Performance Analysis of the Time-Based and Periodogram-Based Energy Detector for Spectrum Sensing

Sadaf Nawaz, Adnan Ahmed Khan, Asad Mahmood, Chaudhary Farrukh Javed

Abstract—Classically, an energy detector is implemented in time domain (TD). However, frequency domain (FD) based energy detector has demonstrated an improved performance. This paper presents a comparison between the two approaches as to analyze their pros and cons. A detailed performance analysis of the classical TD energy-detector and the periodogram based detector is performed. Exact and approximate mathematical expressions for probability of false alarm (Pf) and probability of detection (Pd) are derived for both approaches. The derived expressions naturally lead to an analytical as well as intuitive reasoning for the improved performance of (Pf) and (Pd) in different scenarios. Our analysis suggests the dependence improvement on buffer sizes. Pf is improved in FD, whereas Pd is enhanced in TD based energy detectors. Finally, Monte Carlo simulations results demonstrate the analysis reached by the derived expressions.

Keywords—Cognitive radio, energy detector, periodogram, spectrum sensing.

I. INTRODUCTION

COGNITIVE Radio (CR) is a transceiver which senses the spectrum and utilizes the unused portions of the spectrum efficiently. It monitors the frequency bands, and whenever a vacant slot is detected, it is assigned to the unlicensed (secondary) user, without interfering with the authorized (primary) user [1], [2]. Multiple spectrum sensing techniques have been proposed, including energy detection (ED) [3], matched filtering detection (MF) [4], cyclostationary detection (CSD) [5], eigenvalue-based sensing [6], covariance-based sensing [7], etc. Three main spectrum sensing techniques vigorously used to determine spectrum holes are: MF, CSD, and ED. MF uses a known pattern to correlate the signals. This approach maximizes the received Signal to Noise ratio (SNR) and thus it is the optimum sensing method but it requires a-priori information about the signal waveforms, which at times is not available [8]-[10]. CSD exploits some periodic characteristics of the desired signal to perform detection.

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Noise power and source signal information is required by CSD. It works well over low SNR regime but requires complex calculations [5], [11].

ED method identifies the presence or absence of a signal based on energy estimation. In this detection technique, only noise power information is required. It is not efficient at low SNR, but is the simplest method to implement [8], [12]. ED is usually implemented in TD. Recently, some works demonstrated improved performance of ED by the use of periodogram technique [1], [13]. Although the improved performance for periodogram based ED is claimed by these authors, yet an analytical or intuitive reasoning for this enhanced performance is not available in literature [1], [13]. This work aims to fill this gap by derivations of exact and approximate expressions for P_f and P_d, supported by extensive simulations using various performance parameters like SNR, buffer size and threshold to quantify the performance gap between TD and FD based energy detectors.

The rest of this paper is organized as follows. System model is described in Section II. Then, Section III presents the exact calculations. Next, Section IV gives the approximate calculations for both domains. Theoretical results verifications through Monte Carlo simulations are given in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

Let y (n) be the sampled received signal at time 'n'. Samples at different time are supposed to be independent and identically distributed (i.i.d). The received signal is represented by the two hypotheses, indicating presence and absence of primary user:

$$y(n) = \begin{cases} w(n) & H_0 \\ s(n) + w(n) & H_1 \end{cases} \quad (1)$$

where w(n) $\sim CN(0, \sigma_w^2)$ is an AWGN process, and s(n) is an i.i.d primary user's signal. Binary hypothesis in (1) is used to determine P_f, P_d and Probability of missed detection (P_m) [1]:

$$P_f = Prob[Decision = H_1|H_0] \quad (2)$$

$$P_m = Prob[Decision = H_0|H_1] \quad (3)$$

$$P_d = Prob[Decision = H_1|H_1] \quad (4)$$

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Vol:11, No:6, 2017

III. ED PERFORMANCE ANALYSIS VIA EXACT CLOSED FORM EXPRESSIONS

In the exact analysis, probability density function (PDF) and cumulative density function (CDF) expressions for central and non-central chi-square distribution [14], are used to derive closed form expressions for P_f and P_d. Derivation for TD and FD based energy detectors are as under:

A. Energy Detector in TD

In TD, the decision statistic of ED is expressed as

$$Y = \sum_{n=1}^{N} |y(n)|^2 \tag{5}$$

where y(n) are the samples of received signal as given in (1). Y has a non-central chi-square distribution with N degrees of freedom (DoF), under H_1. Otherwise, it has a central chi-square distribution with N DoF. P_f and P_d, already available [15], [16], are given as

$$P_f = \frac{\Gamma\left(\frac{N}{2}, \frac{\gamma}{2}\right)}{\Gamma\left(\frac{N}{2}\right)} \tag{6}$$

$$P_d = Q_{N/2}(\sqrt{2\lambda}, \sqrt{\gamma}) \tag{7}$$

where $\Gamma(.)$ is gamma function, $\Gamma(a, b)$ is incomplete gamma function, 'N' is number of samples, and 'γ' is sensing threshold [17]. $Q_{N/2}(a, b)$ is the generalized Marcum Q-function with $\lambda = \sum_{i=1}^{N} \left(\frac{\mu_i^2}{\sigma_i^2}\right)$ [18].

B. Energy Detector in FD

Here, the exact analysis is extended over FD and a periodogram based approach is presented. The decision statistic for this case is given as:

$$S(f) = \frac{1}{N} \left| \sum_{k=0}^{K-1} y(k) exp^{-jwk} \right|^2 = \frac{1}{N} |Y(f)|^2 \tag{8}$$

The decision statistic S(f) has exponential distribution and can be considered as chi-squared distributed with two DoF. Under H_0, S(f) has central chi-square distribution with two DoF. Else, it has non-central chi-square distribution with two DoF. CDF and PDF expressions given in [19] are used to derive new expressions for P_f and P_d.

$$P_x(x) = \int_0^\gamma \frac{1}{2\sigma^2} exp^{\frac{-x}{2\sigma^2}} d\gamma = 1 - exp^{\frac{-\gamma}{2\sigma^2}} \tag{9}$$

$$P_f = exp^{\frac{-\gamma}{2\sigma^2}} \tag{10}$$

where γ is the sensing threshold and σ^2 denotes the noise variance. A new, P_d expression is derived using non-central chi-square distribution with two DoF as:

$$P_x(x) = \int_0^y \frac{1}{2\sigma^2} exp^{\frac{-(s^2+y)}{2\sigma^2}} I_0\left(\sqrt{y}\, \frac{s}{\sigma^2}\right) dy \tag{11}$$

$$P_d = 1 - Pm = Q_1(\sqrt{2\lambda}, \sqrt{\gamma}) \tag{12}$$

Marcum Q function is dependent on the modified Bessel function of first kind I_{M-1} which in turn depends on the inverse gamma function having parameter N. Here, M=N/2 and N=DoF. In TD, N independent random variables in (5), show a DoF of N. Whereas in FD, exponential distribution in (8) presents a DoF of two. As the DoF increases in (7), the resultant Bessel function decreases, hence improving probability of detection. So, 'N' DoF in TD gives higher probability of detecting a primary signal as compared to two DoF for FD.

IV. ED PERFORMANCE ANALYSIS VIA APPROXIMATE CLOSED FORM EXPRESSIONS

In conventional TD ED, test statistic in (5) assumes the sum of independent random variables. The test statistics can be approximated by invoking central limit theorem (CLT) when buffer size (N) is large [20].

A. Energy Detector in TD

The TD ED uses the same decision statistic as in (5). For the approximate analysis, Q-functions already available in [21], are used to calculate P_f and P_d as:

$$P_f = Q\left(\frac{\gamma - \mu o}{\sigma 0}\right) \tag{13}$$

$$P_d = Q\left(\frac{\gamma - \mu 1}{\sigma 1}\right) \tag{14}$$

where μ_o and σ_o are the mean and standard deviation for H_0, and μ_1 and σ_1 are the mean and standard deviation for H_1. The mean and standard deviation expressions are easily deduced using the properties of normal distribution [22] as:

$$\mu_0 = N(\mu_w^2 + \sigma_w^2) \tag{15}$$

$$\sigma_0 = \sqrt{2N}(2\mu_w^2 \sigma_w^2 + \sigma_w^4) \tag{16}$$

$$\mu_1 = N[\sigma_s^2 + \sigma_w^2 + (\mu_s + \mu_w)^2] \tag{17}$$

$$\sigma_1 = \sqrt{2N}[2(\mu_s + \mu_w)^2(\sigma_s^2 + \sigma_w^2) + (\sigma_s^2 + \sigma_w^2)^2] \tag{18}$$

Assuming $\mu_w = \mu_s = 0$ for AWGN, P_f and P_d given in (13), (14) are evaluated using (15)-(18) as:

$$P_f = Q\left(\frac{\gamma - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right) \tag{19}$$

$$P_d = Q\left(\frac{\gamma - N(\sigma_w^2 + \sigma_s^2)}{\sqrt{2N}\,(\sigma_w^2 + \sigma_s^2)^2}\right) \tag{20}$$

where σ_w^2 is the noise variance, and σ_s^2 denotes the signal variance.

B. Energy Detector in FD

TD calculations are extended for FD and new expressions for P_f and P_d are derived in this subsection. In FD, the ED is implemented using power spectral density (PSD) estimation.

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The decision statistic in FD is same as in (8). Binary test hypothesis in (1) is used to determine P_f and P_d. New expressions for mean and variance, under H_0 and H_1 are calculated as:

$$\mu_0 = P(e^{jw}) = \frac{1}{N} E\left\{ \left| \sum_{k=0}^{N-1} w(k) exp^{-jwk} \right|^2 \right\} = \sigma_w^2 \quad (21)$$

$$\sigma_0 = Cov[P(e^{jw1})P(e^{jw2})] = \sigma_w^4$$

$$\mu_1 = P_1(e^{jw}) = \frac{1}{N} E\left\{ \left| \sum_{k=0}^{N-1} (w(k)+s(k)) exp^{-jwk} \right|^2 \right\} \quad (22)$$

$$\mu_1 = P_1(e^{jw}) = \sigma_w^2 + \sigma_s^2 \quad (23)$$

$$\sigma_1 = Cov[P_1(e^{jw1})P_1(e^{jw2})] = (\sigma_w^2 + \sigma_s^2)^2 \quad (24)$$

P_f and P_d are derived using (21-24) as:

$$P_f = Q\left(\frac{\gamma - \sigma_w^2}{\sigma_w^2} \right) \quad (25)$$

$$P_d = Q\left(\frac{\gamma - (\sigma_w^2 + \sigma_s^2)}{(\sigma_w^2 + \sigma_s^2)} \right) \quad (26)$$

V. SIMULATION RESULTS

The derived closed form expressions, both for exact and approximate analysis, are verified through Monte Carlo Simulations. A BPSK signal is passed over an AWGN channel. Neyman-Pearson criterion is used to determine the threshold [8]. Theoretical and simulated results can be evaluated for any given value of SNR and noise variance. However, for convenience, SNR=3 dB and $\sigma_w^2 = 1$ dB are frequently used, and results are estimated as following:

A. Probability of False Alarm for Variable N

Fig. 1 shows improved P_f when FD is considered. Exact and approximate expressions in (10) and (25) show that FD does not depend on buffer size N. The buffer size affects the noise variance. If N is increased or decreased, FD results are not affected. However, as N increases in TD, noise variance is enhanced, and performance starts to deteriorate in TD. Hence, FD outperforms TD in terms of P_f, as FD is independent of buffer length.

B. Probability of Detection for Variable Buffer Size

Fig. 2 shows the theoretical and simulated results for P_d. When buffer size is increased in TD, signal variance gets improved. It is evident from (12) and (26) that FD does not depend on buffer length. As the length is increased, signal variance remains unaffected. It is observed that P_d improves as the number of samples increases. Higher the buffer size, better the P_d. Hence, TD performs better than FD in terms of P_d.

C. ROC Analysis

Approximate and exact results are evaluated for both TD and FD as shown in Figs. 3 and 4.
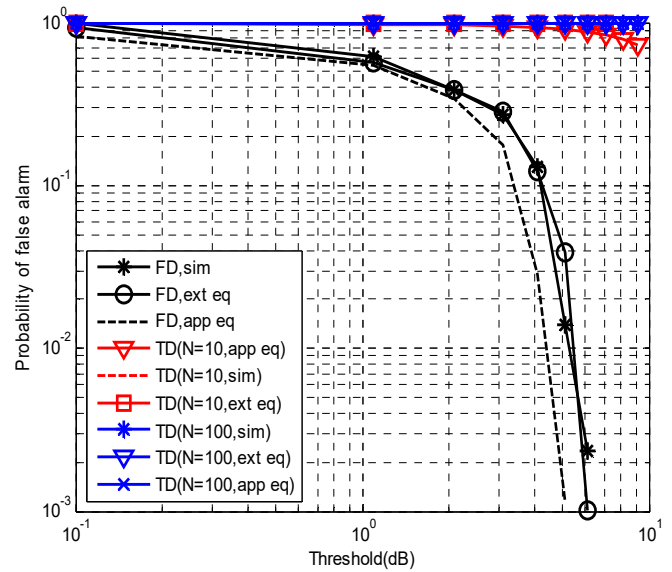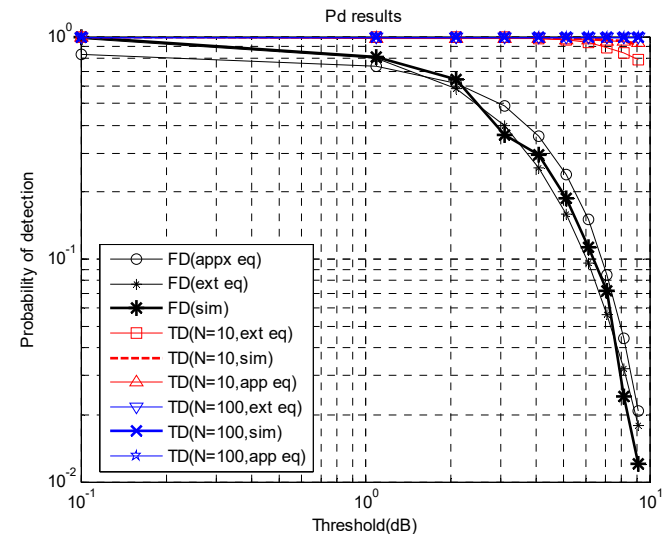

Fig. 1 P_f vs. Threshold for Variable N

Fig. 2 P_d vs. Threshold for Variable N

It is observed that as N increases, TD gives better P_m than FD. TD is dependent on N as stated in (7) and (20), P_m improves as buffer length increases and signal variance rises. The chance of detection gets enhanced as the number of samples increases. The theoretical and simulated results for both analyses are in accordance. Hence, P_m improves in TD, by using Neyman-Pearson detector.

D. Probability of Missed Detection for Variable SNR

Figs. 5 and 6 show the performance of P_m over SNR regime for approximate and exact analysis respectively.

It is evident from (7) and (20) that P_d for TD dependents on the buffer size N. Exact and approximate analysis show that P_m improves as buffer size is increased. On the other hand, FD does not depend on N as given in (12) and (26). SNR improves with increasing N because the signal variance gets enhanced. Hence, TD performs better than FD.
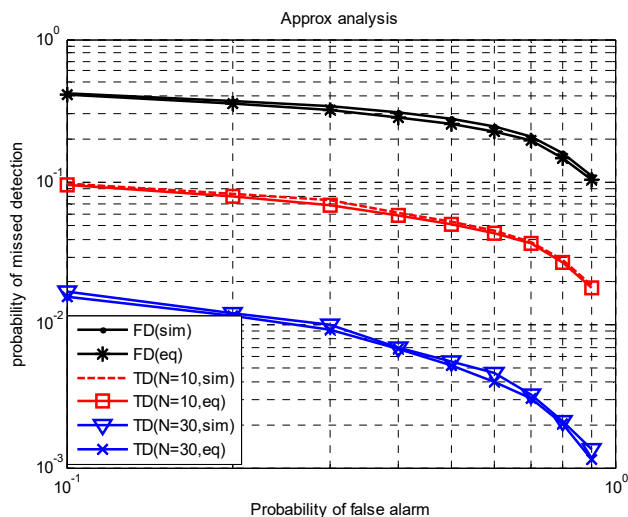
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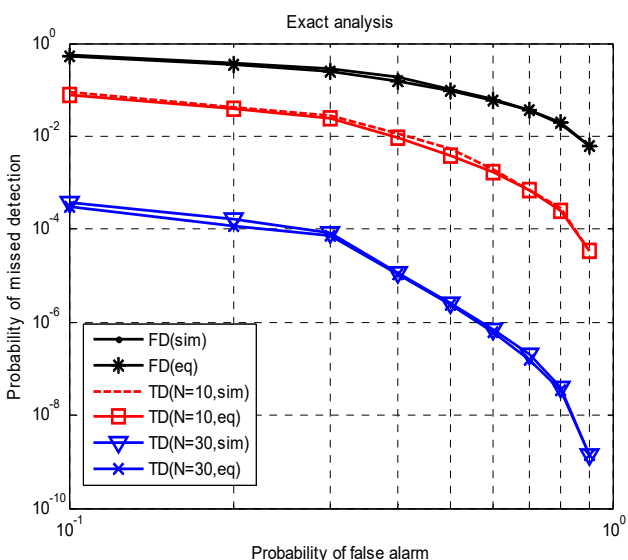Fig. 3 P_m vs. P_f for approximate analysis

Fig. 4 P_m vs. P_f for exact analysis

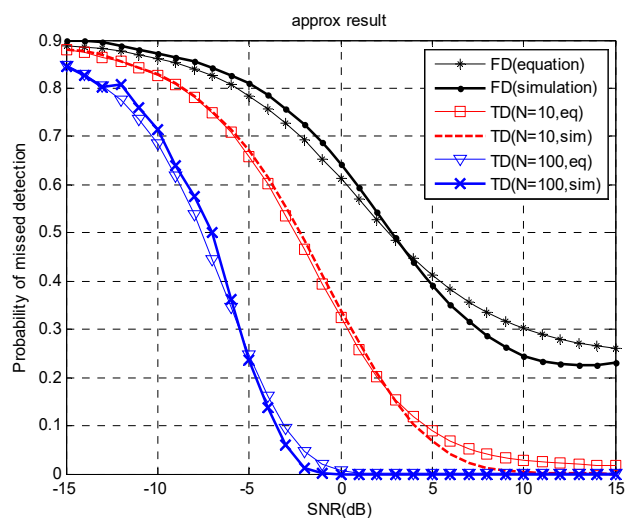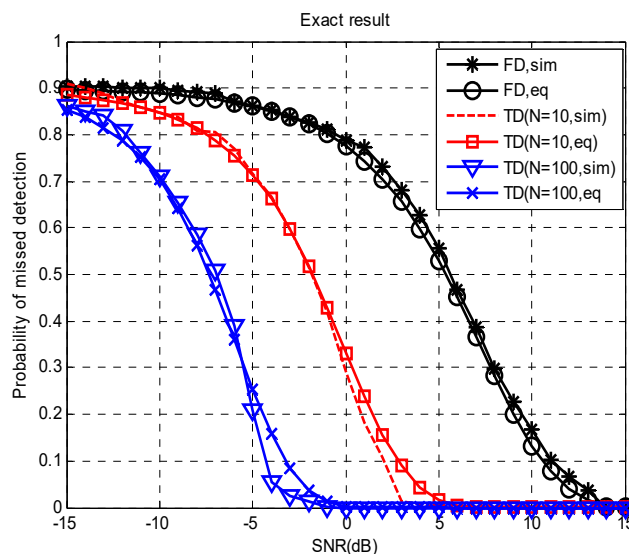Fig. 5 P_m vs. SNR for approximate analysis

Fig. 6 P_m vs. SNR for exact analysis

E. Probability of False Alarm and Detection For Time Averaging

Periodogram based detection performs averaging in FD, whereas the TD decision statistic does not perform averaging of energy samples as given in (5) and (8). Instead of using the simple energy decision statistic for TD, a new averaged TD energy detector is proposed. The decision statistic is given as

$$Y = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2 \qquad (27)$$

The mean and variance of time averaged decision statistic is derived, and P_f and P_d are obtained using (6), (7) as

$$P_f = Q\left(\frac{\gamma - \sigma_w^2}{\sqrt{2}\sigma_w^2}\right) \qquad (28)$$

$$P_d = Q\left(\frac{\gamma - (\sigma_w^2 + \sigma_s^2)}{\sqrt{2}\,(\sigma_w^2 + \sigma_s^2)^2}\right) \qquad (29)$$

The P_f and P_d expressions are evaluated against variable noise variance and SNR values. Fig. 7 shows that P_f improves as noise variance is reduced. Noise variance is low for FD as evident from (25). However, noise variance gets enhanced in case of time averaged ED as given in (28). Higher noise variance deteriorates the performance of P_f. Hence, lower the noise variance, better the P_f. FD ED performs better than averaged TD detector.

It is evident from Fig. 8 that, higher the SNR, better the P_d. Noise variance is reduced by increasing SNR in TD, as shown in (26). However, in FD, noise variance gets enhanced by increasing SNR, and P_d deteriorates as given in (29). Hence, TD gives better P_d.

The theoretical and simulated results discussed in Figs. 1-8 are in accordance. It is evident from the analytical and simulated analysis that P_f gets improved in FD. Whereas, P_d is enhanced in TD.

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Fig. 7 P_f vs. Threshold for time averaging

Fig. 8 P_d vs. Threshold for time averaging

VI. CONCLUSION

An analytical or intuitive reasoning behind improved performance of FD and TD based energy detectors is provided in this paper. Mathematical analysis and simulations are performed for both TD and FD energy detectors over AWGN channel. It is observed that FD gives better P_f when buffer size is increased. TD gives improved P_d when observation length is enhanced, Neyman-Pearson detector is used, and SNR is varied. It is also observed that time averaged ED does not bring any improvement over the classical TD ED. Further analysis in fading channels will be carried out as future work to ascertain the results.

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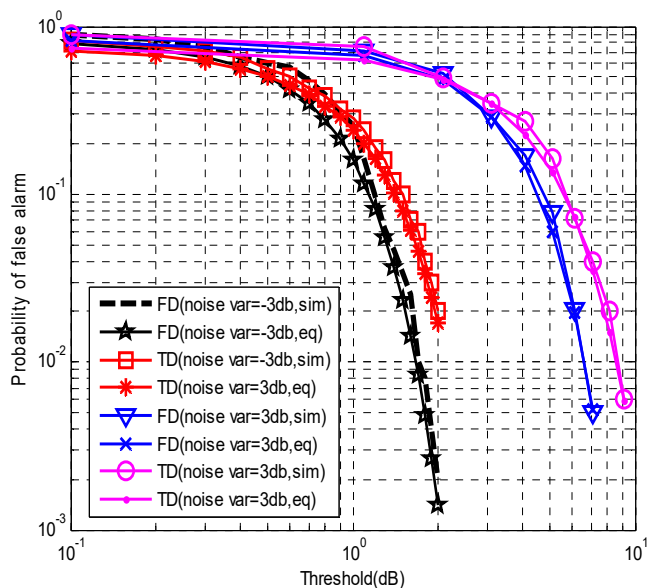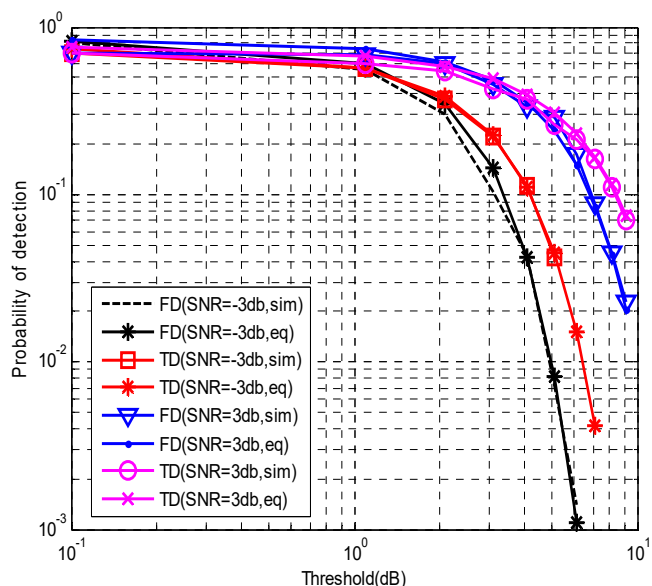[22] W. Brye, "Normal Distributions" in Normal Distribution Characterizations with Applications, 1st ed. Berlin: Springer, 1995, pp. 19-20.