Movies and Dynamic Mathematical Objects on Trigonometry for Mobile Phones

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Abstract—This paper is about movies and dynamic objects for mobile phones. Dynamic objects are the software programmed by JavaScript. They consist of geometric figures and work on HTML5-compliant browsers. Mobile phones are very popular among teenagers. They like watching movies and playing games on them. So, mathematics movies and dynamic objects would enhance teaching and learning processes. In the movies, manga characters speak with artificially synchronized voices. They teach trigonometry together with dynamic mathematical objects. Many movies are created. They are Windows Media files or MP4 movies. These movies and dynamic objects are not only used in the classroom but also distributed to students. By watching movies, students can study trigonometry before or after class.

Keywords—Dynamic mathematical object, JavaScript, Google drive, transfer jet.

I. INTRODUCTION

MOBILE devices are becoming ever more widely available and are increasingly used by students. They give students access to educational material anytime and anywhere. For example, there is a mobile library for books [1]. There are also multimedia contents to teach mathematical principles for mobile phones [2]. Mobile devices are used in classrooms to improve teaching and learning processes [3], and mobile-learning is considered as an effective educational tool [4].

The way of communication of human with the computer was done only via keyboard before. Nowadays, new applications are invented, and we can control computers and mobile phones by our voices. Voice search technology is now applied in the search of book collections [5]. In the near future, we will be able to talk with computers.

There is a software which converts Japanese sentences to artificially synchronized human voices. The speed and intonation of the voices can be arranged. The author made such voices for his lesson and used them instead of reading numbers or mathematical expressions. Students were amazed and very interested. They listened to the synchronized human voices very carefully and studied well.

In Japan, manga comics are very popular among teenagers. The author thought that it would be better to see that a virtual Japanese high school girl shows up and read sentences in movies. Such movies on trigonometry are made. The longest one is about 25 minutes long. The shortest one is about five minutes long. In the movies, two high school students take a

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role of a navigator and her assistant. They are not real students, but they are Japanese manga characters (Fig. 1). They explain how to solve problems of mathematics or how to prove theorems and formulas. Sometimes, they use dynamic objects to solve problems. Students can learn how to use dynamic objects by watching movies. The author created Windows Media files and MP4 movies and distributed to students.
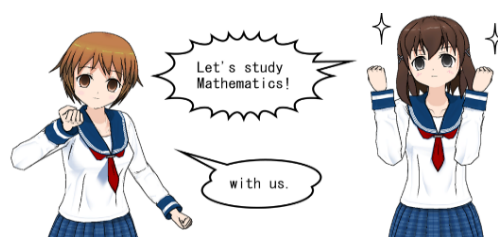

Fig. 1 The navigator and her assistant

II. DYNAMIC OBJECTS

The author had used some software in his mathematics classes. They are called dynamic objects. They consist of geometric figures such as points, lines, circles, and triangles, etc. They were programmed by Microsoft Small Basic, so students could use it only on computers [6].

New objects are made by JavaScript this time. They work on HTML5-compliant browsers. So, students can use them both on computers and on mobile phones. They can drag points, move lines, change radius of a circle, etc. to see how the values of mathematical expressions change [7].

Some examples of dynamic objects on trigonometry will be shown in this chapter.

Example 1. $\sin(180° − \theta) = \sin\theta$

This is a dynamic object to understand the formula well. P and Q are points on the unit circle and they are symmetric. Line of symmetry is y–axis. Students can drag P and see P and Q always have the same y–coordinate (Fig. 2).
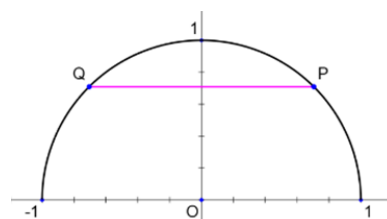

Fig. 2 A dynamic object for the formula

Example 2. A proof of Sine Rule

Let A, B, and C be any points in a plane and they make a

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:11, No:4, 2017

triangle. Let R be the radius of the circumscribed circle of triangle ABC. Then, the following theorem holds.

Theorem (Sine Rule)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \qquad (1)$$

This is a dynamic object to understand a proof of Sine Rule well. Points A, B, and C are arbitrary points on a plane. So, we can drag these points to any place. Let D be the point on the circumscribed circle of triangle ABC, such that BD passes the center of the circle (Fig. 3). As A and D are in the major arc formed by B, C, so angle BAC is equal to angle BDC. Furthermore, the central angle of angle BCD is 180 degrees, so angle BCD is a right angle. This implies $BC = 2R \sin D = 2R \sin A$.

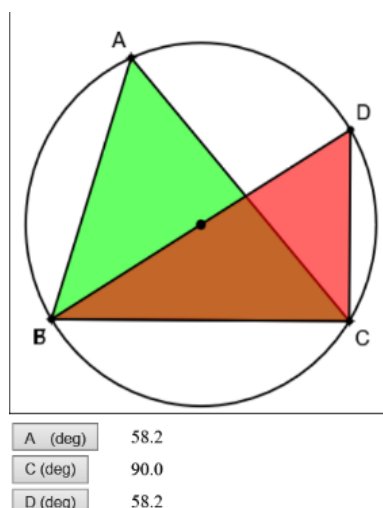Students can examine the proof by changing the shape of triangle ABC.

Fig. 3 A dynamic object for Sine Rule

Example 3. Cosine Rule

Let A, B, and C be any points in a plane and they make a triangle. Then, the following theorem holds.

Theorem (Cosine Rule)

$$a^2 = b^2 + c^2 - 2bc \cos A \qquad (2)$$

This is a dynamic object to understand Cosine Rule well (Fig. 4). Points A, B, and C are arbitrary points on a plane. So we can drag these points to any place. As we drag A, B, and C, the values of $b = AC, c = AB$ and A change in real time. Students were made to calculate a by Cosine Rule, and check the calculations by this object.

Example 4. Circular measure

This is a dynamic object to understand circular measure well.

This object has the unit circle and a point P on it (Fig. 5). As the radius of the unit circle is 1, the length of arc AP is equal to the circular measure of angle AOP. Students drag P and see that they are always the same.
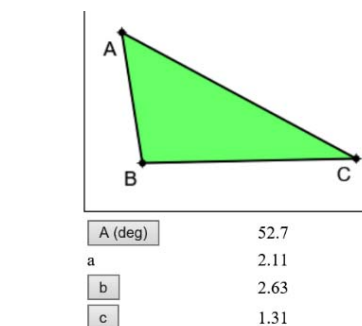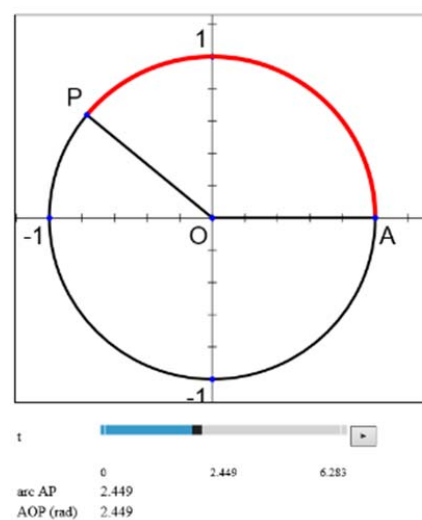

Fig. 4 A dynamic object for Cosine Rule

Fig. 5 A dynamic object for circular measure

Example 5. $\sin(-\theta) = -\sin\theta$

This is a dynamic object to understand the fact that sine function is an odd function well. Points P and Q lie on the unit circle, and P and Q are symmetrical. As the $y-$coodinate of P is $\sin\theta$ and the $y-$coodinate of Q is $\sin(-\theta)$, students drag P and see that $\sin(-\theta) = -\sin\theta$ holds for arbitrary θ (Fig. 6).
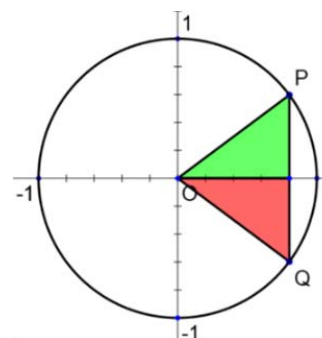

Fig. 6 A dynamic object for the formula

Example 6. $\sin(\theta + 180°) = -\sin\theta$

This is a dynamic object to understand the formula well. Points P and Q lie on the unit circle, and PQ passes the Origin. The $y-$coodinate of P is $\sin\theta$, and the $y-$coodinate of Q is $\sin(\theta + 180°)$. Students drag P and see that $\sin(\theta + 180°) =$

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:11, No:4, 2017

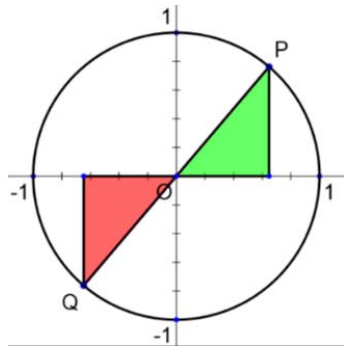$-\sin\theta$ holds for arbitrary θ (Fig. 7).

Fig. 7 A dynamic object for the formula

Example 7. Showing how to draw the graph $y = a\sin\theta$

This is a dynamic object to see how the graph of $y = a\sin\theta$ is drawn. The point P is on a circle of radius 1. Students can move P by dragging the slider below the graph. If they click the 'go' button, then the point P runs one lap in 20 seconds.

Students are also able to vary the value of a, the amplitude of the graph. The locus of point $Q(\theta, a\sin\theta)$ is the graph of $y = a\sin\theta$ (Fig. 8).
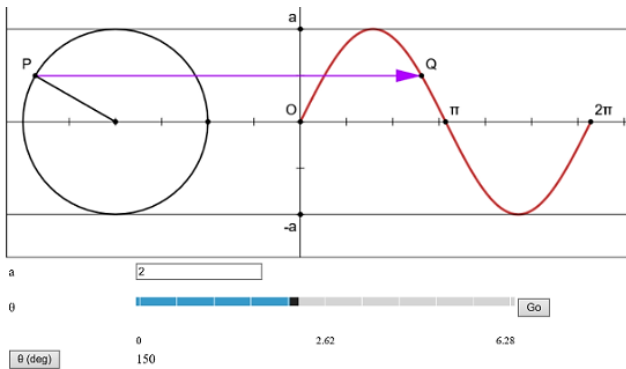

Fig. 8 The graph of $y = a\sin\theta$

Example 8. Showing how to draw the graph $y = a\sin 2\theta$.

This is a dynamic object to see how the graph of $y = a\sin 2\theta$ is drawn. There are four rolling drums. Centers of all drums are on x −axis and A and B have the same center (Fig. 9). Radius of A, C, D is a, and radius of B is $2a$. Drum C locates in the right of B and keeps in contact with B. Drum D locates in the right of C and keeps in contact with C.

Let P be the point on drum A which lies in the right edge of A, and let Q be the point on drum D which lies in the right edge of D. Now, let all the drums roll. Drums A and B roll counterclockwise at the same angular velocity. Drum C rolls twice as fast as B (and A) clockwise. Drum D rolls twice as fast as B (and A) counterclockwise.

As the y-coordinate of P is $a\sin\theta$ and the y-coordinate of Q is $a\sin 2\theta$, we can draw the graphs of $y = a\sin\theta$ and $y = a\sin 2\theta$ simultaneously. To understand the graphs better, students vary a and θ, and watch the result.
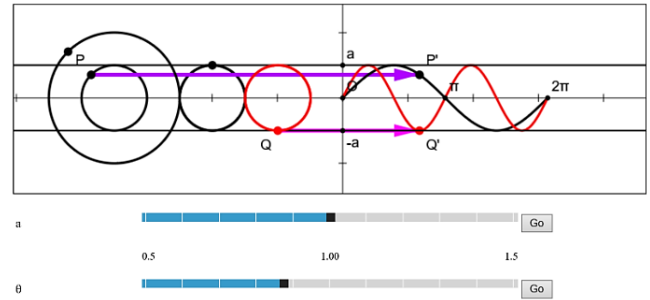

Fig. 9 Graphs of $y = a\sin\theta$ and $y = a\sin 2\theta$

Example 9. Trigonometric equation $\sin\theta = k$.

This is a dynamic object to understand trigonometric equation $\sin\theta = k$ well. If $-1 \le k \le 1$, then there is a real θ such that $\sin\theta = k$. We can find k by using the unit circle or by using the graph of $y = \sin\theta$. This dynamic object includes both the unit circle and the graph. So, students can choose his/her favorite way to solve the equation (Fig. 10).
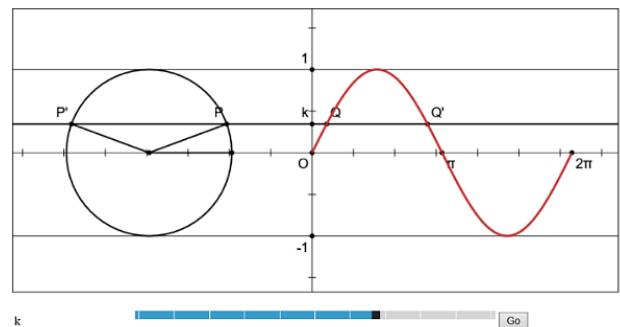

Fig. 10 A dynamic object for trigonometric equation

Example 10. A proof of $\arcsin x + \arccos x = \frac{\pi}{2}$.

This is a dynamic object to understand the formula well. Let x be a real satisfying $-1 \le x \le 1$. There is a point P on the unit circle such that the x −coordinate of P is x (Fig. 11). As the radium of the circle is 1, so $\cos\angle POA = x$, i.e. $\angle POA = \arccos x$.

As $\sin\angle POB = \sin\left(\frac{\pi}{2} - \angle POA\right) = \cos\angle POA = x$, so $\angle POB = \arcsin x$. And $\angle POB + \angle POA = \frac{\pi}{2}$ always holds. This implies $\arcsin x + \arccos x = \frac{\pi}{2}$.
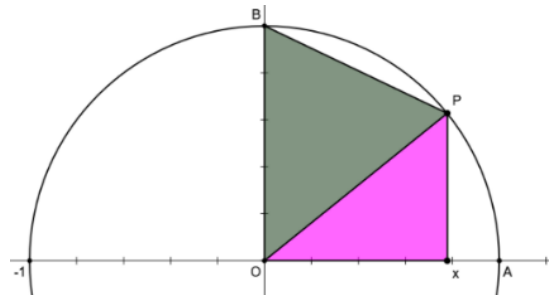

Fig. 11 A dynamic object for the theorem

World Academy of Science, Engineering and Technology
International Journal of Electronics and Communication Engineering
Vol:11, No:4, 2017

Example 11. Trigonometric equation $\sin 2\theta = \sin \theta$.

This equation can be solved geometrically. Let P be the intersecting point of the unit circle, and the line $y = x \tan \theta$.

Then, the coordinate of P is $(\cos \theta , \sin \theta)$. And let Q be the intersecting point of the circle $x^2 + y^2 = 2x$ and the line $y = x \tan \theta$ .Then, the coordinate of Q is $(2 \cos^2 \theta , \sin 2\theta)$ (Fig. 12). If $\sin 2\theta = \sin \theta$ holds, then P is equal to Q (Fig. 13).

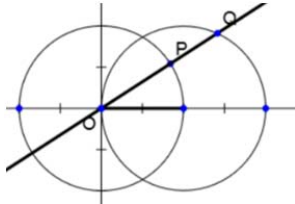Students can find all answers of the equation by rotating P.

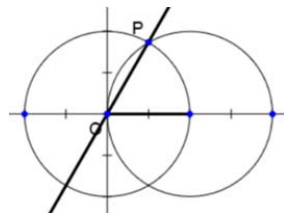

Fig. 12 Two circles and the line $y = x \tan \theta$

Fig. 13 In this case, $\sin 2\theta = \sin \theta$ holds

III. Lessons of Mathematics with Movies

There is a projector and a screen in every classroom at the author's school. Teachers can show students movies in the classroom. There are two ways to use our movies in mathematics lessons. The first way is a normal one. A teacher brings his notebook computer and USB speaker to the classroom. He/she rolls down all blinds for windows and put off all lights of the classroom and projects movies to the screen.

Example 12. A movie on trigonometry

In the movies, two characters teach how to calculate values of $\sin \theta$ and $\cos \theta$ for $\theta = 0°, 15°, 30°, \cdots, 360°$ (Fig. 14). It is a long movie. The author made a short movie for the first quadrant. He rewrote the sentences and transferred them to voices, and merged four short movies. If he used real girls, it would be much harder work.

The other way is a use of Microsoft PowerPoint. Before classes, the author made PowerPoint files which have a movie in each page. In lessons, he showed movies one by one, and added some explanations between movies.

Example 13. Eight new proofs of trigonometric addition formula

In 2012, the author found eight new proofs for trigonometric addition formula [8]. He made eight short movies. Each movie includes one proof, and he made a PowerPoint file which includes all movies. He showed them in his mathematics lesson. Between movies (proofs), he explained how he got the idea for the proof.

$$\cos 30° = \frac{\sqrt{3}}{2} \ , \ \sin 30° = \frac{1}{2} \qquad 30° = \frac{\pi}{6}$$
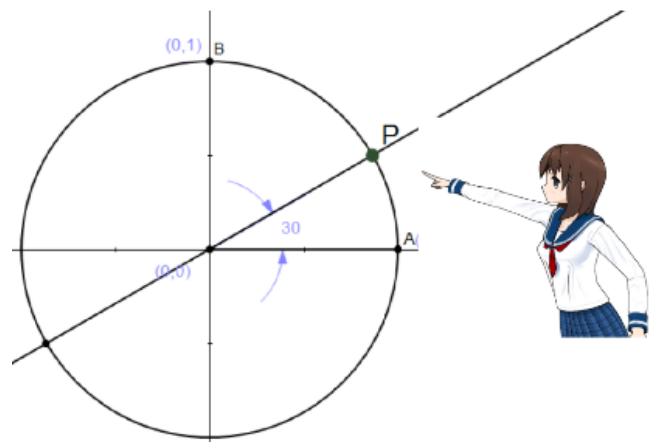

Fig. 14 The navigator is explaining trigonometry

IV. How to Transfer Data to Mobile Phones

There are three different ways to distribute movies and dynamic objects to students. The first way is to use USB memories. They are very handy but it takes much time to distribute many files to many students. They have to make a line to get movies or dynamic objects.

The second way is a use of Toshiba's Transfer Jet technology. It is a Close Proximity Wireless Transfer technology. With this technology, we can transfer files with transmission rate 375 Mbps [9]. This way of transfer needs dedicated adaptors, but we can transfer data not only from computer to mobile but also from mobile to mobile. In the other words, students can exchange their data rapidly with this technology.

The third way is a use of Google drive. This may be the best way. It needs no adaptors. In this way of distribution, students can get movies and dynamic objects not only at school or on weekday but also at home or during their vacations. They can study mathematics by watching movies even if they are somewhere far from school. Now mobile devices give students access to educational material anytime and anywhere.

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