Discrimination of Modes of Double- and Single- Negative Grounded Slab

R. Borghol, T. Aguili

Abstract—In this paper, we investigate theoretically the waves propagation in a lossless double-negative grounded slab (DNG). This study is performed by the Transverse Resonance Method (TRM). The proper or improper nature of real and complex modes is observed. They are highly dependent on metamaterial parameters, i.e. ε_r-negative, μ_r-negative, or both. Numerical results provided that only the proper complex modes (i.e., leaky modes) exist in DNG slab, and only the improper complex modes exist in single-negative grounded slab.

Keywords—Double-negative grounded slab, real and complex modes, single-negative grounded slab, transverse resonance method.

I. INTRODUCTION

THE production of artificial dielectrics with the permittivity and permeability arbitrary (metamaterial) has tremendous interest in the community scientific. These artificial dielectrics may give rise to unexpected and interesting propagation feature for waveguiding structures [1] and they are candidate to reduce edge diffraction effects and enhance radiation efficiency for microstrip antennas [2].

The artificial dielectrics are used in many applications such as microwave/millimeter wave leaky wave antennas [3], where the effective dielectric constants were between zero and unity.

Several groups studied a waveguiding structures based on artificial dielectrics media. Nefedov and Tretyako [4] found that there exist modes whose fields decay exponentially from the interface of the two media for transverse electric (TE) and transverse magnetic (TM) polarizations. These modes, called as evanescent surface modes, were also found by Alù and Engheta [5]. These studies have no study on the complex modes. In 2006, Shu and Song suggested the dispersive propagation of complex modes (leaky waves) on metamaterial grounded slab and their investigations on the Poyting vectors show that these modes do not transport energy in both transverse and longitudinal directions [6]. This study is unfortunately incomplete.

In this paper, we focus to the modal properties of surface modes, volume modes, and complex modes supported by one or both negative parameters of grounded slab using the TRM in order to completely assess their propagation and radiation features. In Section II, we present a simple analysis of the dispersion relation of real and complex modes for TE and TM

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polarizations in our structure. In Section III, we discuss the numerical results of the characteristics of these modes along the considered structure. In Section IV, we conclude our work.

II. ANALYSIS

The structure of interest, as shown in Fig. 1, consists of two media, air and grounded slab. The infinitely extent slab of lossless material is inserted in this structure with thickness d, while the slab is assumed to be of a lossless isotropic DNG material with real parameters $\varepsilon_r < 0$ and $\mu_r < 0$ at the frequency of interest ω.
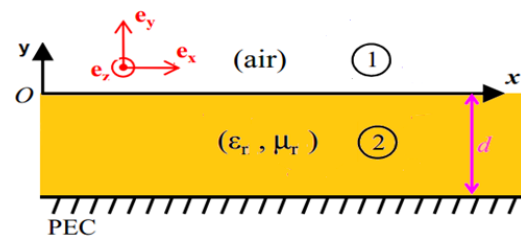

Fig. 1 An infinity grounded slab with negative parameters er and ur

To find the propagation constant for propagation perpendicular to the plane xoz (y direction), we use the well-known TRM in this direction. The structure can be then considered as an equivalent circuit with modes propagating in the z direction, as shown in Fig. 2. These modes are referred to as z-modes because they propagate along the longitudinal z direction with propagation constant k_z. This equivalent circuit leads to the dispersion equation.
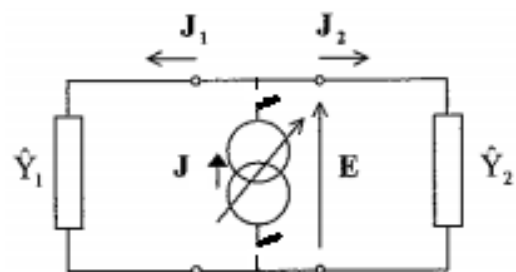
$$(\hat{Y}_1 + \hat{Y}_2)E = 0 \qquad (1)$$

Fig. 2 Equivalent circuit used to illustrate the TRM

We consider here that these modes would be TE and TM. The dispersion equations for both polarizations TE and TM are then written as:

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TE

$$k_{y2}\coth(k_{y2}\,d)+k_{y1}\,\mu_r=0 \qquad (2)$$

TM

$$k_{y2}\,th(k_{y2}\,d)+k_{y1}\,\varepsilon_r=0 \qquad (3)$$

The TE and TM characteristic admittances in the air and slab region are given by [7]:

$$y_{1M}{}^{TE}=\frac{k_{y1}}{j\omega\mu_0} \qquad (4)$$

$$y_{1M}^{TM}=\frac{j\omega\varepsilon_0}{k_{y1}} \qquad (5)$$

$$y_{2M}{}^{TE}=\frac{k_{y2}}{j\omega\mu_0\mu_r} \qquad (6)$$

$$y_{2M}{}^{TM}=\frac{j\omega\varepsilon_0\varepsilon_r}{k_{y2}} \qquad (7)$$

where k_{y1} and k_{y2} are the transverse wavenumber .

The analysis proceeds by first assuming that the propagation constant k_x value is zero (no variation along the x direction).

A. Surface Mode

The surface modes are characterized by exponentially decaying fields in the y-direction [8].

A TM-type surface mode propagating along the z axis is investigated. The non-vanishing magnetic field components which satisfy the Maxwell equations in two regions can be deduced as:

$$\begin{cases} H_{x1}=C_1e^{-jk_z z}e^{-k_{y1} y} \\ H_{x2}=C_1e^{-jk_z z}\cosh(k_{y2}\,y) \end{cases} \qquad (8)$$

The H_x components from the two regions are continuous on the surface y = 0.

For TM surface modes, the surface impedance must be inductive ($real(k_{y1})>0$) in order to provide the field concentration above the surface. The Helmholtz wave equation is written as:

$$k_{y2}^2-k_{y1}^2=-k_0^2(\varepsilon_r\mu_r-1) \qquad (9)$$

The real modes propagate in z direction with a real propagation constant $k_z=\beta_z$.

We aim to obtaining an approximate solution by setting the imaginary parts ε_r and μ_r to zero. Under this assumption,

the wave number k_{y1} is real and, the wave number k_{y2} is real. Let $k_{y1}=\beta_{y1}-j\alpha_{y1}$ and $k_{y2}=\beta_{y2}-j\alpha_{y2}$. In this case, (2) and (8) reduce to

$$\begin{cases} \varepsilon_r\beta_{y1}=-\beta_{y2}\,th(\beta_{y2}d) \\ \beta_{y2}^2-\beta_{y1}^2=-k_0^2(\varepsilon_r\mu_r-1) \\ \beta_{y1}^2-\beta_z^2=-k_0^2 \end{cases} \qquad (10)$$

The equation of system can be solved for the unknowns β_{y1} and β_{y2} using an iterative procedure to arrive at a final set of solutions from some initial trial values. This procedure is released by "fsolve" in MATLAB.

For TE surface mode, the electric field components in two regions can be written as:

$$\begin{cases} E_{x1}=C_1e^{-jk_z z}e^{-k_{y1} y} \\ E_{x2}=C_1e^{-jk_z z}\sinh(k_{y2}\,y) \end{cases} \qquad (11)$$

Assuming the same approximation using in TM mode, we obtain the equation system

$$\begin{cases} \mu_r\beta_{y1}=-\beta_{y2}\coth(\beta_{y2}d) \\ \beta_{y2}^2-\beta_{y1}^2=-k_0^2(\varepsilon_r\mu_r-1) \\ \beta_{y1}^2-\beta_z^2=-k_0^2 \end{cases} \qquad (12)$$

To find the dispersion of the TE surface mode, we will solve this system by the same procedure happening in TM mode.

B. Volume Mode

Volume modes propagate along the longitudinal z direction with a real propagation constant $k_z=\beta_z$. The magnetic field in the TM polarization is represented by these equations in the two regions.

$$\begin{cases} H_{x1}=C_1e^{-jk_z z}e^{-jk_{y1} y} \\ H_{x2}=C_1e^{-jk_z z}\cosh(k_{y2}\,y) \end{cases} \qquad (13)$$

The difference between the two wave equations allows us to obtain this equation:

$$k_{y2}^2+\beta_{y1}^2=-k_0^2(\varepsilon_r\mu_r-1) \qquad (14)$$

We imply that the wave number of the slab k_{y2} is real. Therefore, we can write these two equations:

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$$\begin{cases} \varepsilon_r \beta_{y1} = \beta_{y2}\,\mathrm{th}(\beta_{y2}d) \\ \beta_{y1}^2 + \beta_{y2}^2 = -k_0^2(\varepsilon_r \mu_r - 1) \\ \beta_{y1}^2 + \beta_z^2 = -k_0 \end{cases} \qquad (15)$$

In order to find volume modes, we solve this system using "fsolve" in MATLAB.

In TE polarization, the electric field is written in the two regions by the two equations:

$$\begin{cases} E_{x1} = C_1 e^{-jk_z z} e^{-jk_{y1} y} \\ E_{x2} = C_1 e^{-jk_z z} \sinh(k_{y2}\, y) \end{cases} \qquad (16)$$

Volume mode is characterized by a real wavenumber $k_{y1} = \beta_{y1}$ and a real wavenumber $k_{y2} = \beta_{y2}$. It is represented by the solution of the system

$$\begin{cases} \mu_r \beta_{y1} = -\beta_{y2}\coth(\beta_{y2}d) \\ \beta_{y1}^2 + \beta_{y2}^2 = -k_0^2(\varepsilon_r \mu_r - 1) \\ \beta_{y1}^2 + \beta_z^2 = -k_0 \end{cases} \qquad (17)$$

C. Leaky Mode

The leaky mode is characterized by two complex wave numbers k_z, k_{y1}, and k_{y2}.

For TM and TE polarizations, the system of equations to solve can be written as:

TM
$$\begin{cases} \varepsilon_r k_{y1} = k_{y2}\,\mathrm{th}(k_{y2}\, d) \\ k_{y1}^2 + k_{y2}^2 = -k_0^2(\varepsilon_r \mu_r - 1) \\ k_{y1}^2 + k_z^2 = -k_0 \end{cases} \qquad (18)$$

TE
$$\begin{cases} \mu_r k_{y1} = -k_{y2}\coth(k_{y2}\, d) \\ k_{y1}^2 + k_{y2}^2 = -k_0^2(\varepsilon_r \mu_r - 1) \\ k_{y1}^2 + k_z^2 = -k_0^2 \end{cases} \qquad (19)$$

The resolution of the system can be reduced to the resolution of the single equation of the two polarizations

TM
$$\sqrt{-Z^2 - a^2}\,\varepsilon_r - jZ \tan(Z) = 0 \qquad (20)$$

TE
$$\sqrt{-Z^2 - a^2}\,\mu_r + jZ \cot(Z) = 0 \qquad (21)$$

where $Z = k_{y2}d$ and $a^2 = k_0^2 d^2(\varepsilon_r \mu_r - 1)$.

There is no analytical solution to these equations, then its resolution cannot be done numerically. The resolution of these equations are from the complex variable Z, using a Newton-Raphson algorithm [9]. We chose this method because it is simple and effective.

In our method, we chose to take as initial points, the analytical solutions of (20) and (21) for the particular case $\varepsilon_r = 0$ and $\mu_r = 0$. In this case, these equations will be written as:

TM: $jZ \tan(Z) = 0$

TE: $jZ \cot(Z) = 0$

The solution of the two equations is $Z = (n/2)\,\mathrm{pi}$ where n is an odd integer.

III. NUMERICAL RESULTS AND DISCUSSION

In the numerical study, our structure is made of a metamaterial medium with permeability and permittivity equal to:

$$\mu_r(\omega) = 1 - \frac{F\omega^2}{\omega^2 - \omega_0^2} \qquad \varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

where $F = 0.56$, $\omega_p/2\pi = 10$ GHz and $\omega_0/2\pi = 4$ GHz (parameters chosen as in [7]). These material parameters depend on frequency.
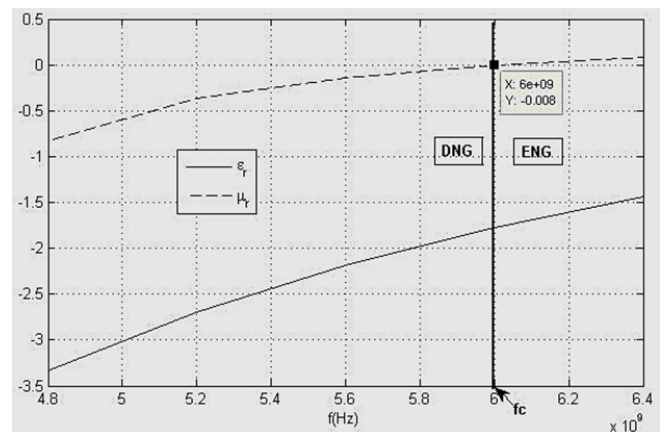

Fig. 3 Relative permittivity ε_r (solid line) and relative permeability μ_r (dashed line) as a function of frequency f for a double-negatives grounded slab

In Fig. 3, we illustrate the permeability and permittivity depending on the frequency of $f = 4.8$ GHz to $f = 6.2$ GHz. The permeability becomes positive above $f = 6$ GHz and it becomes negative below this value. This frequency is the critical frequency.

Graphical representations of the surface mode for TM and TE modes supported by a grounded slab with the above-reported parameters and slab height $d = 60$ mm in a

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frequency range between $f = 4.8$ GHz and $f = 6.2$ GHz are shown in Fig. 4.

In TM polarization, the mode is always real and is properly evanescent above 5.2 GHz (red Dash-dot line) and improper ordinary below (red solid line). But, in TE polarization, the mode is not evanescent above 5.2 GHz, it is only improper ordinary below.

We present in Fig. 5 the characteristics of volume modes propagating along the double-negatives grounded slab.
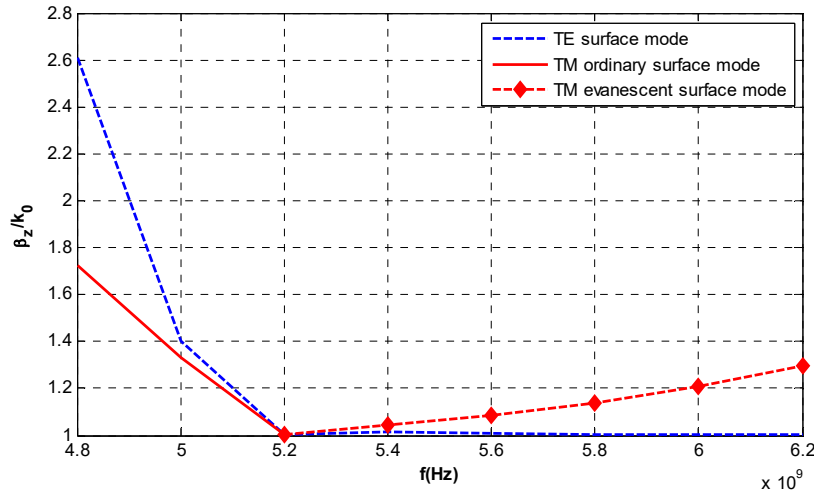

Fig. 4 Dispersion diagrams for TM and TE surface modes of a grounded slab with height d = 60 mm
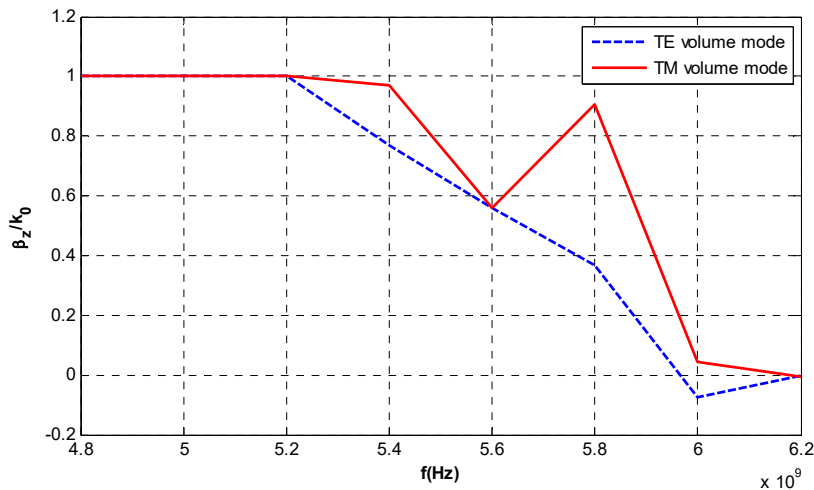

Fig. 5 Normalized real constant of volume mode propagating on a double-negatives grounded slab versus frequency for TM (solid line) and TE (dashed line) polarizations

The real constants of TE and TM volume modes are equal to the constant of free space k_0, $\beta_z = k_0$, above 5.2 GHz and they are decreased below. At the critical frequency $f_c = 6$ GHz, the normalized real constants of TE modes become increased. But, in TM modes, they remain decreased.

As an illustration, Fig. 6 presents the normalized real constant and the normalized attenuation constant of the leaky mode propagating in double-negatives grounded slab.

Two modes for TE and TM are visible in Fig. 6. TM1 and TM2 have two real branches below 5.2 GHz and 5 GHz which to complex branches above those frequencies. TE1 and TE2 have two real branches below 5.22 GHz and 5.02GHz which to complex branches above those frequencies. At $f_c = 6$ GHz (when the metamaterial changes from double-negative to ε_r -negative), the normalized real constants of these two modes for TE and TM become zero and retain their sign. The normalized attenuation constants of TM modes decrease rapidly at f_c. However, the normalized attenuation constants of TE modes increase at this frequency.

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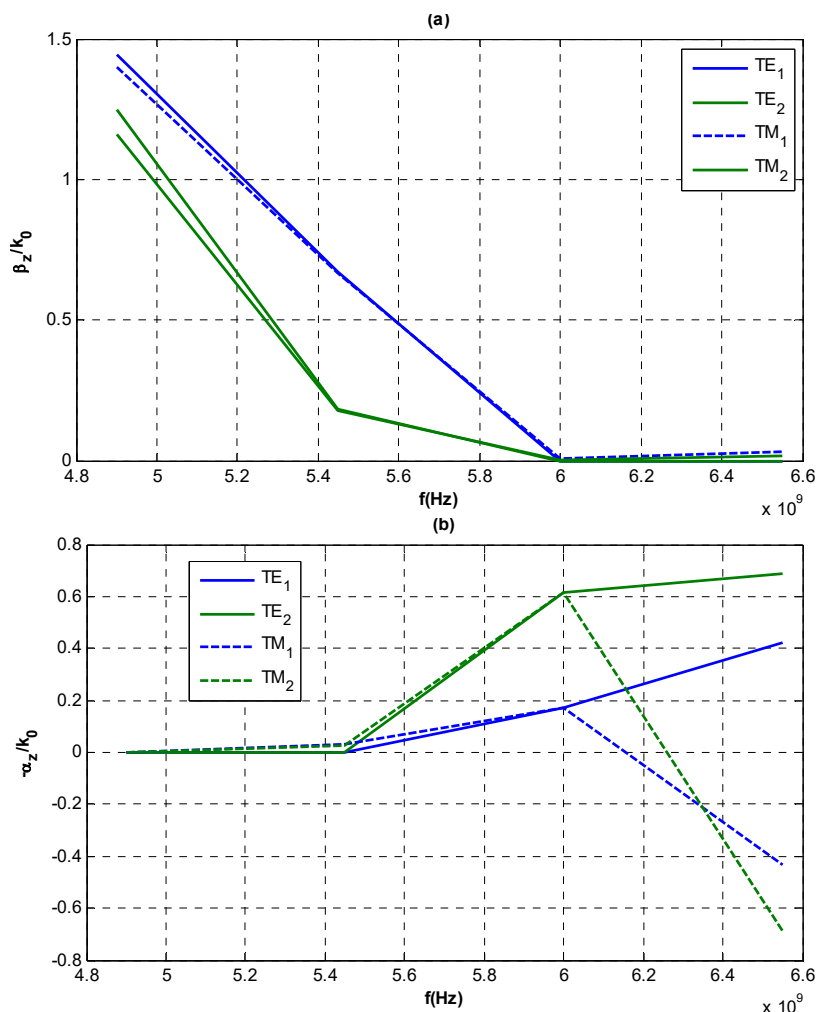International Journal of Electronics and Communication Engineering
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Fig. 6 Characteristics of leaky modes propagating along a double-negatives grounded slab as function frequency. (a) Normalized real constants (b) normalized attenuation constant

IV. CONCLUSION

The dispersion and radiation properties of real and complex modes propagating along double-negative or single-negative grounded slab have been investigated in this paper. The graphical method is used to find the possible real and complex roots. Evanescent surface mode has been shown to occur only in TM modes of DNG case. We found that only the improper leaky modes are observed in DNG, and only the proper leaky modes are observed in SNG.

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