ and $d=(\mu_e/\sigma+\mu_i)(\omega^2-\omega_{cd}^2)$. One would choose the Bessel function $J_\ell(\kappa r)$ for the radial solution $\tilde{f}(r)$ in which κ is obtained through zeros of the Bessel function originated from satisfying [22], [23]

$$a\kappa^6+b\kappa^4+c\kappa^2-d=0, \quad (18)$$

in which we look for those roots which are purely real and positive ($\kappa>0$) because this condition gives us a stable mode. By substituting $\kappa=k^2$ into (18), one can obtain the dispersion relation of rotation mode. Using (16) and (18), one can obtain the density of dust particles as

$$n_d=\left[\kappa^2+\left(\frac{\mu_e}{\sigma}+\mu_i\right)\right]A_\ell J_\ell(\kappa r)e^{i\ell\theta-i\omega t}, \quad (19)$$

From (19), it can be observed that the density of the dust particles of the quantum breathing modes depends on the wavelength, the magnetic field, and the distance from the axis in cylinder devices.

III. Discussion

The normalized frequency ω/ω_{pd} of magnetized and unmagnetized cases in terms of normalized wave numbers $k\lambda_D$ are plotted in Fig. 1. It should be mentioned that this figure is valid only for $\kappa>0$ and the phase velocity of the magnetized waves is always greater than that of the unmagnetized wave, and as the magnetic field increases, the phase velocity increases. Besides, the cut-off frequency in the two cases is not equal. The cut-off frequency of the unmagnetized case is zero while that of the magnetized one is not zero and has the value of $\omega_{cutoff}=\omega_{cd}$. If a small magnetic field is applied, the magnetized modes tend to converge into the unmagnetized mode by increasing the wavenumber.

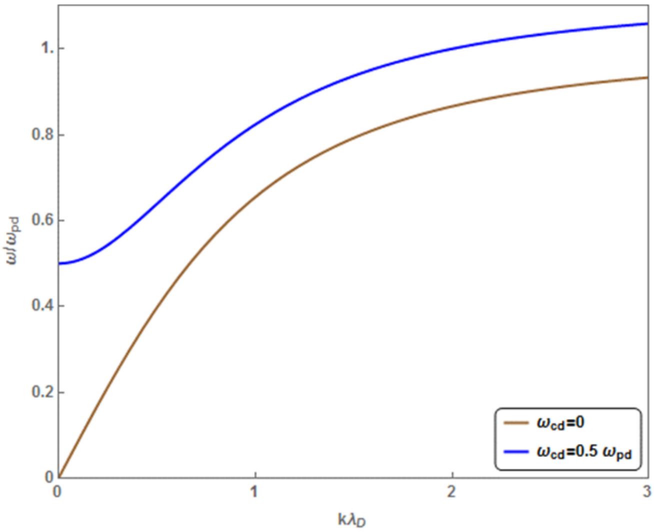

Fig. 1 The normalized frequency ω/ω_{pd} as a function of the normalized wavenumber $k\lambda_D$ for the magnetized ($\omega_{cd}/\omega_{pd}=0.5$) and unmagnetized ($\omega_{cd}/\omega_{pd}=0$) quantum breathing modes

Fig. 2 shows the arbitrary spatial distribution of the normalized density of dust numbers (n_d/n_{d0}) in terms of the normalized distance from the axis of the cylinder (r/λ_D) for $\ell=0$ and $\omega=0.4\,\omega_{pd}$. Although the amplitude of the unmagnetized quantum breathing mode ($\omega_{cd}/\omega_{pd}=0$ i.e. which is shown as a dash blue line) is greater than that of the magnetized one ($\omega_{cd}/\omega_{pd}=0.75$ i.e. which is shown as a red line), the magnetized quantum breathing mode attenuates more slowly in the radial distance than the unmagnetized one.

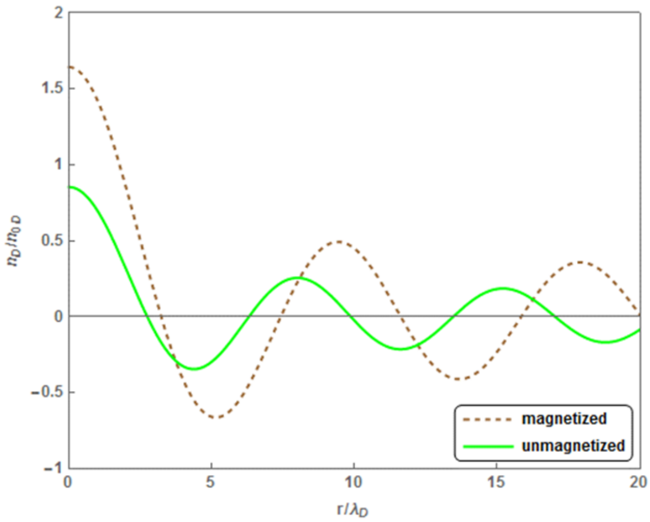

Fig. 2 The normalized spatial distribution of dust density n_D/n_{d0} in terms of the normalized distance r/λ_D for two cases of the magnetized and unmagnetized quantum breathing modes

For the case of $\ell = 0$, fluctuations of the density of number and their contours are depicted for the two cases of the magnetized and the unmagnetized quantum breathing modes in Figs. 3 and 4. It is clear that the magnetic field not only alters the distribution of dust in cylindrical devices but also changes the radial and azimuthal velocities around the axis.

Using Fig. 2, it can be stated that the void structure can exist in a larger radial by applying the magnetic field. In other words, the magnetic field compensates the void-like structure by rotating and gathering the particles.

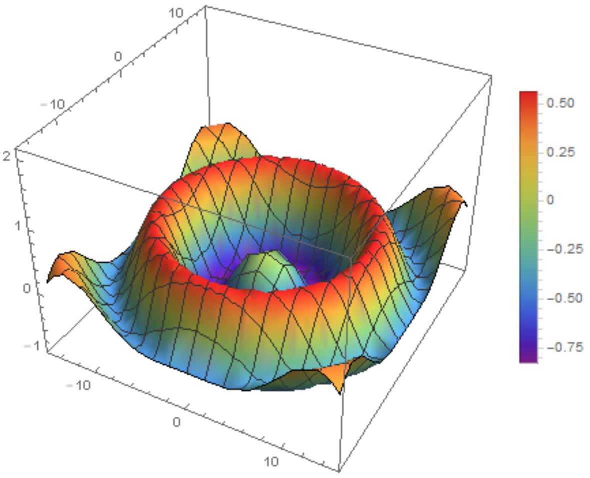

Fig. 3 The spatial distribution of the perturbed density and its contour for the unmagnetized quantum breathing mode

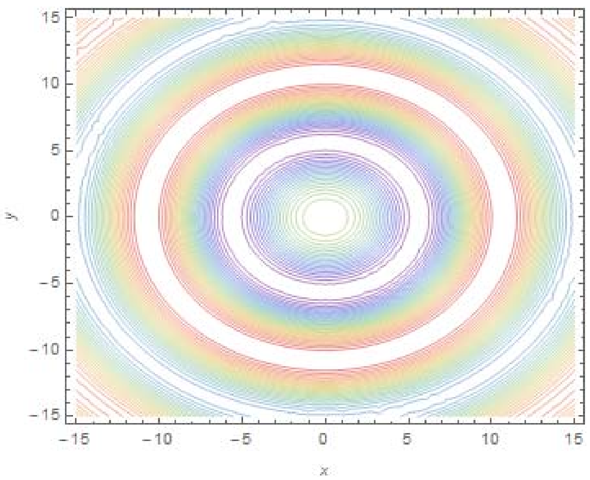

Fig. 4 The spatial distribution of the perturbed density and its contour for the magnetized quantum breathing mode

IV. Conclusion

Using the linear QHDs, the behavior of the quantum breathing mode, the dispersion relation of rotation mode, and the void-like structure in the presence of an external magnetic field have been studied. It was found that the perturbed density function of dust particles depends on the parameters of the wave number, the magnetic field, and the distance from the cylindrical axis. By plotting the dispersion relation, it was shown that the phase velocity of the magnetized quantum breathing mode is greater than that of the unmagnetized one and increases in proportion to the external magnetic field. Besides, the cut-off frequency on the magnetized quantum breathing mode is not zero and has the value of $\omega_{cutoff} = \omega_{cd}$.

In addition, the attenuation of the magnetized quantum breathing mode is more than that of the unmagnetized one and takes places along the radial distance of cylindrical axis and in some radial distances, dust particles cannot exist. By drawing the counter of the magnetized quantum breathing mode it was found that the applied magnetic field alters the distribution of dust particles and tends to change the radial and azimuthal velocities around cylindrical axis because the magnetic field collects the rotated dust particles and compensates void-like structure.

References

[1] P. K. Shukla and A. Mamun, *Introduction to dusty plasma physics*: CRC Press, 2001.

[2] B. Farokhi and A. Abdikian, "Modulational instabilities in two-dimensional magnetized dust-lattice," *Physics of Plasmas,* vol. 18, p. 113705, 2011.

[3] N. Rao, P. Shukla, and M. Y. Yu, "Dust-acoustic waves in dusty plasmas," *Planetary and space science,* vol. 38, p. 543, 1990.

[4] A. Barkan, N. D'Angelo, and R. Merlino, "Experiments on ion-acoustic waves in dusty plasmas," *Planetary and Space Science,* vol. 44, p. 239, 1996.

[5] P. Bandyopadhyay, G. Prasad, A. Sen, and P. Kaw, "Experimental study of nonlinear dust acoustic solitary waves in a dusty plasma," *Physical review letters,* vol. 101, p. 065006, 2008.

[6] F. Chen, *Introduction to plasma physics and controlled fusion*. New York: Plenum Press, 1984.

[7] Y.-y. Wang and J.-f. Zhang, "Cylindrical dust acoustic waves in quantum dusty plasmas," *Physics Letters A,* vol. 372, pp. 3707-3713, 2008.

[8] G. Manfredi and F. Haas, "Self-consistent fluid model for a quantum electron gas," *Physical Review B,* vol. 64, p. 075316, 2001.

[9] P. Holland, "The Quantum Theory of Motion," *Cambridge, New York,* 1993.

[10] G. Manfredi, "How to model quantum plasmas," *Fields Inst. Commun.,* vol. 46, p. 263, 2005.

[11] S. Ali and P. Shukla, "Dust acoustic solitary waves in a quantum plasma," *Physics of Plasmas,* vol. 13, p. 022313, 2006.

[12] L. Wei and Y.-N. Wang, "Quantum ion-acoustic waves in single-walled carbon nanotubes studied with a quantum hydrodynamic model," *Phys. Rev. B,* vol. 75, p. 193407, 05/18/ 2007.

[13] A. Abdikian and M. Bagheri, "Electrostatic waves in carbon nanotubes with an axial magnetic field," *Phys. Plasmas,* vol. 20, p. 102103, 2013.

[14] H. Moritz, T. Stöferle, M. Köhl, and T. Esslinger, "Exciting collective oscillations in a trapped 1D gas," *Physical review letters,* vol. 91, p. 250402, 2003.

[15] S. P. Tewari, H. Joshi, and K. Bera, "Wavevector- and frequency-dependent collective modes in one-component rare hot quantum and classical plasmas," *Journal of Physics: Condensed Matter,* vol. 7, p. 8405, 1995.

[16] S. Bauch, K. Balzer, C. Henning, and M. Bonitz, "Quantum breathing mode of trapped bosons and fermions at arbitrary coupling," *Physical Review B,* vol. 80, p. 054515, 2009.

[17] C. Henning, K. Fujioka, P. Ludwig, A. Piel, A. Melzer, and M. Bonitz, "Existence and vanishing of the breathing mode in strongly correlated finite systems," *Physical review letters,* vol. 101, p. 045002, 2008.

[18] M. R. Geller and G. Vignale, "Quantum breathing mode for electrons with 1/${\mathit{r}}^{2}$ interaction," *Physical Review B,* vol. 53, p. 6979, 03/15/ 1996.

[19] C. Arnas, A. Michau, G. Lombardi, L. Couëdel, and K. Kumar K, "Effects of the growth and the charge of carbon nanoparticles on direct current discharges," *Physics of Plasmas* vol. 20, p. 013705, 2013.

[20] K. Kumar, L. Couëdel, and C. Arnas, "Growth of tungsten nanoparticles in direct-current argon glow discharges," *Physics of Plasmas* vol. 20, p. 043707, 2013.

[21] S. Khan, S. Mahmood, and A. M. Mirza, "Cylindrical and spherical dust ion-acoustic solitary waves in quantum plasmas," *Physics Letters A,* vol. 372, p. 148, 2008.
[22] G. N. Watson, *A treatise on the theory of Bessel functions*: Cambridge university press, 1995.
[23] M. Bagheri and A. Abdikian, "Space-charge waves in magnetized and collisional quantum plasma columns confined in carbon nanotubes," *Phys. Plasmas,* vol. 21, p. 042506, 2014.