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Adaptive Extended Kalman Filter for Ballistic Missile Tracking

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***Abstract*—**In the current work, adaptive extended Kalman filter (AEKF) is presented for solution of ground radar based ballistic missile (BM) tracking problem in re-entry phase with unknown ballistic coefficient. The estimation of trajectory of any BM in re-entry phase is extremely difficult, because of highly non-linear motion of BM. The estimation accuracy of AEKF has been tested for a typical test target tracking problem adopted from literature. Further, the approach of AEKF is compared with extended Kalman filter (EKF). The simulation result indicates the superiority of the AEKF in solving joint parameter and state estimation problems.

***Keywords*—**Adaptive, AEKF, ballistic missile, EKF, re-entry phase, target tracking.

I. Introduction

THE BM tracking is quite crucial for security of any nation against any BM attack from enemy country. In India, a multilayered BM defence has been developed to guard against such attack [1]. Tracking of BM can be done with the help of ground based radar but traced parameters contain certain uncertainty and error. This error can be minimized by Kalman filter [2] which uses a set of equation and consecutive data input (from radar) to estimate the accurate value. However, this filter is only suitable for linear system model and hence, it cannot be used to trace the motion of BM in re-entry phase which is highly nonlinear and unpredictable because of the parameters like drag, drift, ballistic coefficient and atmospheric condition. In case of non-linear process, extended Kalman filter (EKF) is used which is non-linear version of Kalman filter. EKF adopts a technique called Taylor series expansion to linearize a non-linear state model.

EKF estimates the non-linear state by using a priori guess of the process and measurement noise. As the targets enter in the endo-atmospheric reason, the estimation becomes difficult since the physical parameters like ballistic coefficient, drag, lift etc. are not known accurately. Further, it really becomes difficult to estimate the position, when the priori estimated value is too far from its real values. In such situation the estimates offered by EKF may diverge. A modified non-linear state estimator named as adaptive extended Kalman filter (AEKF) has been proposed in [3] for GPS integration. The algorithm updates the process and measurement noise at every measurement to find its proper value and hence reduces the chance of divergence of the estimation.

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In this paper, an attempt has been made to track the BM having unknown ballistic coefficient using EKF and AEKF algorithms. The results obtained have been compared in terms of estimation error and covariance.

In view of the discussions carried out in the above sections, this paper is organized as follows: Section II shows the trajectory as well as the reference system of BM and state model for unknown ballistic coefficient is covered. Moreover, the measurement model of BM is also covered. Ballistic coefficient is different for every flying object. The Jacobian matrix of unknown ballistic coefficient is also shown. Section III covers the EKF and AEKF filters which are used in order to estimate the position of BM. The theory as well as algorithm of both filers is covered. The theory as well as algorithm of both filers is covered. Section IV presents simulation results and discussions. Section V presents the conclusion.

II. Problem Formulation

A. Trajectory

The forces acting on the BM are gravity, drag and other forces like centrifugal acceleration, Coriolis acceleration, wind acceleration, lift force. Spinning motion is ignored because of small effect on its trajectory. The trajectory of BM from launch to impact is mainly divided into three phase boost phase, ballistic flight and re-entry as shown in Fig. 1.

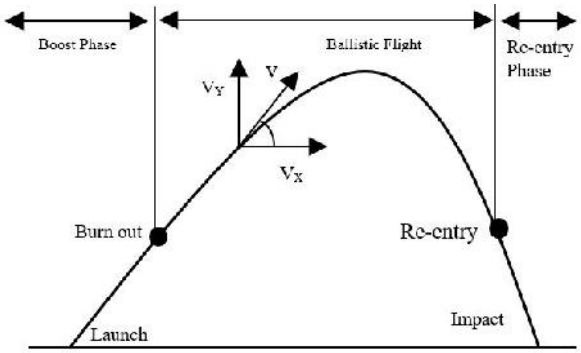


Fig. 1 Different phases of BM Trajectory

Boost phase is the period during which the rockets of a ballistic missile operate to attain its peak velocity. This thrust may vary which makes the trajectory estimation more complex. The total acceleration in boost phase is given by

$$a = a_T + a_d + a_g \quad (1)$$

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where a is total acceleration, a_T is acceleration due to thrust, a_d is acceleration due to drag, a_g is acceleration due to gravity. In this case, the earth rotation can be omitted because of its small duration hence the Coriolis acceleration and centrifugal force is neglected.

Ballistic flight is the travelling of BM between boost phase and re-entry phase. Re-entry phase is when BM re-enters in atmosphere. The forces act on BM in re-entry phase is given by

$$a = a_d + a_g \tag{2}$$

The hypothesis of flat earth is considered here so the orthogonal co-ordinate system depicted in Fig. 2 can be used with following variables as in [4]: x is the abscissa, y is the ordinate, x_0 and y_0 are target co-ordinate at time t_0, v is the velocity module, and ¥ is the angle between the horizontal axis and the direction of motion.

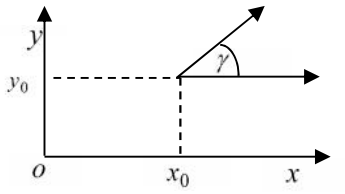


Fig. 2 Co-ordinate reference system of BM

B. State Model of BM for Unknown Ballistic Coefficient

State model of BM during its trajectory in discrete time non- linear dynamic equation is given by [5]

$$s_{k+1} = \Psi_k(s_k) + G\begin{pmatrix} 0 \\ -g \end{pmatrix} + w_k \tag{3}$$

where state vector, $s_k \triangleq [x_k \;\; \dot{x}_k \;\; y_k \;\; \dot{y}_k \;\; s_k[5]]^T$, here x_k is position along x-axis, $\dot{x}_k$ is velocity along x-axis, y_k is position along y-axis, $\dot{y}_k$ is velocity along y axis, $s_k[5]$ is unknown ballistic coefficient.

$$\Psi_k(s_k) = \Phi s_k + G f_k(s_k) \tag{4}$$

$$\Phi \triangleq \begin{bmatrix} 1 & T & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$G \triangleq \begin{bmatrix} \frac{T^2}{2} & 0 \\ T & 0 \\ 0 & \frac{T^2}{2} \\ 0 & T \\ 0 & 0 \end{bmatrix} \tag{6}$$

where T is sampling time.

Drag force is the force that opposes the motion of the body. It can be considered as aerodynamics resistance. Here the drag force will act against the motion of BM and given by [6]

$$D = \frac{1}{2}\left(g \diagup \beta\right) \circ v^2 \tag{7}$$

where β is ballistic co-efficient, g is gravitational force $9.81 m/\sec^2$), ° is air density, v is the velocity module.

Air density (°) is the function of height (exponentially decaying function of height) and it decreases with increases in altitude. It also changes with the variation in temperature and humidity, however in case of tracking BM it is avoided as it hardly affects BM motion. Here air density can be given by

$$\circ = c_1 e^{-c_2 y} \tag{8}$$

where,

$$c_1 = 1.227 kg/m^3, c_2 = 1.4910^{-4}\,\mathrm{m}^{-1} \;\; for \; y < 9144 m$$
$$c_1 = 1.7554 kg/m^3, c_2 = 1.4910^{-4}\,\mathrm{m}^{-1} \;\; for \; y \geq 9144 m$$

Drag force can be written in terms of state vector components which can be given by,

$$f_k(s_k) = -\frac{g}{2} \circ \left(s_k[3]\left(s_k^2[2] + s_k^2[4]\right)\right) \times \begin{bmatrix} \cos\left(arctg\left(\frac{s_k[4]}{s_k[2]}\right)\right) \\ \sin\left(arctg\left(\frac{s_k[4]}{s_k[2]}\right)\right) \end{bmatrix} \tag{9}$$

By exploiting the following identities

$$\sin\left(\mathrm{arctg}\left(\frac{y}{x}\right)\right) = \frac{x}{\sqrt{x^2 + y^2}} \tag{10}$$

$$\cos\left(\mathrm{arctg}\left(\frac{y}{x}\right)\right) = \frac{y}{\sqrt{x^2 + y^2}} \tag{11}$$

Equation (9) can be simplified as

$$f_k(s_k) = -\frac{g}{2s_k[5]} \cdot \circ \cdot \left(s_k[3]\sqrt{\left(s_k^2[2] + s_k^2[4]\right)}\right) \times \begin{bmatrix} s_k[2] \\ s_k[4] \end{bmatrix} \tag{12}$$

The process noise is assumed as Gaussian and it can change dynamically during filter operation and in this case Q is given by

$$Q = \begin{bmatrix} - & 0_2 & 0 \\ 0_2 & - & 0 \\ 0 & 0 & q_2 * T \end{bmatrix} \text{ where } - = q_1 * \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \tag{13}$$

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where q_1[7] in m^2/s^3 and q_2 in $kg^2/m^{-2}s^{-5}$ are tuning parameters which are selected by designers to get the process noise in the target dynamics, $^{-}{}_2$ is 2×2 matrix with elements equal to zero. The better filter operation and result can be achieved by tuning the filter parameters Q and R and its value may change in each time of measurement.

The trajectory of BM depends on the gravity of Earth, aerodynamic forces and aerodynamic moments. The major uncertainty in estimating the trajectory of a ballistic target is due to the aerodynamic forces and moments acting on it.

The initial information matrix J_0 is the inverse of initial covariance matrix and given by

$$J_0 = P_{0/0} \tag{14}$$

The suitable expression for $P_{0/0}$ is

$$P_{0/0} = \begin{bmatrix} \pm_d^2 & -\frac{\pm_d^2}{T} & \pm_{dh} & -\frac{\pm_{dh}}{T} & 0 \\ -\frac{\pm_d^2}{T} & 2\frac{\pm_d^2}{T^2} & -\frac{\pm_{dh}}{T} & 2\frac{\pm_{dh}}{T^2} & 0 \\ \pm_{dh} & -\frac{\pm_{dh}}{T} & \pm_h^2 & -\frac{\pm_h^2}{T} & 0 \\ -\frac{\pm_{dh}}{T} & 2\frac{\pm_{dh}}{T^2} & -\frac{\pm_h^2}{T} & 2\frac{\pm_h^2}{T^2} & 0 \\ 0 & 0 & 0 & 0 & \pm^2 \end{bmatrix} \tag{15}$$

$\pm^2$ is the variance of unknown ballistic coefficient based on the prior value or assumed knowledge of mass of the BM.

C. Measurement Model

The radar is considered to be located at ground level i.e. $x_R = 0$, $y_R = 0$ with range r and elevation £. The error standard deviation of these measurements are denoted as $\circ_r$ (for range) and $\circ_e$ (for elevation).

The measurement equation is given by

$$z_k = Hs_k + v_k \tag{16}$$

where

$$z_k = \begin{bmatrix} d_k & h_k \end{bmatrix}^T,\ H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

The measurement of the radar is in polar coordinate whereas the estimation & prediction equations are in the rectangular coordinates, this requires transformation in one direction. The relationship between the target position in rectangular coordinate and measurement in polar coordinate is given by polar coordinate transformation d= $r\cos$£ and h= $r\sin$£ to make it linear. v_k is the noise measured Cartesian coordinates and it is independent of process noise w_k. Process noise is modeled as a zero-mean white Gaussian process with covariance matrix R_k [8] with element

$$\left.\begin{aligned} d^2 &= \pm_r^2\cos^2(£) + r^2\pm_£^2\sin^2(£) \\ \pm_h^2 &= \pm_r^2\sin(£) + r^2\pm_£^2\cos^2(£) \\ \pm_{dh} &= (\pm_r^2 - r^2\pm_£^2)\sin(£)\cos(£) \end{aligned}\right\} \tag{17}$$

An example of target trajectory is shown with the following parameters are taken into consideration,

$$\beta = 400\ kg.m^{-1}.s^{-2}, q_1 = 1\,m^2.s^3, T = 2s, y_0 = 80km,$$
$$x_0 = 232km, \gamma_0 = 190^\circ, v_0 = 2390m/s.$$

The state model of BM is simulated in MATLAB with above parameters to get the trajectory as well as velocity and acceleration graph. Fig. 3 shows the trajectory of BM in x-y plane, Figs. 4 and 5 show the target velocity and target of BM during its trajectory respectively.

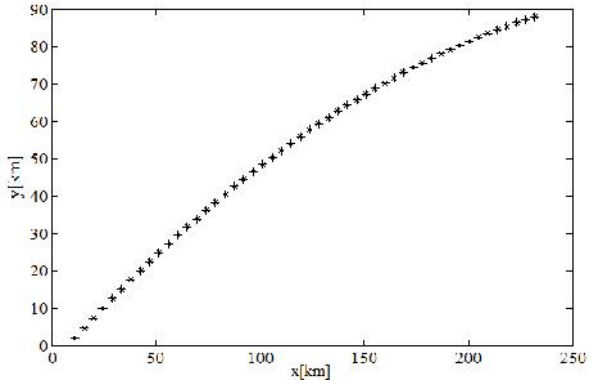


Fig. 3 Trajectory of BM

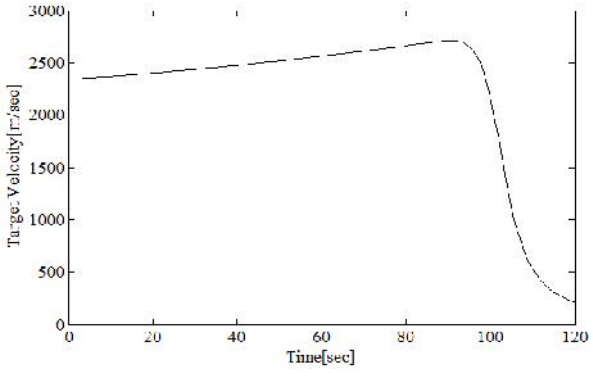


Fig. 4 Velocity of BM

There is strong deceleration because of the drag forces which increases with increase in velocity of the object which can be seen in Fig. 5.

D. Jacobian Matrix for Unknown Ballistic Coefficient

Ballistic coefficient describes how air resistance slows down projectile in its flight [9]. It generally depends on mass, body shape and cross-sectional area. Accurate value of

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Ballistic coefficient is very necessary in order to know the true nature and trajectory of BM and in case of target tracking, ballistic coefficient is unknown to us. So in this paper we have considered it as unknown for all calculation purpose and also estimated the unknown ballistic coefficient.

Jacobian matrix is the matrix of all the first order partial derivative of a vector-valued function. Here Jacobian Ψ can be written as [4]

$$\Psi_k = \Phi + GF_k \tag{18}$$

The elements of the matrix F_k are as:

$$F_k = \left(\nabla_{s_k} f_k^T(s_k)\right)^T \tag{19}$$

are as follows

$$\begin{aligned}
&F_k[1,1] = 0,\\
&F_k[2,1] = 0,\\
&F_k[1,2] = -\frac{1}{2}\cdot\frac{g}{s_k[5]}\cdot{}^\circ{}_k\cdot\frac{2\left(s_k[2]^2\right)+\left(s_k[4]\right)^2}{\sqrt{s_k[2]^2+s_k[4]^2}},\\
&F_k[2,2] = F_k[1,4],\\
&F_k[1,3] = \frac{1}{2}\cdot\frac{g}{s_k[5]}.c_2\cdot{}^\circ{}_k\cdot s_k[2]\sqrt{\left(s_k[2]\right)^2+\left(s_k(4)\right)^2},\\
&F_k[2,3] = \frac{1}{2}\cdot\frac{g}{s_k[5]}.c_2\cdot{}^\circ{}_k\cdot s_k[4]\sqrt{\left(s_k[2]\right)^2+\left(s_k(4)\right)^2},\\
&F_k[1,4] = -\frac{1}{2}\cdot\frac{g}{s_k[5]}\cdot{}^\circ{}_k\cdot\frac{s_k[2]\cdot.s_k[4]}{\sqrt{s_k[2]^2+s_k[4]^2}},\\
&F_k[2,4] = -\frac{1}{2}\cdot\frac{g}{s_k[5]}\cdot{}^\circ{}_k\cdot\frac{2\left(\left(s_k[2]\right)^2\right)+\left(s_k[4]\right)^2}{\sqrt{s_k[2]^2+s_k[4]^2}},\\
&F_k[1,5] = \frac{1}{2}\cdot\frac{g}{\left(s_k[5]\right)^2}\cdot{}^\circ{}_k\cdot s_k[2]\sqrt{\left(s_k[2]\right)^2+\left(s_k(4)\right)^2},\\
&F_k[2,5] = \frac{1}{2}\cdot\frac{g}{\left(s_k[5]\right)^2}\cdot{}^\circ{}_k\cdot s_k[4]\sqrt{\left(s_k[2]\right)^2+\left(s_k(4)\right)^2}.
\end{aligned} \tag{20}$$

III. Filter

A. EKF

EKF is widely used for the position estimation of non-linear system. To estimate the process and measurement noise covariance EKF uses a fixed priori estimates during its whole process. The summary for EKF algorithm is given as [2]:

1) Time update equation

(a) The state Projection

$$\hat{s}_{k+1} = \psi_k\left(\hat{s}_k\right) + G\begin{pmatrix}0\\-g\end{pmatrix} \tag{21}$$

(b) The error covariance projection:

$$P_k^- = \cdot P_{k-1}\cdot{}^T + Q_k \tag{22}$$

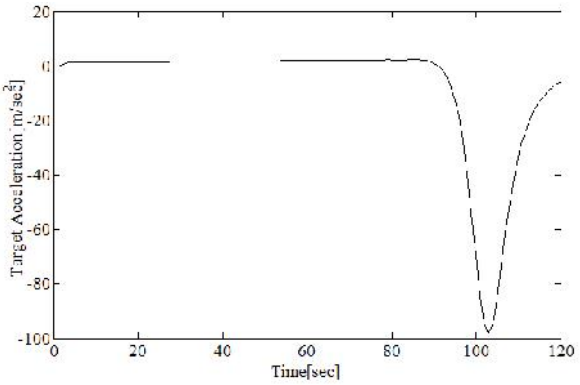


Fig. 5 Acceleration of BM

2) Measurement update equation

(a) Kalman Gain update

$$K_k = P_k^- H^T\left[HP_k^- H^T + R_k\right]^{-1} \tag{23}$$

(b) The error covariance update

$$P_k = (I - K_k H)P_k^- \tag{24}$$

(c) The estimate update with measurement

$$\hat{s}_k = \hat{s}_k^- + K_k\left(z_k - H\,\hat{s}_k^-\right) \tag{25}$$

Here $\hat{s}_k$ is the predicted estimate of state at kth step, $\hat{s}_k^-$,is a posteriori estimate of state at kth state, P_k^- is the error covariance, Ψ_k is the state transition matrix, K_k is the Kalman Gain matrix, Q_k and R_k represent the process and measurement noise covariance respectively.

To implement EKF we require priori knowledge of process and measurement noise. However, there is a chance of high estimation error when the priori noise covariance is presumed with improper value. To overcome this demerit of EKF, AEKF is proposed which adjusts the process and measurement noise at every measurement to find proper noise covariance at each step.

B. AEKF

AEKF helps to reduce the influence of prior Q and R on the estimation by updating it in every measurement. The main two approaches for Q and R estimation are correlations of the innovation sequence and the other one is correlations of the output innovation sequence. Here the correlation of innovation sequence is used to determine Q and R and further the maximum-likelihood estimation for the multivariate normal distribution approach is used to make the actual value of the covariance consistent with its theoretical value [10].

The innovation sequence can be written as

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$$e_k = H(s_k - \hat{s}_k^-) + v_k \tag{26}$$

The covariance can be obtained by taking the variance on both sides of (26)

$$\Pi_k = HP_k^- H^T + R_k \tag{27}$$

The covariance of e_k is written as

$$\Pi_k = E[(e_k)(e_k)^T] \tag{28}$$

According to the maximum-likelihood estimation for the multivariate normal distribution approach, the statistical sample variance Π_k is given as:

$$\Pi_k = \frac{1}{k}\sum_{i=1}^{k}(e_k)(e_k)^T \tag{29}$$

From (27), the estimate of the measurement noise covariance is as

$$R_k = \Pi_k - HP_k^- H^T \tag{30}$$

Process noise covariance Q is the uncertainty in the non-linear dynamic equation measured during measurement update and it is applied to the system in order to get the proper estimate. It can be written as

$$Q_k = \frac{1}{k}\sum_{i=1}^{k}\left(s_k - \hat{s}_k^-\right)\left(s_k - \hat{s}_k^-\right)^T + P_k - \Psi_k P_{k-1}(\Psi_k)^T \tag{31}$$

The process and measurement noise covariance are modified adaptively by using (30) and (31). So, in EKF, it is mandatory that the initial value Q and R should be set correctly otherwise there is high chance of divergence from true trajectory whereas AEKF updates the process and measurement noise at every measurement to find noise covariance proper value and hence reduces the chance of divergence of the estimation.

IV. Simulation Results and Discussions

In this paper, the two non-linear filters namely, EKF and AEKF are used for solution of tracking of a typical ballistic target as considered in [4]. The filtering algorithms are implemented using MATLAB R2012a on a computer with 2.00GHz Intel Core I3-6006U processor with 4 GB RAM. Typical BM tracking problem has been solved and the test run for each case has been performed 30 times and best result of estimated ballistic coefficient and estimation error has been shown in Figs. 8, 9 and 10, respectively. The comparative results for estimation error covariance (as considered in [11]) for both EKF and AEKF are presented in Table I. The mean value for 30 runs is taken into consideration which is shown in Table I.

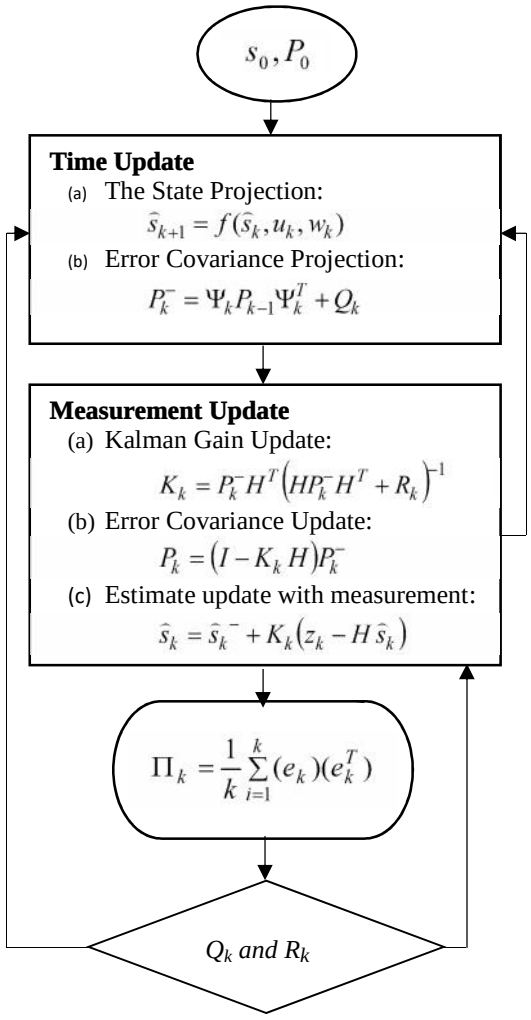


Fig. 6 Block Diagram for Adaptive Kalman filter

TABLE I
Covariance Value For Unknown Ballistic Coefficient

State	Mean Estimation Covariance		Percentage (%) Improvement
	EKF($\times 10^{-5}$)	AEKF($\times 10^{-5}$)	
x_k	0.499	0.395	20.84
$\dot{x}_k$	0.989	0. 573	42.06
y_k	0.289	0.188	34.94
$\dot{y}_k$	0.164	0.102	37.80
s_k[5]	0.321	0.208	35.20

With the % improvement in Table I, the AEKF offers a mean estimation error covariance of 0.395×10^{-5}, 0.573×10^{-5}, 0.188×10^{-5}, 0.102×10^{-5} and 0.208×10^{-5} respectively for the five states under consideration with an improvement 0.104×10^{-5}, 0.416×10^{-5}, 0.101×10^{-5} and 0.062×10^{-5} and 0.113×10^{-5} of respectively, compared to EKF. Thus, AEKF offers more accurate estimation of target trajectory.

The comparative results corresponding to AEKF (*represented by dotted line*) and EKF (*represented by dashed lines*) for typical trajectory is presented in Fig. 7. The

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estimation errors for AEKF and EKF corresponding to position and velocity along the x and y direction are given in Fig. 9. The trajectory offered by AEKF is found to be closer to the true trajectory of BM under consideration.

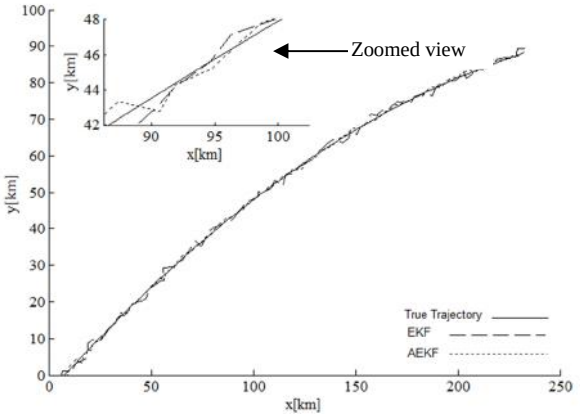


Fig. 7 Estimated Trajectory of BM

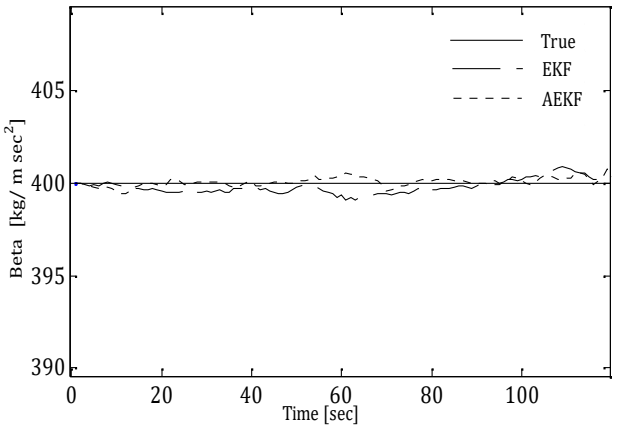


Fig.8 Estimated Ballistic coefficient (beta) of BM

Fig. 8 shows estimated ballistic coefficient (beta) compared with true value of ballistic coefficient (solid line). Here also AEKF able to estimate the unknown beta very close to its true value as compared with EKF. Fig. 10 shows the error of estimated beta as compared to true value and it also proves that estimation error is less in AEKF when compared with EKF.

V. Conclusion

BM tracking problem in re-entry phase with unknown ballistic coefficient is solved using AEKF. In this work, the performance of AEKF is tested using typical target problem and further its result is compared with EKF counterpart. The simulation result obtained using MATLAB shows that AEKF stands superior to EKF in terms of accuracy. Further, it may be observed that, the AEKF not only supersedes EKF in state estimation but also in parameters estimation too. Thus, the AEKF may be recommended as a promising tool for solving similar estimation problems with unknown ballistic coefficient or other more complex estimation problems.

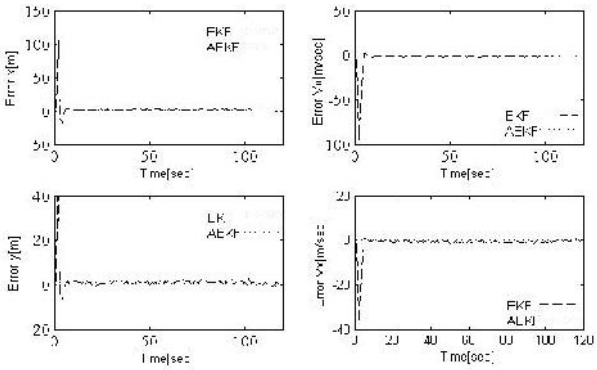


Fig. 9 Error of EKF (dashed line) and AEKF (dotted-line)

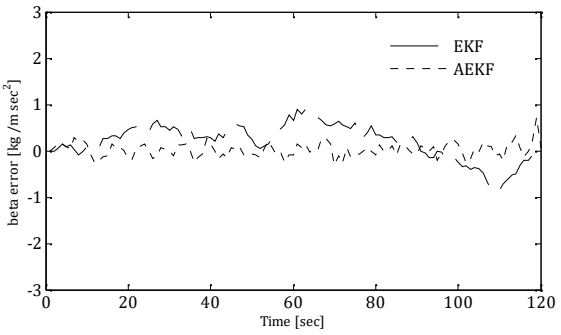


Fig. 10 Error of EKF (dashed line) and AEKF (dotted-line) for beta estimation

References

[1] Raju G. C. Thomas "India's nuclear and space programs: Defense or Development? Cambridge University Press, vol. 38, issue 2, pp. 315-342 Jan 1986.

[2] Greg Welch and Gary Bishop, "An Introduction to the Kalman Filter" Dept. of Computer Science, Univ. of Nort Carolina, TR 95-041 24, July, 2006.

[3] Weidong Ding, Jingling Wang and Chris Rizos, "Improving Adaptive Kalman Estimation in GPS/INS Integration. The Journal of Navigation, vol 60, pp5. 17-529, 2007.

[4] A. Farina, B, Ristic, D. Benvenuti, "Tracking a ballistic target Comparison of Several Filters. IEEE Trans on AES, vol 38, issue no 3, pp. 854-867, July 2002.

[5] K. Vamsi Krishna Reddy. Anand M. Tapas, R. Venkatram Reddy, N. Prabhakar, "State estimation of ballistic missile for homing phase of Exo-atmospheric Interceptor", Int. Conf. on Advances in Control and Optimization of Dynamical System, 2007.

[6] Zarchan, P. (1997) "Tactical and Strategic Missile Guidance" (3rd ed.). AIAA, Progress in Astronautics and Aeronautics, vol.176, 1997.

[7] Farina, A. Studer, F.A. "Radar Data Processing, Introduction and Tracking, Vol. 1.England: Researches Studies Press, May, 1985.

[8] Michael Courtney, Amy Courtney "The Truth about the ballistic coefficient" Popular Physics, Cornell University Library, May 2007.

[9] Bar-Shalom, Y., Li, X-R "Estimation and tracking principle Techniques and software", MA: Artech House, 1993.

[10] Dongkyun Kim, Jihyun Ha and Kwanhi You "Adaptive Extended Kalman Filter Based Geolocation Using TDOA/FDOA" Int, J.Vol-4 No.2 June, 2011.

[11] Karl G. Joreskog, "Structural analysis of covariance and correlation Matrices", Psychometrika, vol., 43,no. 4, Dec. 1978.