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Optimal Performance of Plastic Extrusion Process Using Fuzzy Goal Programming

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Abstract—This study optimized the performance of plastic extrusion process of drip irrigation pipes using fuzzy goal programming. Two main responses were of main interest; roll thickness and hardness. Four main process factors were studied. The L_{18} array was then used for experimental design. The individual-moving range control charts were used to assess the stability of the process, while the process capability index was used to assess process performance. Confirmation experiments were conducted at the obtained combination of optimal factor setting by fuzzy goal programming. The results revealed that process capability was improved significantly from -1.129 to 0.8148 for roll thickness and from 0.0965 to 0.714 and hardness. Such improvement results in considerable savings in production and quality costs.

Keyword**—**Fuzzy goal programming, extrusion process, process capability, irrigation plastic pipes.

I. Introduction

THE plastic industry is a widely growing field of industry since the demand for plastic products has increased rapidly due to its inexpensive raw material and easy processing. There are three types of processes for plastic forming; ignition modeling processes, extrusion process, and blow molding process. Plastics extrusion process produces high-volume of a wide variety of finished or semi-finished products including pipe, profile, sheet, film, and covered wire. One of the main applications in plastic industries that is manufactured by the extrusion process is the manufacturing of drip irrigation pipes. Drip irrigation pipes shown in Fig. 1 are made of polyethylene (PE) and have emitters that are placed at specified spaces along the tube that corresponds with the placement of each plant. For drip pipes production under study, two main quality characteristics are considered; pipe thickness and hardness.

Although the extrusion process provides high efficiency in producing pipes in a continuous manner under certain conditions and process settings, the process attributes variability on the main quality characteristics of the final drip pipe. Typically, customers demand high–quality pipes at minimal variations in the quality production levels and delivery schedules, while in reality the process variations in the drip irrigation pipes from the desired targets lead to produce low quality pipes and to rejection of the production lot, which negatively affects productivity and increases quality costs.

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The Taguchi method is widely used for achieving robust design in a wide range of business applications [1]-[4]. Nevertheless, the past studies showed that this method is found only efficient in optimizing a single quality response [5]-[9]. Recently, optimization of process performance for multiple responses has received significant research attention [10]-[16]. Several formulations of goal programming (GP) models were introduced for solving the fuzzy GP (FGP) problems taking into account the decision maker's (DM's) preferences [17]-[20]. FGP was applied for optimizing process performance in many industrial applications [21]-[24]. It efficiently considers customer and process/product engineers' preferences [21]-[24]. This paper aims at optimizing the performance of direct compression process for multiple quality characteristics using statistical techniques and weighted additive model in fuzzy GP.

II. Process Performance at Initial Factor Settings

A. Control Charts

A sample of 20 rolls of drip irrigation pipes; each of 400 meters, are used to evaluate the process. Pipe's thickness (mm) and hardness (Pa) were measured using a digital caliper and Identec hardness machine, respectively. Since the sample size (n) is equal to 1, the individual moving range (I-MR) control charts are constructed for thickness and hardness as shown in Fig. 2. Obviously, the control charts indicate that the process is in statistical control for both quality responses. Table I summarizes the parameters; upper control limit (UCL), centerline (CL), and lower control limit (LCL), of the I-MR control charts. The estimated values of means and standard deviation are calculated and are also displayed in Table I.

Fig. 1 Drip irrigation pipes

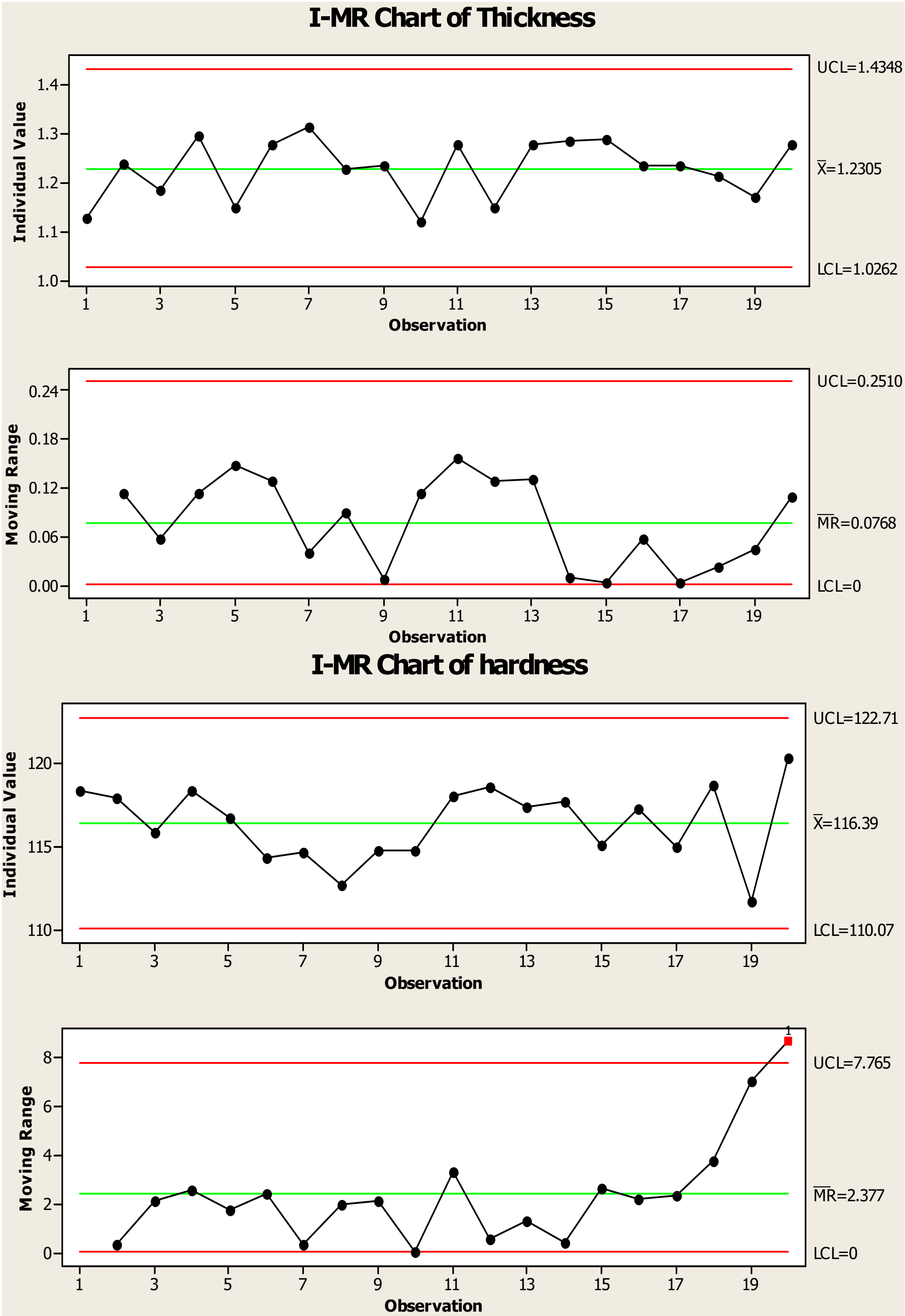

Fig. 2 The I-MR control chart

B. Process Capability Analysis

Capability analysis is usually adopted to assess the ability of a process to meet product specifications. In practice, the process standard deviation, σ, is unknown and is frequently estimated by:

$$\hat{\sigma} = \frac{\overline{MR}}{d_2} \quad (1)$$

where d_2 is a constant related to the sample size (=1), while $\overline{MR}$ is the *CL* value in the *MR* chart. The actual process capability index (C_{pk}) attempts to take the target, T, into account. The C_{pk} estimator, $\hat{C}_{pk}$, can be expressed mathematically by:

$$\hat{C}_{pk} = \min\left\{\frac{\hat{\mu} - LSL}{3\hat{\sigma}}, \frac{USL - \hat{\mu}}{3\hat{\sigma}}\right\} \quad (2)$$

Further, the multivariate process capability (MC_{pk}) is a criterion for selecting an optimal design and is used as a capability measure for a process having multiple performance measures. MC_{pk} is a proposed system capability index for the process which is the geometric mean of performance measure of C_{pk} values:

$$MC_{pk} = \left(\prod_{i=1}^{Q} C_{pki}\right)^{1/Q} \quad (3)$$

where Q (=2) is the number of quality characteristics. For the irrigation pipe under study, the target and specification limit for pipe roll thickness is 0.95 ± 0.5 mm, while the target and specification limit for the hardness in each pipe roll is 116 $\pm$ 1 Pa.

TABLE I
EXPERIMENTAL DATA

Exp.	x_1	x_2	x_3	x_4	y_1	y_2
1	270	13	43	1.5	1.157	118.000
2	270	13	47	2.0	1.202	114.200
3	270	13	52	2.5	1.226	117.333
4	270	15	43	1.5	1.081	116.366
5	270	15	47	2.0	1.433	111.566
6	270	15	52	2.5	1.393	108.300
7	270	17	43	2.0	1.220	113.200
8	270	17	47	2.5	1.343	111.066
9	270	17	52	1.5	1.457	113.600
10	280	13	43	2.5	1.448	117.066
11	280	13	47	1.5	1.452	114.966
12	280	13	52	2.0	1.486	116.000
13	280	15	43	2.0	1.501	114.366
14	280	15	47	2.5	1.534	114.900
15	280	15	52	1.5	1.368	116.300
16	280	17	43	2.5	1.468	116.366
17	280	17	47	1.5	1.520	114.700
18	280	17	52	2.0	1.544	112.233

In Table I, the $\hat{C}_{pk}$ values are 0.58, 3.62, and 0.88 for the averages of tablet's weight, hardness, and thickness respectively. As a result, the tableting process is capable regarding the average tablet hardness, because this value is larger than the accepted level (1.33). However, it is found incapable for the averages of weight and thickness. Moreover, the calculated $\hat{MC}_{pk}$ value (= 0.333) is less than 1. These results indicate that further process improvement is needed.

III. PROCESS OPTIMIZATION

Three main process factors are identified affecting the tablet quality, including: extruder temperature (x_1, ^oC), cooling temperature (x_2, ^oC), feeding rate (x_3, kg/min), and vacuum pressure (x_4, Pa). The appropriate orthogonal array is L_{18}.

Step 1: Formulate the regression models for y_1 and y_2. Tables II and III display the results of test of significance for thickness and hardness, respectively. Mathematically, the regression models are expressed as:

$$y_1 = -49 + 0.191x_1 + 0.841x_2 + 0.655x_3 - 0.206x_4 + 0.002x_2x_3 - 0.0034x_1x_2 - 0.00255x_1x_3 - 0.0278x_2x_4 + 0.0000034x_1x_2x_3x_4$$

$$y_2 = 612 - 2.33x_1 + 3.9x_2 - 3.79x_3 + 27x_4 - 0.691x_2x_3 + 0.0634x_1x_2 + 0.03\ 21x_1x_3 - 4.938x_2x_4 + 0.00021x_2^2x_3^2 + 0.036x_2^2x_4^2 + 0.000075x_1x_2x_3x_4$$

TABLE II
RESULTS OF TEST OF SIGNIFICANCE FOR THICKNESS R^2 =92.7%, R^2(ADJUSTED)=83.4.%

Predictor	Coefficient	Standard Error	T	P
Constant	-49.0500000	16.50000000	-2.97	0.021
x_1	0.191070000	0.059710000	3.20	0.015
x_2	0.840600000	0.608900000	1.38	0.210
x_3	0.655200000	0.287400000	2.28	0.057
x_4	-0.206200000	0.370800000	-0.56	0.595
$x_2 x_3$	0.002007000	0.003035000	0.66	0.530
$x_1 x_2$	-0.003413000	0.001994000	-1.71	0.131
x_1x_3	-0.002547000	0.001017000	-2.50	0.041
$x_2 x_4$	-0.027770000	0.048720000	-0.57	0.586
$x_1 x_2 x_3 x_4$	0.000003330	0.000003330	1.00	0.350

Step 2: Choose the suitable membership function representing each response. That is:

a) For the average tablet thickness, which is of NTB type response, the triangular membership function, μ_{y_1}, is represented by:

$$\mu_{y_1} = \begin{cases} 0, y_1 < 0.9 \\ 1 - \frac{0.95 - y_1}{0.05}, 0.9 \le y_1 < 0.95, \\ 1 - \frac{y_1 - 0.95}{0.05}, 0.95 \le y_1 < 1, \\ 0, y_1 \ge 1 \end{cases}$$

Let $\delta_{y_1}^-$ and $\delta_{y_1}^+$ denote the negative and positive deviation from the thickness target, then the corresponding constrains are:

$$y_1 + \delta_{y_1}^- - \delta_{y_1}^+ = 0.95,$$

$$\mu_{y_1} + \frac{\delta_{y_1}^-}{0.05} + \frac{\delta_{y_1}^+}{0.05} = 1,$$

$$0 \le \delta_{y_1}^- \le 0.05,$$

$$0 \le \delta_{y_1}^+ \le 0.05,$$

Similarly, let $\delta_{y_2}^-$ and $\delta_{y_2}^+$ denote the negative and positive deviation from the hardness target. For the pipe hardness, which is the LTB type, the membership function, μ_{y_2}, is defined by:

$$\mu_{y_2} = \begin{cases} 0, y_2 < 115 \\ 1-\frac{116-y_2}{1}, 115 \le y_2 < 116, \\ 1-\frac{0116-y_2}{1}, 116 \le y_2 < 117, \\ 0, y_2 \ge 117 \end{cases}$$

The goal constraints for y_2 are written as:

$$y_2 + \delta_{y2}^- - \delta_{y_2}^+ = 116,$$
$$\mu_{y_1} + \frac{\delta_{y2}^-}{1} + \frac{\delta_{y2}^+}{1} = 1,$$
$$0 \le \delta_{y_2}^- \le 1,$$
$$0 \le \delta_{y_2}^+ \le 1,$$

Step 3: Since process engineers have no prior information on the exact targets of x_1, x_2, x_3, and x_4, the settings of process factors could be set in ranges for x_1 of 255 to 290^oC, 14 to 20^oC for x_2, 55 to 70 kg\min for x_3, and 1.5 to 2.5 Pa for x_4. Then, the suitable MF, μ_{x_j}, is defined as:

$$\mu_{x_j} = \begin{cases} 0, & x_j < g_{x_j}^l - \Delta_{x_j}^-, \\ 1-\frac{g_{x_j}^l - x_j}{\Delta_{x_j}^-}, & g_{x_j}^l - \Delta_{x_j}^- \le x_j < g_{x_j}^l, \\ 1, & g_{x_j}^l \le x_j < g_{x_j}^u, \\ 1-\frac{x_j - g_{x_j}^u}{\Delta_{x_j}^+}, & g_{x_j}^u \le x_j < g_{x_j}^u + \Delta_{x_j}^+, \\ 0, & x_j \ge g_{x_j}^u + \Delta_{x_j}^+, \end{cases}$$

where $g_{x_j}^l$ and $g_{x_j}^u$ are the lower and the upper limits of x_j, respectively. $\Delta_{x_j}^-$ and $\Delta_{x_j}^+$ are the maximal negative and positive admissible violations from $g_{x_j}^l$ and $g_{x_j}^u$, respectively.

$$x_j + \delta_{x_j}^- \ge g_{x_j}^l,$$
$$x_j - \delta_{x_j}^+ \le g_{x_j}^u,$$
$$\mu_{x_j} + \frac{\delta_{x_j}^-}{\Delta_{x_j}^-} + \frac{\delta_{x_j}^+}{\Delta_{x_j}^+} = 1,$$
$$0 \le \delta_{x_j}^- \le \Delta_{x_j}^-,$$
$$0 \le \delta_{x_j}^+ \le \Delta_{x_j}^+,$$

where $\delta_{x_j}^-$ and $\delta_{x_j}^+$ represent the negative and positive deviations from $g_{x_j}^l$ and $g_{x_j}^u$, respectively. It is decided that the values of $\Delta_{x_j}^-$ and $\Delta_{x_j}^+$ equal 5, 2, 3, and 0.5 for x_1, x_2, x_3, and x_4, respectively. Then,

$$x_1 + \delta_{x1}^- \ge 255, \qquad x_2 + \delta_{x_2}^- \ge 14,$$
$$x_1 - \delta_{x_1}^+ \le 290, \qquad x_2 - \delta_{x_2}^+ \le 20,$$
$$\mu_{x_1} + \frac{\delta_{x_1}^-}{5} + \frac{\delta_{x_1}^+}{5} = 1, \qquad \mu_{x_2} + \frac{\delta_{x_2}^-}{2} + \frac{\delta_{x_2}^+}{2} = 1,$$
$$0 \le \delta_{x_1}^- \le 5, \qquad 0 \le \delta_{x_2}^- \le 2,$$
$$0 \le \delta_{x_1}^+ \le 5, \qquad 0 \le \delta_{x_2}^+ \le 2,$$

$$x_3 + \delta_{x3}^- \ge 55, \qquad x_4 + \delta_{x_4}^- \ge 1.5,$$
$$x_3 - \delta_{x_3}^+ \le 70, \qquad x_4 - \delta_{x_4}^+ \le 2.5,$$
$$\mu_{x_3} + \frac{\delta_{x_3}^-}{3} + \frac{\delta_{x_3}^+}{3} = 1, \qquad \mu_{x_4} + \frac{\delta_{x_4}^-}{0.5} + \frac{\delta_{x_4}^+}{0.5} = 1,$$
$$0 \le \delta_{x_3}^- \le 3, \qquad 0 \le \delta_{x_4}^- \le 0.5,$$
$$0 \le \delta_{x_3}^+ \le 3, \qquad 0 \le \delta_{x_4}^+ \le 0.5,$$

Step 4: The objective function of is to minimize the sum of the weighted positive and negative deviations for the two responses and four process factors. Accordingly, the objective function is to minimize:

$$Z = (\delta_{y_1}^+ + \delta_{y_1}^-)/0.05 + (\delta_{y_2}^+ + \delta_{y_2}^-) + (\delta_{x_1}^+ + \delta_{x_1}^-)/5 + (\delta_{x_2}^+ + \delta_{x_2}^-)/2$$
$$+ (\delta_{x_3}^+ + \delta_{x_3}^-)/3 + (\delta_{x_4}^+ + \delta_{x_4}^-)/0.5$$

The obtained optimal process conditions of extruder temperature (x_1, ^oC), cooling temperature (x_2, ^oC), feeding rate (x_3, kg/min), and vacuum pressure (x_4, Pa) are 290, 17.92, 70, and 1.6, respectively. The expected values for the thickness and hardness are calculated 0.95 and 116, respectively.

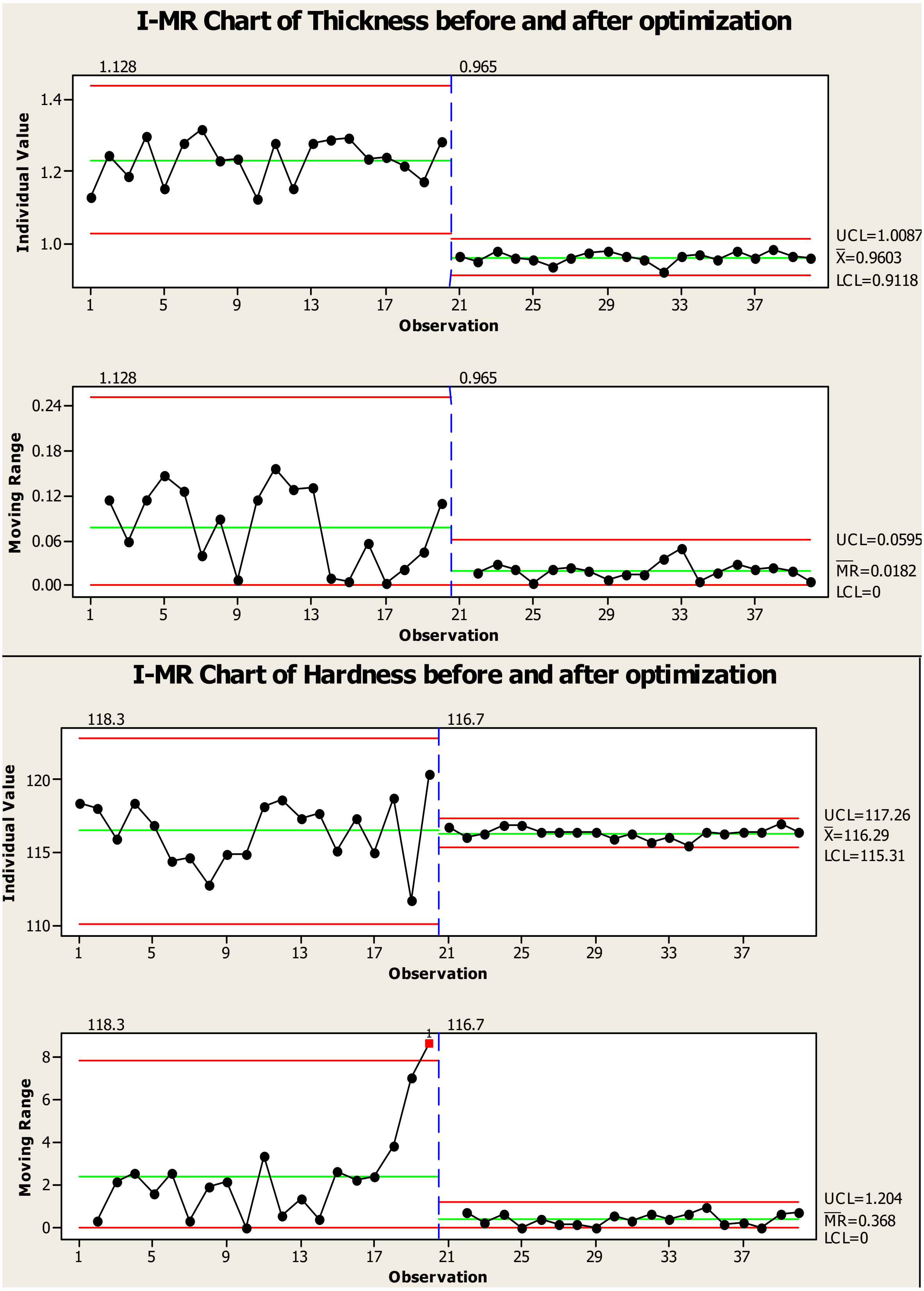

Fig. 3 Comparison between the I-MR charts

TABLE III
RESULTS OF TEST OF SIGNIFICANCE FOR HARDNESS. R^2 =97%, R^2(ADJUSTED)=90.5%

Predictor	Coefficient	Standard Error	*T*	*P*
Constant	611.80000000	400.1000000	1.53000000	0.187000000
x_1	-2.33200000	1.45600000	-1.60000000	0.17000000
x_2	3.91000000	13.71000000	0.28000000	0.78700000
x_3	-3.78600000	6.91200000	-0.55000000	0.60700000
x_4	26.96000000	13.6100000	1.98000000	0.10400000
$x_2 x_3$	-0.69080000	0.17480000	-3.95000000	0.01100000
$x_1 x_2$	0.06338000	0.04239000	1.5000000	0.11500000
$x_1 x_3$	0.03212000	0.02346000	1.37000000	0.22900000
$x_2 x_4$	-4.92800000	1.62300000	-3.04000000	0.02900000
$(x_2 x_3)^2$	0.00020755	0.00005587	3.71000000	0.01400000
$(x_2 x_4)^2$	0.03602000	0.00997900	3.61000000	0.01500000
$x_1 x_2 x_3 x_4$	0.00007521	0.00006422	1.17000000	0.29400000

TABLE IV
THE ESTIMATED PARAMETERS OF THE I-MR CONTROL CHARTS

Response	Process settings	*I-Chart*			*MR-Chart*			$\hat{\sigma}$	$\hat{\mu}$
		UCL	CL	LCL	UCL	CL	LCL		
Thickness (mm)	Initial	1.4348	1.2305	1.0262	0.25100	0.07680	0	0.0680	1.2305
	Optimal	1.0091	0.9604	0.9116	0.05985	0.01832	0	0.0162	0.9604
Hardness (Pa)	Initial	122.71	116.39	110.070	7.76500	2.37700	0	2.1070	116.39
	Optimal	117.284	116.289	115.294	1.22200	0.37400	0	0.3315	116.289

IV. RESULTS

Confirmation experiments are conducted in the combination of optimal factor settings. The corresponding I-MR control charts are then established as shown in Fig. 3. It is obvious that the I-MR charts are in statistical control for both responses. The related parameters and the values of the estimated means and standard deviations are also displayed in Table IV. Finally, the process capability index, $\hat{C}_{pk}$, values are calculated and found to be 0.8148 and 0.7140 for thickness and hardness, respectively. The estimated value of $\hat{MC}_{pk}$ is 0.5817.

V. CONCLUSIONS

Fuzzy GP was implemented to optimize two quality responses of irrigation pipes. The L_{18} array was utilized for conducting the experimental work. Confirmation results showed that: (1) the process means for roll thickness and hardness at optimal factor settings are closer to the desired values of 0.95 mm and 116 Pa, respectively, (2) process variability is significantly reduced by, and (3) the $\hat{C}_{pk}$ is improved significantly from -1.129 to 0.8148 for roll thickness and from 0.0965 to 0.714 for hardness. In conclusion, the fuzzy GP model is found to be an efficient approach for enhancing the performance of plastic extrusion processes with multiple responses, taking into consideration the engineers' preferences about process settings.

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