Optimal Performance of Plastic Extrusion Process Using Fuzzy Goal Programming
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Abstract—This study optimized the performance of plastic extrusion process of drip irrigation pipes using fuzzy goal programming. Two main responses were of main interest; roll thickness and hardness. Four main process factors were studied. The L_18 array was then used for experimental design. The individual-moving range control charts were used to assess the stability of the process, while the process capability index was used to assess process performance. Confirmation experiments were conducted at the obtained combination of optimal factor setting by fuzzy goal programming. The results revealed that process capability was improved significantly from -1.129 to 0.8148 for roll thickness and from 0.0965 to 0.714 and hardness. Such improvement results in considerable savings in production and quality costs.

Keyword—Fuzzy goal programming, extrusion process, process capability, irrigation plastic pipes.

I. INTRODUCTION

The plastic industry is a widely growing field of industry since the demand for plastic products has increased rapidly due to its inexpensive raw material and easy processing. There are three types of processes for plastic forming; ignition modeling processes, extrusion process, and blow molding process. Plastics extrusion process produces high-volume of a wide variety of finished or semi-finished products including pipe, profile, sheet, film, and covered wire. One of the main applications in plastic industries that is manufactured by the extrusion process is the manufacturing of drip irrigation pipes. Drip irrigation pipes shown in Fig. 1 are made of polyethylene (PE) and have emitters that are placed at specified spaces along the tube that corresponds with the placement of each plant. For drip pipes production under study, two main quality characteristics are considered; pipe thickness and hardness.

Although the extrusion process provides high efficiency in producing pipes in a continuous manner under certain conditions and process settings, the process attributes variability on the main quality characteristics of the final drip pipe. Typically, customers demand high-quality pipes at minimal variations in the quality production levels and delivery schedules, while in reality the process variations in the drip irrigation pipes from the desired targets lead to produce low quality pipes and to rejection of the production lot, which negatively affects productivity and increases quality costs.

II. PROCESS PERFORMANCE AT INITIAL FACTOR SETTINGS

A. Control Charts

A sample of 20 rolls of drip irrigation pipes; each of 400 meters, are used to evaluate the process. Pipe's thickness (mm) and hardness (Pa) were measured using a digital caliper and Identec hardness machine, respectively. Since the sample size (n) is equal to 1, the individual moving range (I-MR) control charts are constructed for thickness and hardness as shown in Fig. 2. Obviously, the control charts indicate that the process is in statistical control for both quality responses. Table I summarizes the parameters; upper control limit (UCL), centerline (CL), and lower control limit (LCL), of the I-MR control charts. The estimated values of means and standard deviation are calculated and are also displayed in Table I.

Fig. 1 Drip irrigation pipes
B. Process Capability Analysis

Capability analysis is usually adopted to assess the ability of a process to meet product specifications. In practice, the process standard deviation, $\sigma$, is unknown and is frequently estimated by:

$$\hat{\sigma} = \frac{\overline{MR}}{d_2}$$ (1)

where $d_2$ is a constant related to the sample size (=1), while $\overline{MR}$ is the CL value in the MR chart. The actual process...
capability index \( C_{pk} \) attempts to take the target, \( T \), into account. The \( C_{pk} \) estimator, \( \hat{C}_{pk} \), can be expressed mathematically by:

\[
\hat{C}_{pk} = \min \left\{ \frac{\hat{\mu} - LSL}{3\sigma}, \frac{USL - \hat{\mu}}{3\sigma} \right\} \tag{2}
\]

Further, the multivariate capability index \( MC_{pk} \) is a criterion for selecting an optimal design and is used as a capability measure for a process having multiple performance measures. \( MC_{pk} \) is a proposed system capability index for the process which is the geometric mean of performance measure values:

\[
MC_{pk} = \left( \prod_{i=1}^{Q} C_{pk_i} \right)^{\frac{1}{Q}} \tag{3}
\]

where \( Q (=2) \) is the number of quality characteristics. For the irrigation pipe under study, the target and specification limit for pipe roll thickness is 0.95 ± 0.5 mm, while the target and specification limit for the hardness in each pipe roll is 116 ± 1 Pa.

In Table I, the \( \hat{C}_{pk} \) values are 0.58, 3.62, and 0.88 for the averages of tablet's weight, hardness, and thickness respectively. As a result, the tableting process is capable regarding the average tablet hardness, because this value is larger than the accepted level (1.33). However, it is found incapable for the averages of weight and thickness. Moreover, the calculated \( MC_{pk} \) value (= 0.333) is less than 1. These results indicate that further process improvement is needed.

### III. PROCESS OPTIMIZATION

Three main process factors are identified affecting the tablet quality, including: extruder temperature \((x_1, ^\circ C)\), cooling temperature \((x_2, ^\circ C)\), feeding rate \((x_3, \text{kg/min})\), and vacuum pressure \((x_4, \text{Pa})\). The appropriate orthogonal array is L18.

**Step 1:** Formulate the regression models for \( y_1 \) and \( y_2 \).

Tables II and III display the results of test of significance for thickness and hardness, respectively. Mathematically, the regression models are expressed as:

\[
y_1 = -49 + 0.191x_1 + 0.841x_1 + 0.655x_1 - 0.206x_1 - 0.002x_1 - 0.003x_1, \quad y_2 = 0.00255x_1 - 0.0278x_1 + 0.000034x_1 \quad \text{for } 21 \leq x_1 \leq -4.938x_1 + 0.00021x_1 \quad \text{for } 21 \leq x_1 \leq -4.938x_1 + 0.00021x_1 \quad \text{for } 21 \leq x_1 \leq
\]

**Table II**

<table>
<thead>
<tr>
<th>Predictor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>( T )</th>
<th>( P )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-49.0500000</td>
<td>16.5000000</td>
<td>-2.97</td>
<td>0.021</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.191070000</td>
<td>0.059710000</td>
<td>3.20</td>
<td>0.015</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>0.840600000</td>
<td>0.068900000</td>
<td>13.38</td>
<td>0.210</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>0.655200000</td>
<td>0.287400000</td>
<td>2.28</td>
<td>0.057</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>-0.206200000</td>
<td>0.370800000</td>
<td>-0.56</td>
<td>0.595</td>
</tr>
</tbody>
</table>

**Step 2:** Choose the suitable membership function representing each response. That is:

a) For the average tablet thickness, which is of NTB type response, the triangular membership function, \( \mu_{\text{NTB}} \), is represented by:

\[
\mu_{\text{NTB}}(y) = \begin{cases} 
0, & y \leq 0.9 \\
1 - 0.95 + 0.95y, & 0.9 \leq y < 0.95, \\
\frac{0.95 - y}{0.05}, & 0.95 \leq y < 1, \\
0, & y \geq 1 
\end{cases}
\]

Let \( \delta^- \) and \( \delta^+ \) denote the negative and positive deviation from the thickness target, then the corresponding constrains are:

\[
y_i + \delta^-_i - \delta^+_i = 0.95, \\
\mu_{\text{NTB}}(y_i) = \frac{\delta^-_i + \delta^+_i}{0.05 + 0.05} = 1, \\
0 \leq \delta^-_i \leq 0.05, \\
0 \leq \delta^+_i \leq 0.05,
\]

where \( \delta^-_i \) and \( \delta^+_i \) are the negative and positive deviations from the thickness target.
Similarly, let $\delta_j^-$ and $\delta_j^+$ denote the negative and positive deviation from the hardness target. For the pipe hardness, which is the LTB type, the membership function, $\mu_{y_j}$, is defined by:

$$
\mu_{y_j} = \begin{cases} 
0, & y_j < 115 \\
\frac{1 - 116 - y_j}{1}, & 116 \leq y_j < 116, \\
\frac{1 - 0116 - y_j}{1}, & 116 \leq y_j < 117, \\
0, & y_j \geq 117 
\end{cases}
$$

The goal constraints for $y_j$ are written as:

$$
y_j + \delta_j^- - \delta_j^+ = 116, \\
\mu_{y_j} + \frac{\delta_j^-}{1} + \frac{\delta_j^+}{1} = 1, \\
0 \leq \delta_j^- \leq 1, \\
0 \leq \delta_j^+ \leq 1,
$$

**Step 3:** Since process engineers have no prior information on the exact targets of $x_1, x_2, x_3$, and $x_4$, the settings of process factors could be set in ranges for $x_1$ of 255 to $290 \, ^\circ C$, 14 to $20 \, ^\circ C$ for $x_2$, 55 to 70 kg/min for $x_3$, and 1 to 2.5 Pa for $x_4$.

Then, the suitable MF, $\mu_{x_j}$, is defined as:

$$
\mu_{x_j} = \begin{cases} 
0, & x_j < g_{x_j}^l - \Delta_j^- \\
\frac{g_{x_j}^l - x_j}{\Delta_j^-}, & g_{x_j}^l - \Delta_j^- \leq x_j < g_{x_j}^l, \\
1, & g_{x_j}^l \leq x_j < g_{x_j}^u, \\
\frac{1 - x_j - g_{x_j}^u}{\Delta_j^+}, & g_{x_j}^u \leq x_j < g_{x_j}^u + \Delta_j^+, \\
0, & x_j \geq g_{x_j}^u + \Delta_j^+
\end{cases}
$$

where $g_{x_j}^l$ and $g_{x_j}^u$ are the lower and the upper limits of $x_j$, respectively. $\Delta_j^-$ and $\Delta_j^+$ are the maximal negative and positive admissible violations from $g_{x_j}^l$ and $g_{x_j}^u$, respectively.

$$
x_j + \delta_j^- \geq g_{x_j}^l, \\
x_j - \delta_j^+ \leq g_{x_j}^u,
$$

$$
\mu_{x_j} + \frac{\delta_j^-}{\Delta_j^-} + \frac{\delta_j^+}{\Delta_j^+} = 1, \\
0 \leq \delta_j^- \leq \Delta_j^-, \\
0 \leq \delta_j^+ \leq \Delta_j^+,
$$

where $\delta_j^-$ and $\delta_j^+$ represent the negative and positive deviations from $g_{x_j}^l$ and $g_{x_j}^u$, respectively. It is decided that the values of $\Delta_j^-$ and $\Delta_j^+$ equal 5, 2, 3, and 0.5 for $x_1, x_2, x_3$, and $x_4$, respectively. Then,

$$
x_1 + \delta_1^- \geq 255, \\
x_1 - \delta_1^+ \leq 290, \\
\mu_{x_1} + \frac{\delta_1^-}{5} + \frac{\delta_1^+}{2} = 1, \\
0 \leq \delta_1^- \leq 5, \\
0 \leq \delta_1^+ \leq 2,
$$

$$
x_3 + \delta_3^- \geq 55, \\
x_3 - \delta_3^+ \leq 70, \\
\mu_{x_3} + \frac{\delta_3^-}{3} + \frac{\delta_3^+}{0.5} = 1, \\
0 \leq \delta_3^- \leq 3, \\
0 \leq \delta_3^+ \leq 0.5.
$$

**Step 4:** The objective function is to minimize the sum of the weighted positive and negative deviations for the two responses and four process factors. Accordingly, the objective function is to minimize:

$$
Z = \frac{\delta_1^-}{0.05} + \frac{\delta_3^-}{0.5} + \frac{\delta_1^+}{2} + \frac{\delta_3^+}{0.5} + \frac{1}{0.5}
$$

The obtained optimal process conditions of extruder temperature ($x_1, ^\circ C$), cooling temperature ($x_2, ^\circ C$), feeding rate ($x_3, \, \text{kg/min}$), and vacuum pressure ($x_4, \, \text{Pa}$) are 290, 17.92, 70, and 1.6, respectively. The expected values for the thickness and hardness are calculated 0.95 and 116, respectively.
Fig. 3 Comparison between the I-MR charts
Conclusions

The fuzzy GP model is found to be an efficient approach for enhancing the performance of plastic extrusion processes with multiple responses, taking into consideration the engineers’ preferences about process settings.

REFERENCES


