World Academy of Science, Engineering and Technology
International Journal of Geological and Environmental Engineering
Vol:9, No:10, 2015

Assessment of the Energy Balance Method in the Case of Masonry Domes

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***Abstract*—**Masonry dome structures had been widely used for covering large spans in the past. The seismic assessment of these historical structures is very complicated due to the nonlinear behavior of the material, their rigidness, and special stability configuration. The assessment method based on energy balance concept, as well as the standard pushover analysis, is used to evaluate the effectiveness of these methods in the case of masonry dome structures. The Soltanieh dome building is used as an example to which two methods are applied. The performance points are given from superimposing the capacity, and demand curves in Acceleration Displacement Response Spectra (ADRS) and energy coordination are compared with the nonlinear time history analysis as the exact result. The results show a good agreement between the dynamic analysis and the energy balance method, but standard pushover method does not provide an acceptable estimation.

***Keywords*—**Energy balance method, pushover analysis, time history analysis, masonry dome.

I. Introduction

MANY buildings of holy places and other types of structures have been built using masonry domes. According to the historical and cultural importance of these buildings, their seismic evaluations and rehabilitations are important and noticeable. Seismic assessment of historical structures generally involves linear static analysis and modal analysis, and for some particular buildings, the nonlinear dynamic analyses were carried out. In this research, two new methods of conventional pushover and energy balance were presented and applied to an existing building.

In pushover analysis, the performance point of structure is obtained from the intersection between the capacity and demand curves in ADRS coordination. The method expects to provide information on many response characteristics that cannot be obtained from a linear static or nonlinear dynamic analysis, the characteristics such as estimation of ultimate capacity, deformation demands, and identification of critical regions [1].

Roof displacement in majority case of pushover analysis is used as the index to represent the behavior of structure; nevertheless, for some cases, wrong results are obtained by using this point as the index. A solution for this misleading feature is introduced by considering the absorbed energy as a better index to establish the capacity and demand curves. In this method, the basic assumption which assumes that input energy must be equal to absorbed energy, was used to estimate the behavior of structure. The intersection between capacity curves (absorbed energy) and demand curves (input energy) is the performance point of building under given earthquake.

The object of this research was to develop new seismic evaluation methods for masonry dome buildings. First, the pushover analysis and evaluation method based on energy are presented briefly, and the methods are applied to a historical dome named Soltanieh which is well-known building from 700 years ago. The results obtained from the procedures are compared with the results from nonlinear dynamic analysis as the exact results.

II. Nonlinear Static Procedure (NSP)

One of the useful tools for evaluation and design of structures is nonlinear static procedure (NSP) that has been widely used in practice. In this method, several analysis approaches have been proposed for evaluating the existing buildings, and in most of cases, performance point is given by superimposing the capacity and demand curves in ADRS coordination. Capacity curve is given by pushing structure with constant load pattern and it is illustrated by plotting the base shear versus the target displacement. The demand curve is obtained from the response of several single degree of freedom structures under the ground motion records. The evaluations imply that the demand should be compared to the capacity and performance point is the resultant of this comparison. In fact, the performance point of a structure is its expected target displacement under a given earthquake. Once determined, the performance point can be projected back to the target displacement. Thus, the remaining capacities of each component are calculated. Then, re-designing of the structure can be conducted based on the mentioned remained capacities.

The NSP, pushover analysis, is relatively simple but more realistic and more comprehensive than linear static analysis procedure. Moreover, the pushover analysis provides some special results which cannot be obtained by using nonlinear time history analysis [1].

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III. Energy Balance Method

Because of practical reasons, the roof displacement of buildings is used as a target displacement in the most assessments; however, it is not always the most suitable criteria to represent the characteristic of capacity. In some cases, for example, when higher modes of structure are involved, the results of pushover analysis based on the roof displacement criteria may not be representative of the real behavior of the structure. Absorbed energy which is the work of external forces, instead of the roof displacement, has been proposed recently as a better index to represent the behavior of a structure [2], [3].

New seismic evaluation based on energy is developed recently by assuming that absorbed and input energy for a system are equal. This assumption is more clear and reasonable than NSP. For seismic evaluation, the maximum response of structure is obtained from the maximum input energy; thus, at the performance point in the capacity curve, the absorbed energy is equal to the maximum input energy.

The basic assumption of energy method is given by

$$E = \frac{1}{2} M S_V^2 = \frac{1}{2} V_y D_y + V_y (D_m - D_y) \quad (1)$$

where M is the mass of the system, S_v is the pseudo velocity, V_y is the yield strength, Dy is the yield displacement, and Dm is the maximum inelastic displacement. As can be seen, the maximum input energy (left side of (1)) is obtained from the pseudo velocity spectra that are given from response of structure under ground motion record and mass of the system, and the absorbed energy (right side of (1)) is given from behavior of structure.

Equation (1) is valid only for systems with particular time period ranges; Lee and Goel introduce the energy factor γ to modify the equation as follows [4]

$$\gamma \frac{1}{2} M {S_v}^2 = \frac{1}{2} V_y D_y + V_y \left(D_m - D_y\right) \quad (2)$$

γ is obtained from the assumption of energy balance that implies that energy computed from monotonic load deformation response of inelastic system and the one computed from the corresponding elastic system are the same and γ is defined as

$$\gamma = \frac{2\mu - 1}{{R_y}^2} \quad (3)$$

where μ is the displacement ductility factor and R_y is the yield strength reduction factor.

$$\mu = \frac{max|u|}{u_y} \quad (4)$$

$$R_y = \frac{V_e}{V_y} \quad (5)$$

where V_e is the strength which is required for the system to remain elastic.

The energy factor can be determined if the relationship between R_y-μ-T is provided.

IV. Seismic Evaluation Based on Energy Balance Concept

The seismic assessment of structure can be done if the energy balance relationship is applied correctly into this case. By analyzing the both sides of (1), the performance of a structure can be predicted. The absorbed energy (right side of (1)) is obtained from the capacity curve that is used in the pushover analysis. This curve is called the conventional capacity curve that is illustrated by base shear versus target displacement. The area under the conventional capacity curve is represented the amount of energy that is absorbed by the system. Absorbed energy can be calculated from the work done by the external force at each floor or at each level for continuous system.

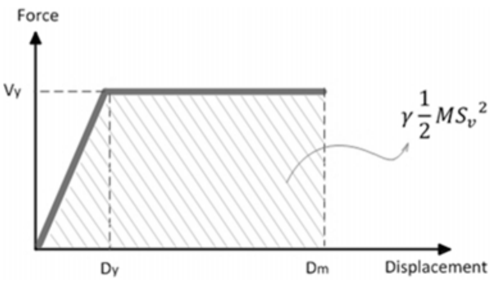


Fig. 1 Illustration of modified energy balance concept [4]

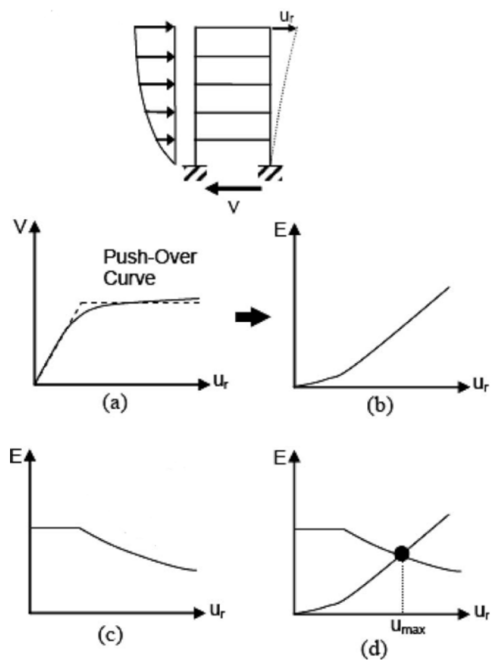


Fig. 2 Seismic evaluation based on energy balance: (a) pushover curve; (b) absorbed energy curve; (c) input energy curve; (d) intersection of capacity and demand curve [6]

For the left side of (1), the demand curve is a function of three parameters which are the mass of the system, energy factor, and pseudo velocity. Two types of mass are proposed for this analysis [5]. The first one is the total mass, and the second one is the effective mass in first mode of structure. For determining the energy factor, the relationship between R_y-μ-T is required. This relationship is obtained from analyzing various single degree of freedom structures with different frequencies and strength reduction factor, under a specific ground motion record.

The expected displacement in this method is the point at which the input energy and absorbed energy are equal. Therefore, the intersection between capacity and demand curves in energy coordination is the performance point. The evaluation method based on energy balance concept is illustrated in Fig. 2.

Choosing the target point can affect the results of the pushover analysis; however, it does not affect the results which are based on energy method. The reason is that the target point in energy method is used just as a visualization index for the response of the structure.

V. Seismic Evaluation of Masonry Dome

Lots of studies have been focused on evaluation of masonry domes under the seismic loads. Parts of the studies consist of linear static analysis [7], modal analysis [8], and nonlinear dynamic analysis for some special buildings like Hagia Sophia [9]. In this study, two new methods are presented and applied to an existing masonry dome building in order to determine the maximum displacement response under three ground motion records. The building that is evaluated in this paper is a well-known dome in Iran named Soltanieh dome (Oljeitu mausoleum (1302-12 A.D)) which has been studied before [7]; the best prototype, which shows the specific development of Persian domes in the Ilkhanid epoch. The dome of the building consists of two thin shells which are divorced at angle of 22.5°. The base diameter and the height of the dome are 25.5 and 52 meters, respectively (Fig. 3). Using regular brick connector, two layers of dome are connected to each other in some special positions. The mentioned bilayer dome is an innovative structure, made up of semi-connected shells, which was created in Soltaniyeh dome for the first time. Professor Piero Sanpaolesi strongly believed that this dome might be the origin of the dome of the Cathedral of Santa Maria del Fiore in Florence, [7]. Soltaniyeh is the largest dome type building in Iran and the largest of the world after the famous Cathedral of Santa Maria del Fiore in Florence and the Hagia Sophia mosque in Turkey [10].

In this study, only the main building is analyzed and the burial chamber and minarets are disregarded. In addition, because the aim of this study is to introduce new methods for evaluating masonry domes, the complicated details of structure, that are not important for this research, are ignored. Fig. 4 shows the finite element model of the dome structure. Structural solid elements are used for modeling the structure. For a lot of similarities between masonry and plain concrete, concrete smeared cracking model is used to simulate the material of the structure. The yield and failure surface of this kind of non-linear material is shown in Fig. 4. Parameters that are used for modeling the building by concrete smeared cracking material are shown in Table I. The fundamental period of the building was determined as 0.31 s which is close to 0.34 s which is calculated by Vasseghi for this structure [7].

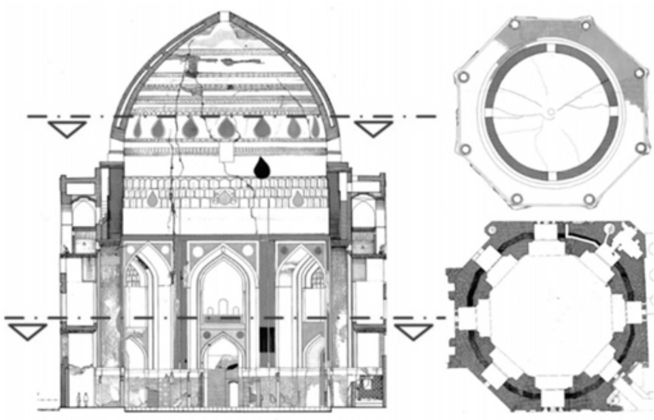


Fig. 3 The dome of Soltanieh, section and plans [7]

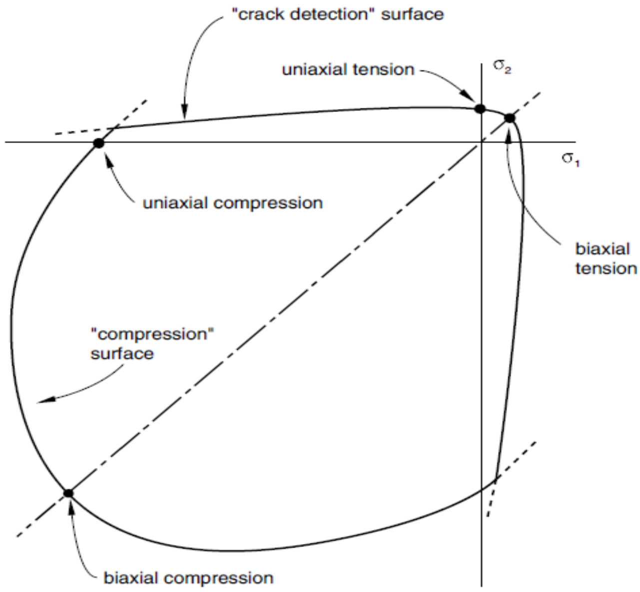


Fig. 4 Yield and failure surface in plain stress of concrete smeared cracking

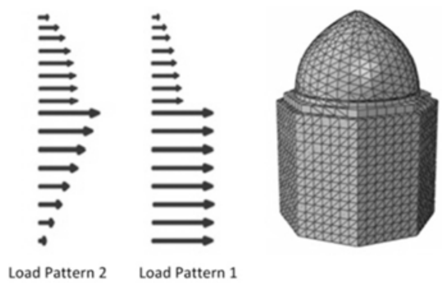


Fig. 5 Finite element and load patterns representations of the dome structure

TABLE I
PROPERTIES OF MATERIAL OF THE MASONRY DOME

Quantity	Value
Young's Modulus	2000 MPa
Poisson's Ratio	0.15
Mass Density	1600 kg/m^3
Damping	5%
Compressive Strength	8 MPa
Tensile Strength	0.8 MPa

The modeled dome was analyzed to determine the displacement under three selected ground motions which are El Centro, Northridge, and El Centro multiple of 2. The Peak Ground Acceleration (PGA) of the records are 0.32g, 0.67g, and 0.63g, respectively. For seismic evaluation of the building, two load patterns are proposed which are shown in Fig. 5. Load pattern 1 is given from constant body force, and load pattern 2 is given from triangular body force.

The conventional capacity curves were obtained from pushing the building under two load patterns and are shown in Fig. 6. The curves are represented by top point of dome displacement versus base shear.

Using Bispec computer program, and two ground motion records of El Centro and Northridge, the demand curves in T-Sa coordination were obtained, and after that, they converted to ADRS coordination. The demand curves for three records in ADRS coordination are shown in Fig. 7.

Intersection between demand and capacity curves in the same coordination is the performance point that represents the maximum response of the structure for a given ground motion. In Fig. 8, two superimposing curves for three ground motion of El Centro, Northridge and multiple of 2 El Centro are shown. The expected displacement for given ground motions are provided in Table II.

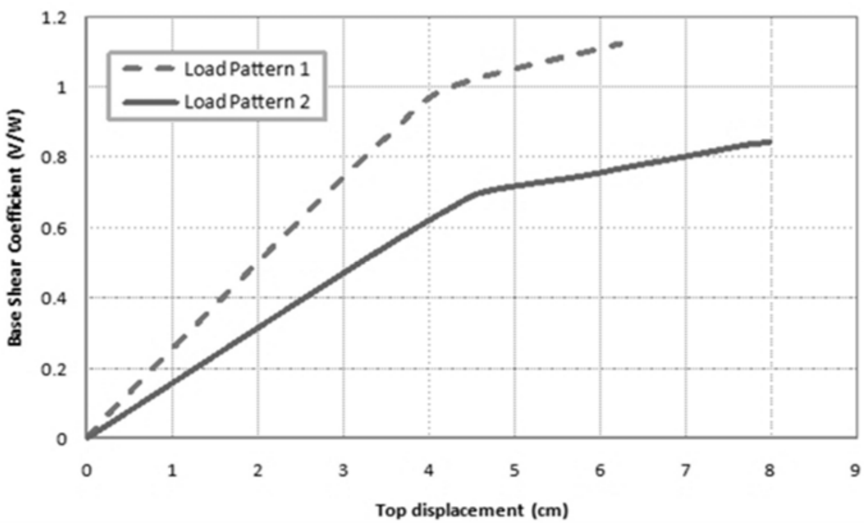


Fig. 6 Plot of base shear versus the top displacement for the dome of Soltanieh

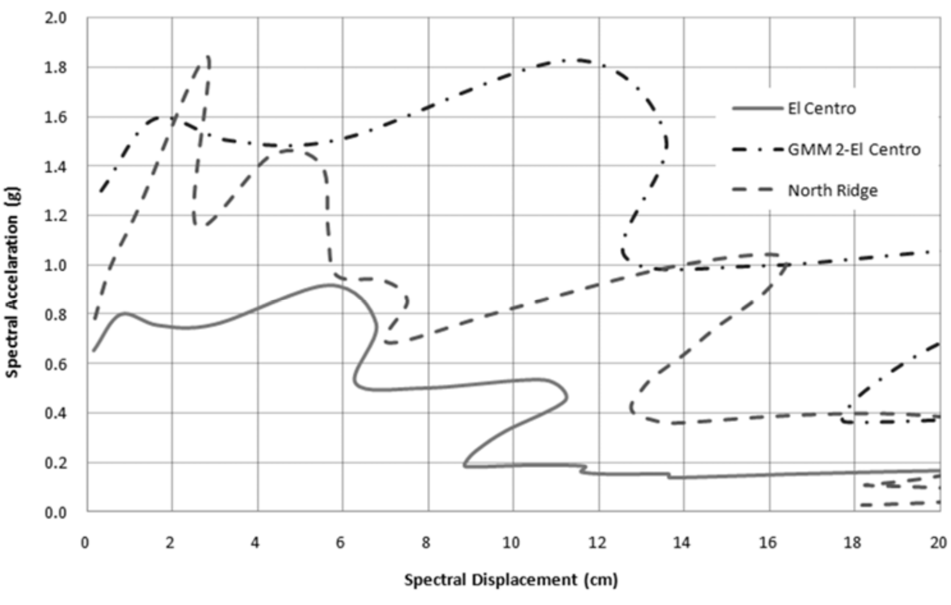


Fig. 7 Response spectra of three ground motion records

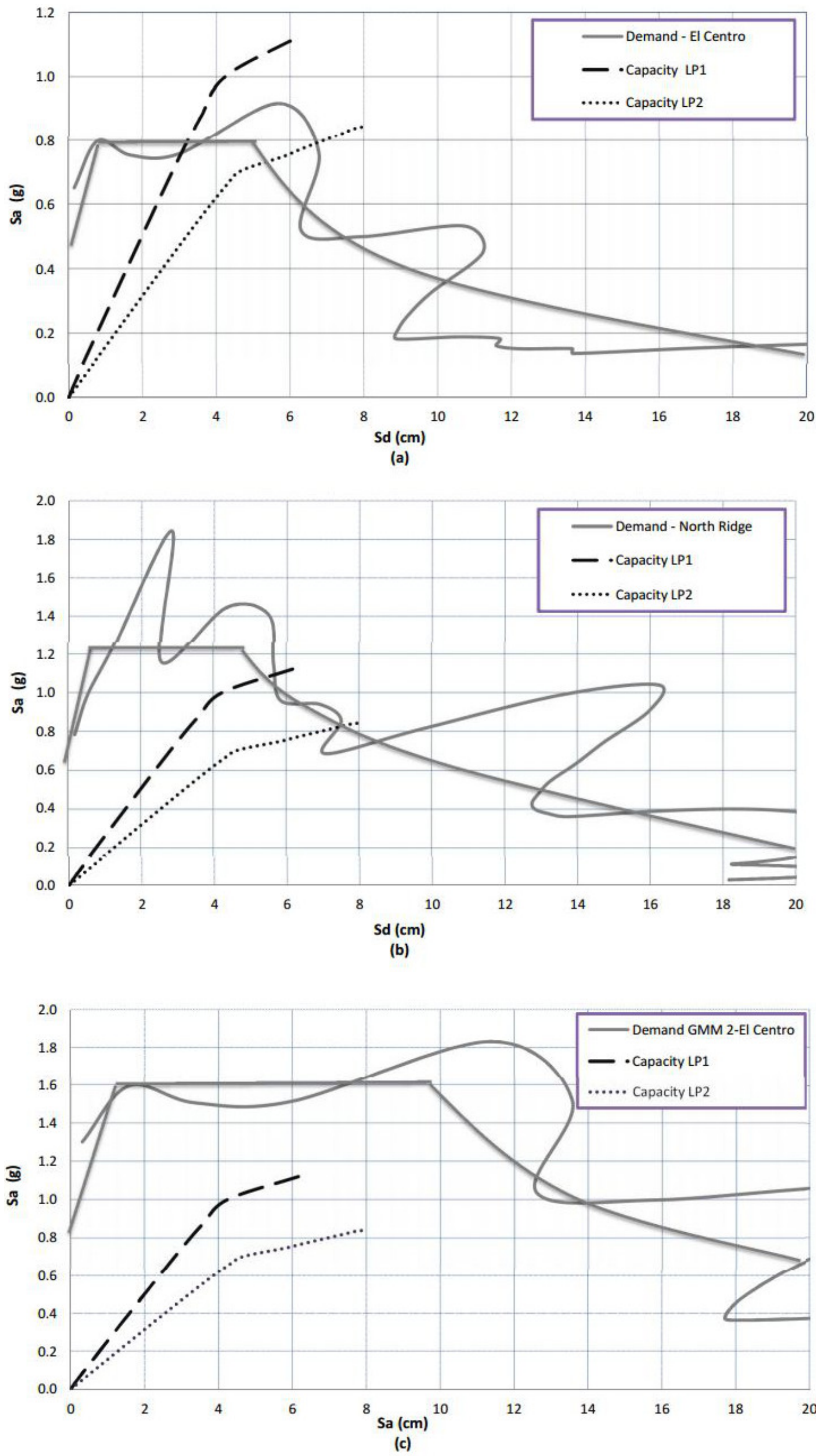


Fig. 8 Intersection of demand and capacity curves for three ground motion records of El Centro, Northridge and multiple 2 of El Centro, respectively

For seismic evaluation based on energy balance concept, input energy and absorbed energy must be calculated. For demand side, three parameters of mass, the energy factor and pseudo velocity are required to determine the input energy. Total mass of the building is considered as mass of the system, and for calculating the energy factor, the relationship between R_y-μ-T is required. Using the mentioned program, Bispec, and El Centro ground motion record, the relationships for strength reduction factor of 2, 4, and 8 are shown in Fig. 9.

As noted in (3), the energy factor can be calculated if the relationship between R_y-μ-T is provided. The energy factor for El Centro ground motion is shown in Fig. 8.

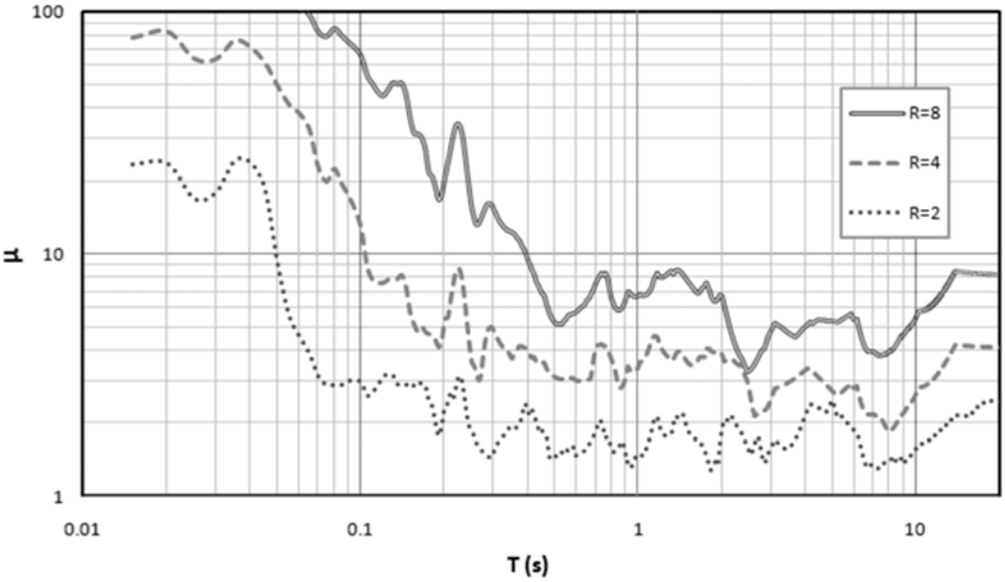


Fig. 9 Relationship between Rv-μ-T

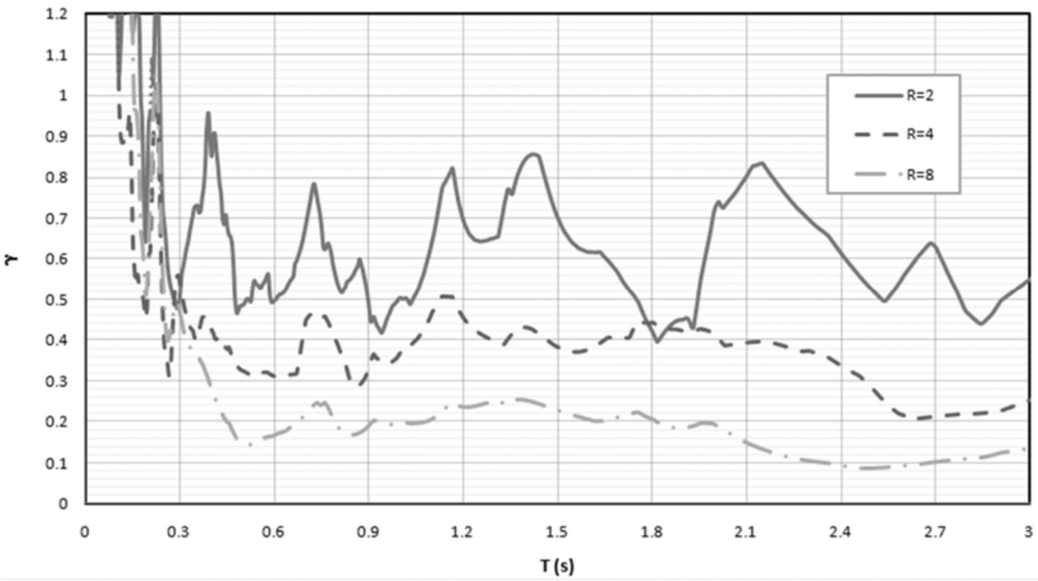


Fig. 10 Energy factor plot

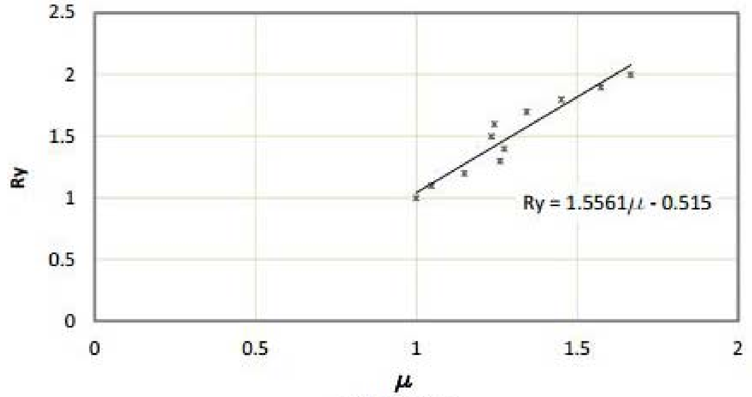
For seismic evaluation of the building with fundamental period 0.31 s, a series of dynamic analysis of SDOF system with the same period and different strength reduction factor were carried out to obtain the relationship between R_y and μ. Plot of R_y versus μ is generated for a system with period of 0.31s and is shown in Fig. 11. By Least-square fitting, the relationship is obtained and shown in Fig. 11 for El Centro and Northridge ground motion records.

Pseudo velocity curves are given from the response of series single degree of freedom structures under given ground motion records. The analyses were carried out by using Bispec and assuming the damping ratio to be 5%. The results are shown in Fig. 12.

For capacity side, the absorbed energy was calculated by numerically integration work done due to lateral loads. The absorbed energy curves are shown in Fig. 13. The curves are provided using the absorbed energy versus the displacement at the top point of the building.

The capacity-demand curves in energy coordination for two load patterns are shown in Figs. 14 and 15. The intersection between two curves of absorbed and input energy represents the maximum displacement value of the building that are shown in Table II.

In order to verify the results, the nonlinear dynamic procedures were done for comparing the maximum response displacement of each method with the exact results that are obtained by NDP. Top point displacement time history from NDP of example dome for three given ground motion record are shown in Fig. 16. The maximum displacements of each time history analysis are provided in Table II.



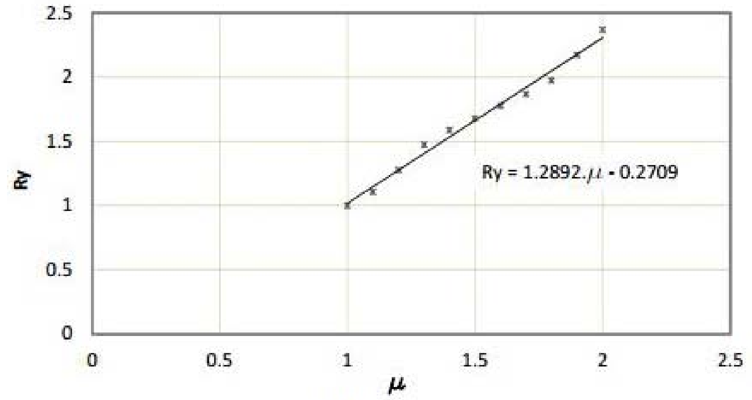


Fig. 11 Relationship between R_y-μ

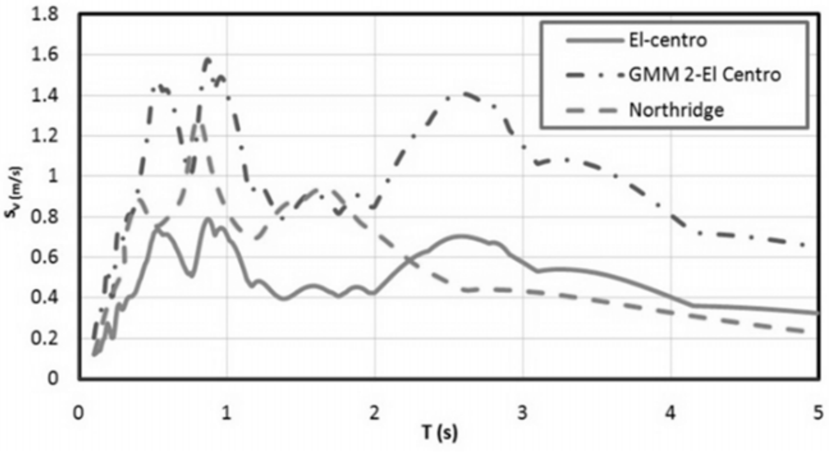


Fig. 12 Pseudo velocity spectra for three ground motion records

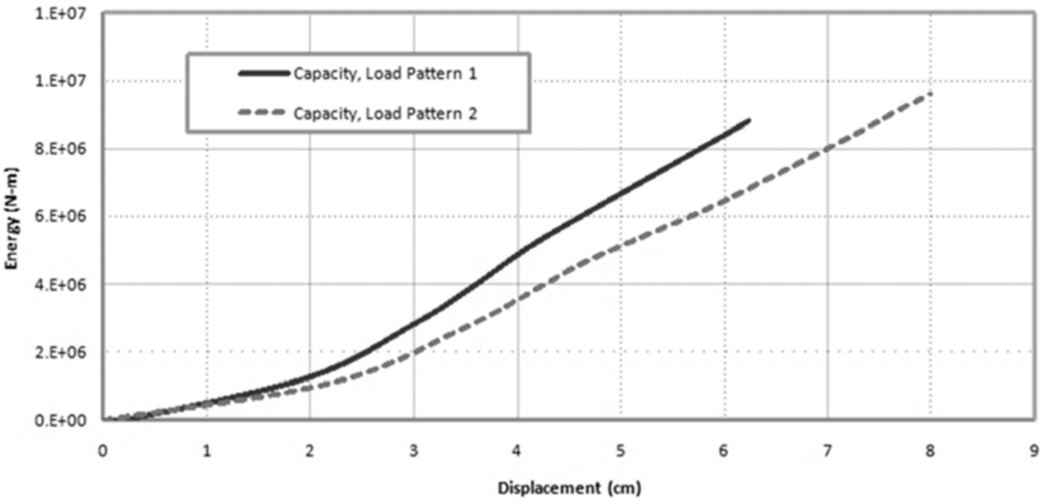


Fig. 13 Absorbed energy curves of Soltanieh dome

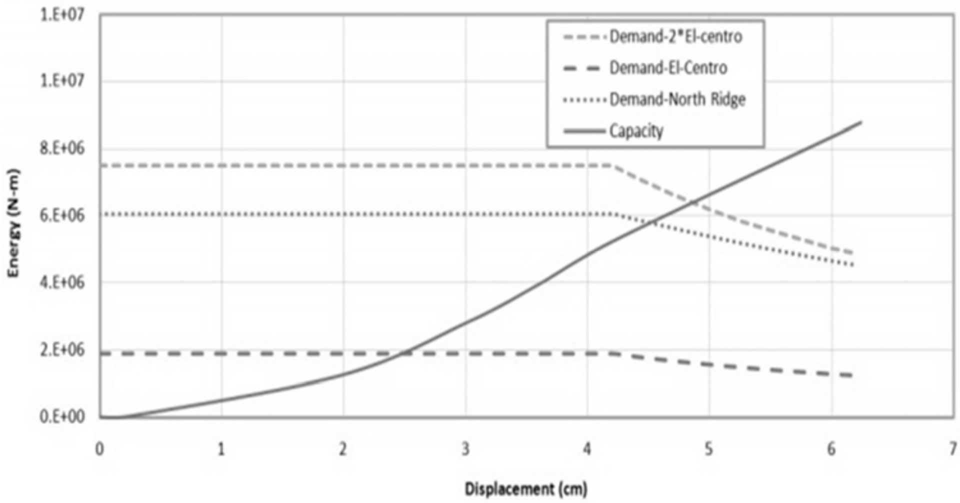


Fig. 14 Plot of capacity (based on load pattern 1) and demand curves of example dome for different ground motion record

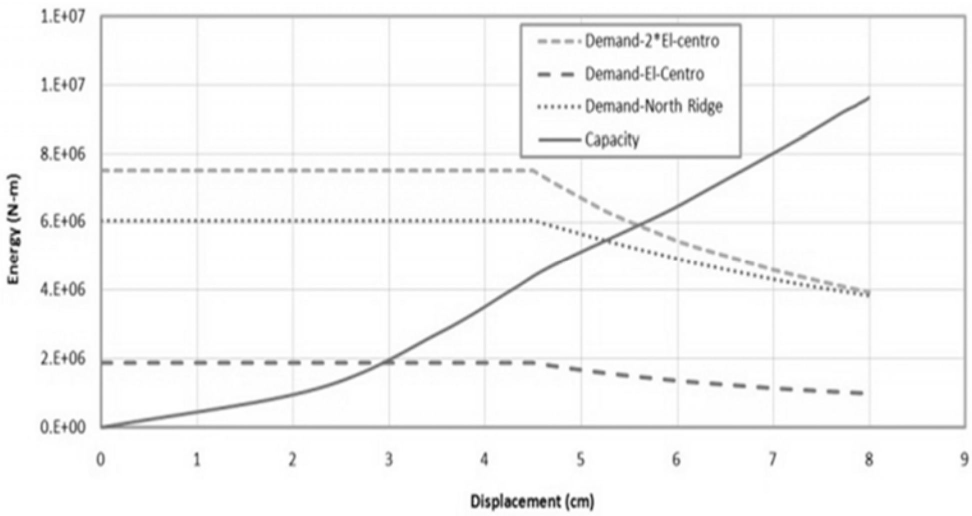


Fig. 15 Plot of capacity (based on load pattern 2) and demand curves of example dome for different ground motion record

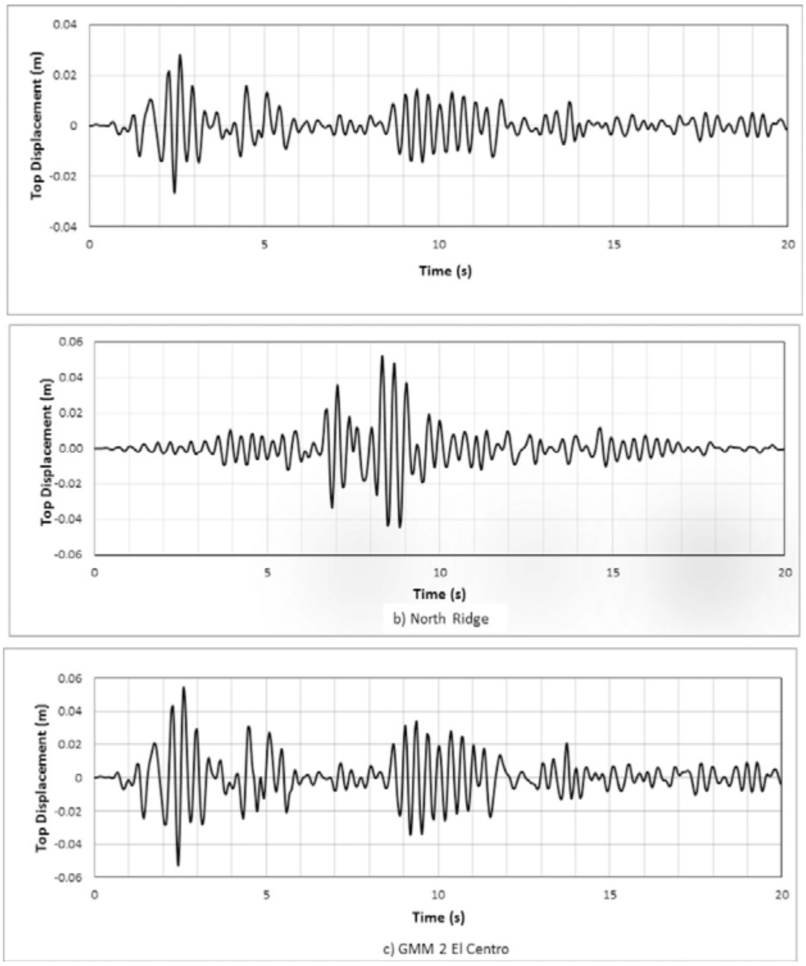


Fig. 16 Top displacement time history from nonlinear dynamic analyses for El Centro, Northridge and multiple 2 of El Centro, respectively

VI. Conclusion

This study is done for introducing the methods of nonlinear static analysis in order to evaluate the masonry dome, and for the aim of the research, the dome of Soltanieh is used for seismic evaluation. The procedure of pushover analysis that is used for multi-story building, is developed for the masonry dome structure. In addition, the evaluation method based on energy balance concept that is introduced recently, is used for assessment the structure of masonry dome. The main findings of the research are:

The comparison between the results are stated as follows

- The conventional pushover analyses provide the results that are not sufficiently accurate. Top displacement in masonry dome structure cannot represent the behavior of the structure; therefore, the usage of this point as the target displacement can lead to wrong results.
- The sufficient accurate results are obtained by using the evaluation method based on energy balance. This method can consider the total absorbed energy and it avoids the arbitrary use of conventional capacity and demand curves. In addition, there is no need to consider the structure as a SDOF in energy method.
- The concept of balance between the absorbed and input energy is more clear and reasonable than pushover analysis. In addition, practical application of this method is not very complicated. Consequently, the method is suitable for practical purpose.

TABLE II
Comparison of Results

	NSP				NSP (based on energy balance)				NDP
	Load Pattern1		Load Pattern 2		Load Pattern1		Load Pattern 2		
	Dis (cm)	Diff (%)	Dis (cm)	Diff (%)	Dis (cm)	Diff (%)	Dis (cm)	Diff (%)	Dis (cm)
El Centro	3.17	14.0%	5.23	88.1%	2.51	9.7%	2.98	7.2%	2.78
Northridge	5.44	4.2%	7.35	40.8%	4.45	14.8%	5.28	1.1%	5.22
GMM 2-El Centro	-	-	-	-	4.87	9.8%	5.53	2.4%	5.4

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