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Four Positive Almost Periodic Solutions to an Impulsive Delayed Plankton Allelopathy System with Multiple Exploit (or Harvesting) Terms

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Abstract—In this paper, we obtain sufficient conditions for the existence of at least four positive almost periodic solutions to an impulsive delayed periodic plankton allelopathy system with multiple exploited (or harvesting) terms. This result is obtained through the use of Mawhins continuation theorem of coincidence degree theory along with some properties relating to inequalities.

Keywords—Almost periodic solutions, plankton allelopathy system, coincidence degree, impulse.

I. Introduction

The study of large fluctuations in the population size and density of phytoplankton communities is an important subject in aquatic ecology. Workers have attributed these fluctuations to several factors, such as physical factors, variation of necessary nutrients, and a combination of the two. Another important observation made is that, by the production of allelopathic toxins or stimulators, the increased population of one species might affect the growth of another species, thus influencing seasonal succession [7].

Maynard Smith [17] incorporated the effect of toxic substances in a two-species Lotka-Volterra competitive system by considering that each species produces a substance that is toxic to the other but only when the other is present. The model was

$$\begin{cases} y_1'(t) = y_1(t)[r_1 - \alpha_1 y_1(t) - \beta_1 y_2(t) - \\ \qquad\qquad \gamma_1 y_1(t) y_2(t)], \\ y_2'(t) = y_2(t)[r_2 - \alpha_2 y_2(t) - \beta_2 y_1(t) - \\ \qquad\qquad \gamma_2 y_1(t) y_2(t)], \end{cases} \tag{1}$$

However, Mukhopadhyay et al. [1] suggested that a species needs some time to mature to produce a substance that will be toxic (or stimulatory) to another; i.e., the production of a toxic substance by the competing species is not instantaneous. Therefore, a delay term in the system is necessary to capture the time lag required for such a maturity. They studied the revised model

$$\begin{cases} y_1'(t) = y_1(t)[r_1 - \alpha_1 y_1(t) - \beta_1 y_2(t) - \\ \qquad\qquad \gamma_1 y_1(t) y_2(t-\tau_2)], \\ y_2'(t) = y_2(t)[r_2 - \alpha_2 y_2(t) - \beta_2 y_1(t) - \\ \qquad\qquad \gamma_2 y_1(t-\tau_1) y_2(t)], \end{cases} \tag{2}$$

A species might also experience abrupt changes of state. This can occur due to certain seasonal effects, such as weather change, food supply, and mating habits. As a result, the population levels of a species repeatedly undergo changes of relatively short duration at certain moments of time due to the existence of these external forces. However, the duration of these changes is often negligible in comparison with that of the entire evolution process and thus these abrupt changes can be well-approximated as impulses. To accurately describe this ecological system, one may use impulsive differential equations and many papers investigate impulsive ecological systems in this way (see [11]-[13], [18], [20], [23], [25], [31], [35], [37]-[39], [41]).

Since biological and environmental parameters are naturally subject to fluctuations over time, the effects of a periodically varying environment are considered important selective forces on systems in a fluctuating environment. The ecological system is often deeply perturbed by the activities of human exploitation, such as planting and harvesting. It is more realistic to consider almost periodic systems than periodic systems and, over all kinds of population models, many excellent results have been obtained from the study of positive almost periodic solutions (see [3], [8], [9], [19], [32], [33], [40]). However, few results are available for the existence of positive almost periodic solutions to the impulsive delayed plankton allelopathy system with multiple exploited (or harvesting) terms.

In 2013, Li & Ye [32] studied the existence multiple positive almost periodic solutions to an impulsive non-autonomous Lotka-Volterra predator-prey system with harvesting terms. The authors first introduced a new method to discuss the existence multiple positive almost periodic solutions to population models by using Mawhin's continuation theorem of coincidence degree. Moreover, their method can be used to study other types of population systems.

Motivated by the above and applying the method analogous to the one used by Li & Ye, the purpose of this paper is to study the existence of multiple positive almost periodic solutions of a delayed plankton allelopathy system with multiple exploited (or harvesting) terms. In addition, we consider the impact generated by the coexistence of multiple generations of a species. To the best of our knowledge, there are few results of the existence of four positive almost periodic solutions for this kind of system.

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$$\begin{cases} x_1'(t) = x_1(t)\big[r_1(t) - a_1(t)x_1(t) - \sum_{i=1}^{n} b_{1i}(t)\cdot \\ \quad x_2(t-\tau_{2i}(t)) - \sum_{i=1}^{n} c_{1i}(t)x_1(t)x_2(t- \\ \quad \sigma_{2i}(t))\big] - h_1(t) \\ x_2'(t) = x_2(t)\big[r_2(t) - \sum_{i=1}^{n} a_{2i}(t)x_1(t- \\ \quad \tau_{1i}(t)) - b_2(t)x_2(t) - \sum_{i=1}^{n} c_{2i}(t)x_2(t)\cdot \\ \quad x_1(t-\sigma_{1i}(t))\big] - h_2(t) \\ x_m(t_k^+) = (1+\Gamma_{mk})x_m(t_k), m = 1,2. \end{cases} \tag{3}$$

where $x_1(t), x_2(t)$ are the population densities of two competing species. $r_1(t), r_2(t)$ are the first and the second specific intrinsic rates of increase, $a_1(t), b_2(t)$ are the rates of intra specific competition of the first and second species respectively. $b_{1i}(t), a_{2i}(t), (i = 1, 2, \cdots, n)$ stand for the ith generation's inter specific competition rates of the first and the second species. $\tau_{1i}(t), \tau_{2i}(t)$ are the time delays required for the maturities of the ith generation of the first and second species. $c_{1i}(t)$ and $c_{2i}(t)$ are the rates of toxic inhibition about the ith generation of the first species by the second and vice versa. $\sigma_{1i}(t)$ and $\sigma_{2i}(t)$ are the time delays required for making inhibition toxins of the ith generation of the first and the second species. $h_1(t)$ and $h_2(t)$ are the harvesting rate of the first and the second specifies; $r_1(t), r_2(t), a_1(t), a_{2i}(t), b_{1i}(t), b_2(t), c_{1i}(t), c_{2i}(t), h_1(t), h_2(t), (i = 1, 2\cdots, n)$ are all continuous positive ω-almost periodic functions.

The organization of this paper is as follows. In Section II, we make some preparations and state lemmas that are useful in the following sections. In Section III, we apply Mawhins continuation theorem of coincidence degree theory to establish sufficient conditions for the existence of multiple positive almost periodic solutions to system. A conclusion is given in Section IV.

II. Preliminaries

We first introduce some basic notations. Let $AP(\mathbf{R}) = \{p(t) : p(t)$ is a continuous, real valued, almost periodic function on $\mathbf{R}\}$. Suppose that $f(t, \phi)$ is almost periodic in t, uniformly with respect to $\phi \in C([-\sigma, 0], R)$. $T(f, \epsilon, S)$ will denote the set of ϵ-almost periods with respect to $S \subset C([-\sigma, 0], R), l(\epsilon, S)$ the inclusion interval, $\Lambda(f)$ the set of Fourier exponents, mod(f) the module of f, and $m(f)$ the mean value.

Lemma 1: If $f(t) \in AP(\mathbf{R})$, then there exists $t_0 \in R$ such that $f(t_0) = m(f)$.

Lemma 2: Assume that $x(t) \in AP(\mathbf{R})$, then $x(t)$ is bounded on $\mathbf{R}$.

Lemma 3: [32] Assume that $x(t) \in AP(\mathbf{R}) \cap C^1(\mathbf{R}, \mathbf{R})$, then there exist two points sequences $\{\xi_k\}_{k=1}^{\infty}, \{\eta_k\}_{k=1}^{\infty}$ such that $N'(\xi_k) = N'(\eta_k) = 0, \lim_{k\to\infty} \xi_k = \infty$ and $\lim_{k\to\infty} \eta_k = -\infty$.

Lemma 4: [32] Assume that $N(t) \in AP(\mathbf{R}) \cap C^1(\mathbf{R}, \mathbf{R})$, then $N(t)$ falls into one of the following four cases:

(i) There are $\xi, \eta \in R$ such that $N(\xi) = \sup_{t\in\mathbf{R}} N(t)$ and $N(\eta) = \inf_{t\in\mathbf{R}} N(t)$. In this case, $N'(\xi) = N'(\eta) = 0$.

(ii) There are no $\xi, \eta \in \mathbf{R}$ such that $N(\xi) = \sup_{t\in\mathbf{R}} N(t)$ and $N(\eta) = \inf_{t\in\mathbf{R}} N(t)$. In this case, for any $\epsilon > 0$, there are exist two points $\xi, \eta \in \mathbf{R}$ such that $N'(\xi) = N'(\eta) = 0, N(\xi) > \sup_{t\in\mathbf{R}} N(t) - \epsilon$ and $N(\eta) < \inf_{t\in\mathbf{R}} N(t) + \epsilon$.

(iii) There is a $\xi \in \mathbf{R}$ such that $N(\xi) = \sup_{t\in\mathbf{R}} N(t)$ and there is no $\eta \in \mathbf{R}$ such that $N(\eta) = \inf_{t\in\mathbf{R}} N(t)$. In this case, $N'(\xi) = 0$ and for any $\epsilon > 0$, there exists an η such that $N'(\xi) = N'(\eta) = 0$ and $N(\eta) < \inf_{t\in\mathbf{R}} N(t) + \epsilon$.

(iv) There is an $\eta \in \mathbf{R}$ such that $N(\eta) = \inf_{t\in\mathbf{R}} N(t)$ and there is no $\xi \in \mathbf{R}$ such that $N(\xi) = \sup_{t\in\mathbf{R}} N(t)$. In this case, $N'(\eta) = 0$ and for any $\epsilon > 0$, there exists an ξ such that $N'(\xi) = N'(\eta) = 0$ and $N(\xi) > \sup_{t\in\mathbf{R}} N(t) - \epsilon$.

Let $PC(\mathbf{R}, \mathbf{R}) = \{\varphi : \mathbf{R} \to \mathbf{R}, \varphi$ is a piecewise continuous function with points of discontinuity of the first kind at $t_k, k = 1, 2, \ldots,$ at which $\varphi(t_k^-)$ and $\varphi(t_k^+)$ exit and $\varphi(t_k^-) = \varphi(t_k)\}$.

Since the solutions of system (3) belong to the space $PC(\mathbf{R}, \mathbf{R}^2)$, we adopt the following definitions for almost periodicity.

Definition 1: The family of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in \mathbf{Z}\}$ is said to be equipotentially almost periodic if for arbitrary $\epsilon > 0$, there exists a relatively dense set of ϵ-almost periods, that are common for any sequences.

Definition 2: The function $\varphi \in PC(\mathbf{R}, \mathbf{R})$ is said to be almost periodic, if the following conditions hold:

(1) the set of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in \mathbf{Z}\}$ is equipotentially almost periodic;

(2) for any $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that if the points t_1 and t_2 belong to the same interval of continuity of $\varphi(t)$ and $|t_1 - t_2| < \delta$, then $|\varphi(t_1) - \varphi(t_2)| < \epsilon$;

(3) for any $\epsilon > 0$ there exists a relatively dense set T of ω-almost periodic such that if $\tau \in T$, then $|\varphi(t+\tau) - \varphi(t)| < \epsilon$ for all $t \in \mathbf{R}$ which satisfy the condition $|t - t_k| > \epsilon, k \in \mathbf{Z}$.

Consider the following system

$$\begin{cases} y_1'(t) = y_1(t)\big[r_1(t) - \bar{a}_1(t)y_1(t) - \sum_{i=1}^{n} \bar{b}_{1i}(t)\cdot \\ \quad y_2(t-\tau_{2i}(t)) - \sum_{i=1}^{n} \bar{c}_{1i}(t)y_1(t) \\ \quad y_2(t-\sigma_{2i}(t))\big] - \bar{h}_1(t) \\ y_2'(t) = y_2(t)\big[r_2(t) - \sum_{i=1}^{n} \bar{a}_{2i}(t)y_1(t- \\ \quad \tau_{1i}(t)) - \bar{b}_2(t)y_2(t) - \sum_{i=1}^{n} \bar{c}_{2i}(t)y_2(t)\cdot \\ \quad y_1(t-\sigma_{1i}(t))\big] - \bar{h}_2(t) \end{cases} \tag{4}$$

where

$$\begin{aligned} \bar{a}_1(t) &= a_1(t)\textstyle\prod_{0<t_k<t}(1+\Gamma_{1k}), \\ \bar{a}_{2i}(t) &= a_{2i}(t)\textstyle\prod_{0<t_k<t}(1+\Gamma_{1k}), \\ \bar{b}_2(t) &= b_2(t)\textstyle\prod_{0<t_k<t}(1+\Gamma_{2k}), \\ \bar{b}_{1i}(t) &= b_{1i}(t)\textstyle\prod_{0<t_k<t}(1+\Gamma_{2k}), \end{aligned}$$

$$\begin{aligned}\bar{c}_{1i}(t) = &\ c_{1i}(t)\prod_{0<t_k<t}(1+\Gamma_{1k})(1+\Gamma_{2k}),\\ \bar{c}_{2i}(t) = &\ c_{2i}(t)\prod_{0<t_k<t}(1+\Gamma_{1k})(1+\Gamma_{2k}),\\ \bar{h}_1(t) = &\ h_1(t)\prod_{0<t_k<t}\frac{1}{1+\Gamma_{1k}},\\ \bar{h}_2(t) = &\ h_2(t)\prod_{0<t_k<t}\frac{1}{1+\Gamma_{2k}},\end{aligned}$$

Lemma 5: For systems (3) and system (4), the following results hold:

(1) if $(y_1(t), y_2(t))^T$ is a solution of (4), then

$$(x_1(t), x_2(t))^T = \Big(\prod_{0<t_k<t}(1+\Gamma_{1k})y_1(t), \prod_{0<t_k<t}(1+\Gamma_{2k})y_2(t)\Big)^T$$

is a solution of (3).

(2) if $(x_1(t), x_2(t))^T$ is a solution of (3), then

$$(y_1(t), y_2(t))^T = \Big(\prod_{0<t_k<t}(1+\Gamma_{1k})^{-1}x_1(t), \prod_{0<t_k<t}(1+\Gamma_{2k})^{-1}x_2(t)\Big)^T$$

is a solution of (4).

Proof: Suppose that $(y_1(t), y_2(t))^T$ is a solution of (4). Let

$$x_m(t) = \prod_{0<t_k<t}(1+\Gamma_{mk})y_m(t), m = 1, 2,$$

then for any $t \neq t_k, k \in \mathbf{Z}^+$, by substituting

$$y_m(t) = \prod_{0<t_k<t}(1+\Gamma_{mk})^{-1}x_m(t), m = 1, 2,$$

into system (3), we can easily verify that the first and the second equations of system (3) holds.

For $t = t_k, k \in \mathbf{Z}^+, m = 1, 2,$ we obtain

$$\begin{aligned}x_m(t_k^+) &= \lim_{t\to t_k^+}\prod_{0<t_k<t}(1+\Gamma_{mk})y_m(t)\\ &= \prod_{0<t_s\le t_k}(1+\Gamma_{ms})y_m(t_k)\\ &= (1+\Gamma_{mk})\prod_{0<t_s<t_k}(1+\Gamma_{ms})y_m(t_k)\\ &= (1+\Gamma_{mk})x_m(t_k).\end{aligned}$$

Hence, the second equation of system (3) also holds. Thus $(x_1(t), x_2(t))^T$ is a solution of system (3).

(2) We first show that $y_m(t), m = 1, 2$ are continuous. Since $y_m(t), m = 1, 2$ are continuous on each interval $(t_k, t_{k+1}]$, it is sufficient to check the continuity of $y_s(t)$ at the impulse points $t_k, k \in \mathbf{Z}^+$. Since $y_m(t) = \prod_{0<t_k<t}(1+\Gamma_{mk})^{-1}x_m(t), m = 1, 2,$ we have

$$\begin{aligned}y_m(t_k^+) &= \prod_{0<t_s\le t_k}(1+\Gamma_{ms})^{-1}x_m(t_k^+)\\ &= \prod_{0<t_s<t_k}(1+\Gamma_{ms})^{-1}x_m(t_k) = y_m(t_k),\\ y_m(t_k^-) &= \prod_{0<t_s<t_k}(1+\Gamma_{ms})^{-1}x_m(t_k^-)\\ &= \prod_{0<t_s<t_k}(1+\Gamma_{ms})^{-1}x_m(t_k) = y_m(t_k).\end{aligned}$$

Thus $y_m(t), m = 1, 2$ is continuous on $[0, \infty)$. It is easy to prove that $(y_1(t), y_2(t))^T$ satisfies system (3). Therefore, it is a solution of system (4). This completes the proof of lemma 7. ■

For the sake of convenience, we denote $f^l = \inf_{t\in\mathbf{R}} f(t),\ f^M = \sup_{t\in\mathbf{R}} f(t),$ here $f(t)$ is a positive continuous almost periodic function.

For simplicity, we need to introduce some notations as:

$$\begin{aligned}l_1^{\pm} &= \frac{r_1^M \pm \sqrt{(r_1^M)^2 - 4\bar{a}_1^l\bar{h}_1^l}}{2\bar{a}_1^l},\\ l_2^{\pm} &= \frac{r_2^M \pm \sqrt{(r_2^M)^2 - 4\bar{b}_2^l\bar{h}_2^l}}{2\bar{b}_2^l},\\ A_1^{\pm} &= \frac{r_1^l - \sum_{i=1}^n \bar{b}_{1i}^M l_2^+ - \sum_{i=1}^n \bar{c}_{1i}^M l_1^+ l_2^+ \pm \sqrt{S_1}}{2\bar{a}_1^M},\\ S_1 &= (r_1^l - \sum_{i=1}^n \bar{b}_{1i}^M l_2^+ - \sum_{i=1}^n \bar{c}_{1i}^M l_1^+ l_2^+)^2 - 4\bar{a}_1^M\bar{h}_1^m,\\ A_2^{\pm} &= \frac{r_2^l - \sum_{i=1}^n \bar{a}_{2i}^M l_1^+ - \sum_{i=1}^n \bar{c}_{2i}^M l_1^+ l_2^+ \pm \sqrt{S_2}}{2\bar{b}_2^M},\\ S_2 &= (r_2^l - \sum_{i=1}^n \bar{a}_{2i}^M l_1^+ - \sum_{i=1}^n \bar{c}_{2i}^M l_1^+ l_2^+)^2 - 4\bar{b}_2^M\bar{h}_2^m\end{aligned}$$

Throughout this paper, we need the following assumptions.

(H_1) $r_1^l - \sum_{i=1}^n \bar{b}_{1i}^M l_2^+ - \sum_{i=1}^n \bar{c}_{1i}^M l_1^+ l_2^+ > 2\sqrt{\bar{a}_1^M\bar{h}_1^m}$ and $r_2^l - \sum_{i=1}^n \bar{a}_{2i}^M l_1^+ - \sum_{i=1}^n \bar{c}_{2i}^M l_1^+ l_2^+ > 2\sqrt{\bar{b}_2^M\bar{h}_2^m}$;

(H_2) The set of sequences $\{t_k^j = t_{k+j} - t_k, k, j \in \mathbf{Z}^+\}$ is uniformly almost periodic.

(H_3) $\prod_{0<t_k<t}(1+\Gamma_{mk})$ is almost periodic, $m = 1, 2$.

Lemma 6: [17] Let $x > 0, y > 0, z > 0$ and $x > 2\sqrt{yz}$, for the functions $f(x, y, z) = \frac{x+\sqrt{x^2-4yz}}{2z}$ and $g(x, y, z) = \frac{x-\sqrt{x^2-4yz}}{2z}$, the following assertions hold.

(1) $f(x, y, z)$ and $g(x, y, z)$ are monotonically increasing and monotonically decreasing on the variable $x \in (0, \infty)$, respectively.

(2) $f(x, y, z)$ and $g(x, y, z)$ are monotonically decreasing and monotonically increasing on the variable $y \in (0, \infty)$, respectively.

(3) $f(x, y, z)$ and $g(x, y, z)$ are monotonically decreasing and monotonically increasing on the variable $z \in (0, \infty)$, respectively.

Lemma 7: For the following equations

$$\begin{aligned}r_1(t) - \bar{a}_1(t)e^{u_1(t)} - \bar{h}_1(t)e^{-u_1(t)} = 0\\ r_2(t) - \bar{b}_2(t)e^{u_2(t)} - \bar{h}_2(t)e^{-u_2(t)} = 0\end{aligned}$$

if assumption(H_1) holds,then we have the following inequalities

$$\ln l_i^- < \ln u_i^- < \ln A_i^- < \ln A_i^+ < \ln u_i^+ < \ln l_i^+,$$

$i = 1, 2$, for all $t \in R$.

where

$$u_1^{\pm} = \frac{r_1(t) \pm \sqrt{r_1^2(t) - 4\bar{a}_1(t)\bar{h}_1(t)}}{2\bar{a}_1(t)},$$

$$u_2^{\pm} = \frac{r_2(t) \pm \sqrt{r_2^2(t) - 4\bar{b}_2(t)\bar{h}_2(t)}}{2\bar{b}_2(t)}.$$

Proof: Using lemma 6, it is easily that

$$\begin{aligned}
&\frac{r_1^M + \sqrt{(r_1^M)^2 - 4\bar{a}_1^l\bar{h}_1^l}}{2\bar{a}_1^l} \\
&> \frac{r_1(t) + \sqrt{r_1^2(t) - 4\bar{a}_1(t)\bar{h}_1(t)}}{2\bar{a}_1(t)} \\
&> \frac{r_1^l - \sum_{i=1}^n \bar{b}_{1i}^M l_2^+ - \sum_{i=1}^n \bar{c}_{1i}^M l_1^+ l_2^+ + \sqrt{S_1}}{2\bar{a}_1^M} \\
&> \frac{r_1^l - \sum_{i=1}^n \bar{b}_{1i}^M l_2^+ - \sum_{i=1}^n \bar{c}_{1i}^M l_1^+ l_2^+ - \sqrt{S_1}}{2\bar{a}_1^M} \\
&> \frac{r_1(t) - \sqrt{r_1^2(t) - 4\bar{a}_1(t)\bar{h}_1(t)}}{2\bar{a}_1(t)} \\
&> \frac{r_1^M - \sqrt{(r_1^M)^2 - 4\bar{a}_1^l\bar{h}_1^l}}{2\bar{a}_1^l}
\end{aligned}$$

where So we can get
$l_1^+ > u_1^+ > A_1^+ > A_1^- > u_1^- > l_1^-$ or
$\ln l_1^+ > \ln u_1^+ > \ln A_1^+ > \ln A_1^- > \ln u_1^- > \ln l_1^-$.
Analogously, we have

$$\ln l_2^+ > \ln u_2^+ > \ln A_2^+ > \ln A_2^- > \ln u_2^- > \ln l_2^-.$$

The proof of this lemma is completed. ■

III. Existence of at Least Four Positive Almost Periodic Solutions

We first summarize a few concepts from the book by Gaines and Mawhin [22].

Let X and Z be real normed vector spaces. Let $L : \operatorname{Dom} L \subset X \to Z$ be a linear mapping and $N : X \times [0, 1] \to Z$ be a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker L = codim Im $L < \infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero, then there exists continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $\operatorname{Im} P = \operatorname{Ker} L$ and $\operatorname{Ker} Q = \operatorname{Im} L = \operatorname{Im}(I - Q)$, and $X = \operatorname{Ker} L \bigoplus \operatorname{Ker} P, Z = \operatorname{Im} L \bigoplus \operatorname{Im} Q$. It follows that $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I - P)X \to \operatorname{Im} L$ is invertible and its inverse is denoted by K_P. If Ω is a bounded open subset of X, the mapping N is called L-compact on $\bar{\Omega} \times [0, 1]$, if $QN(\bar{\Omega} \times [0, 1])$ is bounded and $K_P(I-Q)N : \bar{\Omega} \times [0, 1] \to X$ is compact. Because $\operatorname{Im} Q$ is isomorphic to $\operatorname{Ker} L$, there exists an isomorphism $J : \operatorname{Im} Q \to \operatorname{Ker} L$.

Lemma 8: [22] Let L be a Fredholm mapping of index zero and let N be L-compact on $\bar{\Omega} \times [0, 1]$. Assume

(a) for each $\lambda \in (0, 1)$, every solution x of $Lx = \lambda N(x, \lambda)$ is such that $x \notin \partial\Omega \cap \operatorname{Dom} L$;

(b) $QN(x, 0)x \neq 0$ for each $x \in \partial\Omega \cap \operatorname{Ker} L$;

(c) $\deg(JQN(x, 0), \Omega \cap \operatorname{Ker} L, 0) \neq 0$.

Then $Lx = N(x, 1)$ has at least one solution in $\overline{\Omega} \cap \operatorname{Dom} L$.

Let T be a given positive constant and a finite number of points of the sequence $\{\tau_k\}$ lies in the interval $[0, T]$. Let $PC([0, T], \mathbf{R}^n)$ be the set of functions $x : [0, T] \to \mathbf{R}^n$ which are piecewise continuous in $[0, T]$ and have points of discontinuous $\tau_k \in [0, T]$, where they are continuous from the left. In the set $PC([0, T], \mathbf{R}^n)$ introduce the norm $\|x\| = \sup |x(t)| : t \in [0, T]$ with which $PC([0, T], \mathbf{R}^n)$ becomes a Banach space with the uniform convergence topology.

In our case, we shall consider $\mathbf{X} = \mathbf{Z} = V_1 \bigoplus V_2$, where

$$V_1 = \{z(t) = (z_1(t), z_2(t))^T : z_s(t) \in AP(\mathbf{R}),$$

$\mathrm{mod}(z_s(t)) \subset \mathrm{mod}\ (F_s), \forall \mu \in \Lambda(z_s(t))$ satisfies $|\mu| \geq \alpha, s = 1, 2\}$ gets that

$$V_1 \cup \{r_s(t), \bar{a}_1(t), \bar{b}_{1i}(t), \bar{b}_2(t), \bar{a}_{1i}(t), \bar{c}_{si}(t),$$

$\bar{h}_s(t), \tau_{si}(t), \sigma_{si}(t), i = 1, 2, \cdots, n, s = 1, 2\}$ is equi-almost-periodic,

$$V_2 = \{z(t) \equiv (c_1, c_2) \in \mathbf{R}^2\},$$

where

$$F_1(t, \phi_1, \phi_2) = r_1(t) - \bar{a}_1(t)e^{\phi_1(0)} - \sum_{i=1}^n \bar{b}_{1i}(t) \cdot e^{\phi_2(-\tau_{2i}(t))} - \sum_{i=1}^n \bar{c}_{1i}(t)e^{\phi_1(0)}e^{\phi_2(-\sigma_{2i}(t))} - \bar{h}_1(t)e^{-\phi_1(0)},$$

$$F_2(t, \phi_1, \phi_2) = r_2(t) - \sum_{i=1}^n \bar{a}_{2i}(t)e^{\phi_1(-\tau_{1i}(t))} - \bar{b}_2(t) \cdot e^{\phi_2(0)} - \sum_{i=1}^n \bar{c}_{2i}(t)e^{\phi_2(0)}e^{\phi_1(-\sigma_{1i}(t))} - \bar{h}_2(t)e^{-\phi_2(0)}.$$

in which $\phi_s \in C([-\sigma, 0], \mathbf{R}), s = 1, 2, \sigma = \max_{1 \leq i \leq n} sup\{\tau_{1i}(t), \tau_{2i}(t), \sigma_{1i}(t), \sigma_{2i}(t)\}$, and α is a given positive constant.

Define the norm

$$\|z\| = \sum_{s=1}^2 \sup_{t \in \mathbf{R}} |z_s(t)| \ \ \forall z \in \mathbf{X} = \mathbf{Z}.$$

By making the substitution

$$y_s(t) = e^{z_s(t)}, \ s = 1, 2,$$

system (3) is reformulated as

$$\begin{cases}
z_1'(t) = r_1(t) - \bar{a}_1(t)e^{z_1(t)} - \sum_{i=1}^n \bar{b}_{1i}(t) \cdot \\
\quad e^{z_2(t-\tau_{2i}(t))} - \sum_{i=1}^n \bar{c}_{1i}(t)e^{z_1(t)}e^{z_2(t-\sigma_{2i}(t))} \\
\quad -\bar{h}_1(t)e^{-z_1(t)}, \\
z_2'(t) = r_2(t) - \sum_{i=1}^n \bar{a}_{2i}(t)e^{z_1(t-\tau_{1i}(t))} \\
\quad -\bar{b}_2(t)e^{z_2(t)} - \sum_{i=1}^n \bar{c}_{2i}(t)e^{z_2(t)}e^{z_1(t-\sigma_{1i}(t))} \\
\quad -\bar{h}_2(t)e^{-z_2(t)}.
\end{cases} \quad (5)$$

Similar to the proofs of lemma 2 and lemma 7 in [30], one can easily prove the following three lemmas, respectively.

Lemma 9: $\mathbf{X}$ and $\mathbf{Z}$ are Banach spaces equipped with the norm $\|\cdot\|$.

Lemma 10: Let $L:\mathbf{X}\to\mathbf{Z}, Lz=u'=(z_1^{'},z_2^{'})^T$. Then L is a Fredholm mapping of index zero.

Lemma 11: Let $N:\mathbf{X}\times[0,1]\to\mathbf{Z}$,

$$N(u(t),\lambda)=(N_1(u(t),\lambda),N_2(u(t),\lambda))^T,$$

where

$N_1(z(t),\lambda)=r_1(t)-\bar{a}_1(t)e^{z_1(t)}-\lambda\sum_{i=1}^n\bar{b}_{1i}(t)\cdot$
$e^{z_2(t-\tau_{2i}(t))}-\lambda\sum_{i=1}^n\bar{c}_{1i}(t)e^{z_1(t)}e^{z_2(t-\sigma_{2i}(t))}$
$-\bar{h}_1(t)e^{-z_1(t)}$,
$N_2(z(t),\lambda)=r_2(t)-\lambda\sum_{i=1}^n\bar{a}_{2i}(t)e^{z_1(t-\tau_{1i}(t))}$
$-\bar{b}_2(t)e^{z_2(t)}-\lambda\sum_{i=1}^n\bar{c}_{2i}(t)e^{z_2(t)}e^{z_1(t-\sigma_{1i}(t))}$
$-\bar{h}_2(t)e^{-z_2(t)}$.

and $P:\mathbf{X}\to\mathbf{X}, Px=m(x); Q:Z\to\mathbf{Z}, Qu=m(u)$.
Then N is L-compact on $\bar{\Omega}$ (Ω is a open bounded subset of $\mathbf{X}$.

Theorem 1: Assume that (H_1),(H_2) and (H_3) hold, then system (3) has at least four positive almost periodic solutions.

Proof: In order to use lemma 8, we have to find at least four appropriate open bounded subsets in $\mathbf{X}$. Corresponding to the operator $Lz=\lambda N(z,\lambda),\lambda\in(0,1)$, we have

$$\begin{cases} z_1^{'}(t)=\lambda(r_1(t)-\bar{a}_1(t)e^{z_1(t)}-\lambda\sum\limits_{i=1}^n\bar{b}_{1i}(t)\cdot \\ \quad e^{z_2(t-\tau_{2i}(t))}-\lambda\sum\limits_{i=1}^n\bar{c}_{1i}(t)e^{z_1(t)}e^{z_2(t-\sigma_{2i}(t))} \\ \quad -\bar{h}_1(t)e^{-z_1(t)}), \\ z_2^{'}(t)=\lambda(r_2(t)-\lambda\sum\limits_{i=1}^n\bar{a}_{2i}(t)e^{z_1(t-\tau_{1i}(t))} \\ \quad -\bar{b}_2(t)e^{z_2(t)}-\lambda\sum\limits_{i=1}^n\bar{c}_{2i}(t)e^{z_2(t)}e^{z_1(t-\sigma_{1i}(t))} \\ \quad -\bar{h}_2(t)e^{-z_2(t)}). \end{cases} \tag{6}$$

Assume that $z\in X$ is an almost periodic solution of system (6) for some $\lambda\in(0,1)$. By lemma 4, for any $\epsilon>0$, there exist $\xi_s,\eta_s\in\mathbf{R}$ such that $z_s(\xi_s)>z_s^M-\epsilon, z_s(\eta_s)<z_s^l+\epsilon$ and $\dot{z}_s(\xi_s)=0,\dot{z}_s(\eta_s)=0, s=1,2$. From this and system (6), we obtain

$$\begin{cases} r_1(\xi_1)-\bar{a}_1(\xi_1)e^{z_1(\xi_1)}-\lambda\sum\limits_{i=1}^n\bar{b}_{1i}(\xi_1)e^{z_2(\xi_1-\tau_{2i}(\xi_1))} \\ -\lambda\sum\limits_{i=1}^n\bar{c}_{1i}(\xi_1)e^{z_1(\xi_1)}e^{z_2(\xi_1-\sigma_{2i}(\xi_1))} \\ -\bar{h}_1(\xi_1)e^{-z_1(\xi_1)}=0, \\ r_2(\xi_2)-\lambda\sum\limits_{i=1}^n\bar{a}_{2i}(\xi_2)e^{z_1(\xi_2-\tau_{1i}(\xi_2))} \\ -\bar{b}_2(\xi_2)e^{z_2(\xi_2)}-\lambda\sum\limits_{i=1}^n\bar{c}_{2i}(\xi_2)e^{z_2(\xi_2)}e^{z_1(\xi_2-\sigma_{1i}(\xi_2))} \\ -\bar{h}_2(\xi_2)e^{-z_2(\xi_2)}=0. \end{cases} \tag{7}$$

and

$$\begin{cases} r_1(\eta_1)-\bar{a}_1(\eta_1)e^{z_1(\eta_1)}-\lambda\sum\limits_{i=1}^n\bar{b}_{1i}(\eta_1)e^{z_2(\eta_1-\tau_{2i}(\eta_1))} \\ -\lambda\sum\limits_{i=1}^n\bar{c}_{1i}(\eta_1)e^{z_1(\eta_1)}e^{z_2(\eta_1-\sigma_{2i}(\eta_1))} \\ -\bar{h}_1(\eta_1)e^{-z_1(\eta_1)}=0, \\ r_2(\eta_2)-\lambda\sum\limits_{i=1}^n\bar{a}_{2i}(\eta_2)e^{z_1(\eta_2-\tau_{1i}(\eta_2))} \\ -\bar{b}_2(\eta_2)e^{z_2(\eta_2)}-\lambda\sum\limits_{i=1}^n\bar{c}_{2i}(\eta_2)e^{z_2(\eta_2)}e^{z_1(\eta_2-\sigma_{1i}(\eta_2))} \\ -\bar{h}_2(\eta_2)e^{-z_2(\eta_2)}=0. \end{cases} \tag{8}$$

On the one hand, according to the first equation of (7), we have

$$\begin{aligned} &\bar{a}_1^le^{2z_1(\xi_1)}-r_1^Me^{z_1(\xi_1)}+\bar{h}_1^l \\ &\le\bar{a}_1(\xi_1)e^{2z_1(\xi_1)}-r_1(\xi_1)e^{z_1(\xi_1)}+\bar{h}_1(\xi_1) \\ &=-\lambda e^{z_1(\xi_1)}(\sum_{i=1}^n\bar{b}_{1i}(\xi_1)e^{z_2(\xi_1-\tau_{2i}(\xi_1))} \\ &\qquad+\sum_{i=1}^n\bar{c}_{1i}(\xi_1)e^{z_1(\xi_1)}e^{z_2(\xi_1-\sigma_{2i}(\xi_1))}) \\ &<0, \end{aligned}$$

namely,

$$a_1^le^{2z_1(\xi_1)}-r^Me^{z_1(\xi_1)}+h_1^l<0,$$

which implies that

$$\ln l_1^-<z_1(\xi_1)<\ln l_1^+. \tag{9}$$

Similarly, by the first equation of (8), we obtain

$$\ln l_1^-<z_1(\eta_1)<\ln l_1^+. \tag{10}$$

From the second equation of (7), we obtain

$$\begin{aligned} &\bar{b}_2^le^{2z_2(\xi_2)}-r_2^Me^{z_2(\xi_2)}+\bar{h}_2^l \\ &\le\bar{b}_2(\xi_2)e^{2z_2(\xi_2)}-r_2(\xi_2)e^{z_2(\xi_2)}+\bar{h}_2(\xi_2) \\ &=-\lambda e^{z_2(\xi_2)}(\sum_{i=1}^n\bar{a}_{2i}(\xi_2)e^{z_1(\xi_2-\tau_{1i}(\xi_2))} \\ &\qquad+\sum_{i=1}^n\bar{c}_{2i}(\xi_2)e^{z_2(\xi_2)}e^{z_1(\xi_2-\sigma_{1i}(\xi_2))}) \\ &<0. \end{aligned}$$

That is

$$b_2^le^{2z_2(\xi_2)}-r_2^Me^{z_2(\xi_2)}+\bar{h}_2^l<0.$$

which imply that

$$\ln l_2^-<z_2(\xi_2)<\ln l_2^+. \tag{11}$$

Similarly, by the second equation of (8), we get

$$\ln l_2^-<z_2(\eta_2)<\ln l_2^+. \tag{12}$$

On the other hand, by (7), (10), (11) we have

$$r_1^l \le r_1(\xi_1)$$
$$= \bar{a}_1(\xi_1)e^{z_1(\xi_1)} + \lambda\sum_{i=1}^{n}\bar{b}_{1i}(\xi_1)e^{z_2(\xi_1-\tau_{2i}(\xi_1))}$$
$$+\lambda\sum_{i=1}^{n}\bar{c}_{1i}(\xi_1)e^{z_1(\xi_1)}e^{z_2(\xi_1-\sigma_{2i}(\xi_1))}$$
$$+\bar{h}_1(\xi_1)e^{-z_1(\xi_1)}$$
$$\le \bar{a}_1^M e^{z_1(\xi_1)} + \sum_{i=1}^{n}\bar{b}_{1i}^M l_2^+ + \sum_{i=1}^{n}\bar{c}_{1i}^M l_1^+ l_2^+$$
$$+\bar{h}_1^M e^{-z_1(\xi_1)}$$

and

$$r_2^l \le r_2(\xi_2)$$
$$= \lambda\sum_{i=1}^{n}\bar{a}_{2i}(\xi_2)e^{z_1(\xi_2-\tau_{1i}(\xi_2))} + \bar{b}_2(\xi_2)e^{z_2(\xi_2)}$$
$$+\lambda\sum_{i=1}^{n}\bar{c}_{2i}(\xi_2)e^{z_2(\xi_2)}e^{z_1(\xi_2-\sigma_{1i}(\xi_2))}$$
$$+\bar{h}_2(\xi_2)e^{-z_2(\xi_2)}$$
$$\le \sum_{i=1}^{n}\bar{a}_{2i}^M l_1^+ + \bar{b}_2^M e^{z_2(\xi_2)} + \sum_{i=1}^{n}\bar{c}_{2i}^M l_1^+ l_2^+$$
$$+\bar{h}_2^M e^{-z_2(\xi_2)}$$

namely,

$$\bar{a}_1^M e^{2z_1(\xi_1)} - (r_1^l - \sum_{i=1}^{n}\bar{b}_{1i}^M l_2^+ - \sum_{i=1}^{n}\bar{c}_{1i}^M l_1^+ l_2^+)e^{z_1(\xi_1)}$$
$$+\bar{h}_1^M > 0,$$

and

$$\bar{b}_2^M e^{2z_2(\xi_2)} - (r_2^l - \sum_{i=1}^{n}\bar{a}_{2i}^M l_1^+ - \sum_{i=1}^{n}\bar{c}_{2i}^M l_1^+ l_2^+)e^{z_2(\xi_2)}$$
$$+\bar{h}_2^M > 0,$$

which imply that

$$z_1(\xi_1) < \ln(A_1^-) \quad or \quad z_1(\xi_1) > \ln(A_1^+), \tag{13}$$

and

$$z_2(\xi_2) < \ln(A_2^-) \quad or \quad z_2(\xi_2) > \ln(A_2^+), \tag{14}$$

according to (8), similarly we can get for each $s = 1, 2$.

$$z_s(\eta_s) < \ln(A_s^-) \quad or \quad z_s(\eta_s) > \ln(A_s^+), \tag{15}$$

It follows from (9)-(14), lemma 4, lemma 7 and the arbitrariness of ϵ that for any $t \in \mathbf{R}$,

$$\ln l_1^- \le z_1(t) \le \ln A_1^- \quad or \quad \ln A_1^+ \le z_1(t) \le \ln l_1^+,$$

and

$$\ln l_2^- \le z_2(t) \le \ln A_2^- \quad or \quad \ln A_2^+ \le z_2(t) \le \ln l_2^-.$$

For convenience, we denote

$$G_s = (\Theta_s^1 \ln l_s^-, \ln A_s^- + \Theta_s^2),$$

$$H_s = (\ln A_s^+ - \Theta_s^3, \Theta_s^4 \ln l_s^+), s = 1, 2,$$

where $\Theta_s^1 \in (0,1), \Theta_s^2, \Theta_s^3 \in (0, \frac{\ln A_s^- + \ln A_s^+}{2}), \Theta_s^4 \in (1,\infty), s = 1, 2$. Clearly, $\ln l_s^{\pm}, s = 1, 2$, are independent of λ. For each $s = 1, 2$, we choose one of interval among the two intervals G_s and H_s and denote it as Δ_s, then define the set

$$\{z = (z_1, z_2)^T : z_s(t) \in \Delta_s, t \in R, s = 1, 2\}.$$

It is obvious the number of the above sets is 4. We denote these sets as $\Omega_k, k = 1, 2, 3, 4$. $\Omega_k, k = 1, 2, 3, 4$ are bounded open sunsets of X, $\Omega_m \bigcap \Omega_n = \emptyset, m \ne n$. Thus $\Omega_k, k = 1, 2, 3, 4$ satisfies the requirement (a) in lemma 8.

Now we show that (b) of lemma 8 holds, i.e., we prove when $z \in \partial\Omega_k \cap \ker L = \partial\Omega_k \cap R^2, QN(z, 0) \ne (0, 0)^T, k = 1, 2, 3, 4$. If it is not true, then when $z \in \partial\Omega_k \cap \ker L = \partial\Omega_k \cap R^2, k = 1, 2, 3, 4$, constant vector $z = (z_1, z_2)^T$ with $z \in \partial\Omega_k, k = 1, 2, 3, 4$ satisfies

$$m(r_1(t) - \bar{a}_1(t)e^{z_1} - \bar{h}_1 e^{-z_1}) = 0,$$

and

$$m(r_2(t) - \bar{b}_2(t)e^{z_2} - \bar{h}_2 e^{-z_2}) = 0.$$

Using the mean value theorem of calculous, there exist two points $\zeta_s (s = 1, 2)$ such that

$$r_1(\zeta_1) - \bar{a}_1(\zeta_1)e^{z_1} - \bar{h}_1(\zeta_1)e^{-z_1} = 0, \tag{16}$$

and

$$r_2(\zeta_2) - \bar{b}_2(\zeta_2)e^{z_2} - \bar{h}_2(\zeta_2)e^{-z_2} = 0. \tag{17}$$

By (16) and (17), we have

$$u_1^{\pm} = \frac{r_1(\zeta_1) \pm \sqrt{(r_1(\zeta_1))^2 - 4\bar{a}_1(\zeta_1)\bar{h}_1(\zeta_1)}}{2\bar{a}_1(\zeta_1)}$$

and

$$u_2^{\pm} = \frac{r_2(\zeta_2) \pm \sqrt{(r_2(\zeta_2))^2 - 4\bar{b}_2(\zeta_2)\bar{h}_2(\zeta_2)}}{2\bar{b}_2(\zeta_2)}.$$

According to lemma 7, we obtain for each $s = 1, 2$.

$$\ln l_s^- < \ln z_s^- < \ln A_s^- < \ln A_s^+ < \ln zs^+ < \ln l_s^+.$$

Then u belongs to one of $\Omega_k \cap R^2$, $k = 1, 2, 3, 4$. This contradicts the fact that $z \in \partial\Omega_k \cap R^2, k = 1, 2, 3, 4$. This proves (b) in lemma 8 holds. Finally, we show that (c) in lemma 8 holds. Because(H_1) holds, the algebraic equations of the system

$$\begin{cases} r_1(\zeta_1) - a_1(\zeta_1)e^{z_1} - h_1(\zeta_1)e^{-z_1} = 0, \\ r_2(\zeta_2) - b_2(\zeta_2)e^{z_2} - h_2(\zeta_2)e^{-z_2} = 0, \end{cases}$$

has four distinct solutions.

$$(z_1^*, z_2^*) = (\ln \hat{z}_1, \ln \hat{z}_2),$$

in the above situations $\hat{z}_1 = z_1^-$ or $\hat{z}_1 = z_1^+, z_1^{\pm} = \frac{r_1(\zeta_1) \pm \sqrt{(r_1(\zeta_1))^2 - 4a_1(\zeta_1)h_1(\zeta_1)}}{2a_1(\zeta_1)}$, and $\hat{z}_2 = z_2^-$ or $\hat{z}_2 = z_2^+, z_2^{\pm} = \frac{r_2(\zeta_2) \pm \sqrt{(r_2(\zeta_2))^2 - 4b_2(\zeta_2)h_2(\zeta_2)}}{2b_2(\zeta_2)}$. By lemma 7, it is easy to verify that for each $s = 1, 2$,

$$\ln l_s^- < \ln z_s^- < \ln A_s^- < \ln A_s^+ < \ln z_s^+ < \ln l_s^+,$$

Therefore, (z_1^*, z_2^*) uniquely belongs to the corresponding Ω_k. Since $\mathrm{Ker}L = \mathrm{Im}Q$, we can take $J = I$. A direct computation gives, for $k = 1, 2, 3, 4$,

$$\deg\Big\{JQN(u,0), \Omega_k \cap \mathrm{Ker}L, (0,0)^T\Big\}$$
$$= \mathrm{sign}\Big[\big(-a_1(\zeta_1)z_1^* + \tfrac{h_1(\zeta_1)}{z_1^*}\big)\cdot \big(-b_2(\zeta_2)z_2^* + \tfrac{h_2(\zeta_2)}{z_2^*}\big)\Big].$$

Since

$$r_1(\zeta_1) - a_1(\zeta_1)z_1^* - \frac{h_1(\zeta_1)}{z_1^*} = 0$$

and

$$r_2(\zeta_2) - b_2(\zeta_2)z_2^* - \frac{h_2(\zeta_2)}{z_2^*} = 0,$$

then

$$\deg\Big\{JQN(u,0), \Omega_k \cap \ker L, (0,0)^T\Big\}$$
$$= \mathrm{sign}\Big[\big(r_1(\zeta_1) - 2a_1(\zeta_1)z_1^*\big)\cdot \big(r_2(\zeta_2) - 2b_2(\zeta_2)z_2^*\big)\Big]$$
$$= \pm 1.$$

So far, we have prove that $\Omega_k (k = 1, 2, 3, 4)$ satisfies all the assumptions in lemma 8. Hence, system (5) has at least four different almost periodic solutions. If $z^*(t) = (z_1^*, z_2^*)^T$ is an almost periodic solution of system (4), by applying lemma 5, we known that

$$(x_1(t), x_2(t))^T = \Big(e^{z_1^*(t)} \prod_{0<t_k<t}(1+\Gamma_{1k}), e^{z_2^*(t)} \prod_{0<t_k<t}(1+\Gamma_{2k})\Big)^T$$

is almost periodic solution of system (3). Since conditions (H_2) and (H_3) hold, similar to the proofs of lemma 31 and theorem 79 in [2], we can prove that $\bar{x}_s(t) = \prod\limits_{0<t_k<t}(1+\Gamma_{sk})e^{\bar{z}_s(t)}$ is almost periodic in the sense of definition 2. Therefore, system (3) has at least four different positive almost periodic solutions. This completes the proof of theorem 1. ■

Consider the following delayed plankton allelopathy system on time scales with exploited (or harvesting) terms

$$\begin{cases} x_1'(t) = x_1(t)[r_1(t) - a_1(t)x_1(t) \\ \quad - \sum\limits_{i=1}^{n} b_{1i}(t)x_2(t-\tau_{2i}(t)) - \sum\limits_{i=1}^{n} c_{1i}(t)\cdot \\ \quad x_1(t)x_2(t-\sigma_{2i}(t))] - h_1(t) \\ x_2'(t) = x_2(t)[r_2(t) - \sum\limits_{i=1}^{n} a_{2i}(t)x_1(t-\tau_{1i}(t)) \\ \quad -b_2(t)x_2(t) - \sum\limits_{i=1}^{n} c_{2i}(t)x_2(t)x_1(t-\sigma_{1i}(t))] \\ \quad -h_2(t) \end{cases} \quad (18)$$

where $a_1(t), b_2(t), a_{si}(t), b_{si}(t), c(st), h_s(t), (s = 1, 2)$, are all positive continuous almost periodic functions,the time delays $\tau_{si}(t), \sigma_{si}(t), s = 1, 2$ are all nonnegative continuous almost periodic functions.

Similar to the proof of theorem 1, we may easily obtain,

Corollary 1: Assume that the following condition holds

$$(H_1') \quad r_1^l - \sum_{i=1}^{n} b_{1i}^M l_2^+ - \sum_{i=1}^{n} c_{1i}^M l_1^+ l_2^+ > 2\sqrt{a_1^M h_1^m}$$

and $r_2^l - \sum\limits_{i=1}^{n} a_{2i}^M l_1^+ - \sum\limits_{i=1}^{n} c_{2i}^M l_1^+ l_2^+ > 2\sqrt{b_2^M h_2^m}$.

Then system (18) has at least four different positive almost periodic solutions.

IV. Conclusion

By applying Mawhins continuation theorem of coincidence degree theory, we study an impulsive delayed plankton allelopathy system o time scales with multiple exploited or harvesting terms and give some sufficient conditions for the existence of four positive almost periodic solutions of this system (3).

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