Entropy Measures on Neutrosophic Soft Sets and Its Application in Multi Attribute Decision Making

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Abstract—The focus of the paper is to furnish the entropy measure for a neutrosophic set and neutrosophic soft set which is a measure of uncertainty and it permeates discourse and system. Various characterization of entropy measures are derived. Further we exemplify this concept by applying entropy in various real time decision making problems.

Keywords—Entropy measure, Hausdorff distance, neutrosophic set, soft set.

I. Introduction

THE decision making problems of the real world will involve many imprecision and inadequate data. Such type of uncertainties was dealt with the idea of fuzzy logic which was introduced by L. A. Zadeh [10] in the year 1965, it uses a membership function in the interval [0, 1]. This idea was further refined by Atanassov [2] in 1983 which include the grade of membership and the grade of non membership. The Neutrosophic set was first of its kind to introduce the idea of neutralities, as the decision making models will involve some indeterminate data. It was defined by Smarandache [6], [7]. This logic introduces a component called indeterminacy to the concept of fuzzy logic. Problems which involve imprecision, indeterminacy and inconsistency can be treated with the Neutrosophic logic which has degree of truth (T), a degree of indeterminacy (I) and a degree of falsity (F), where T, I, F takes the values from real standard or non-standard subsets of]-0, 1+[. Entropy, similarity measure and distance measure are three important notions for measuring uncertainty. Several researchers like Huang [3], Hung and Yang [4], Majumdar and Samanta [5], Szmidt and Kacprzy [8], Wang and Qu [9] have studied the similarity measures distance measure and entropy on fuzzy sets, intuitionistic fuzzy set, vague soft set and neutrosophic set. In this paper we have introduced some new entropy and distance measures for neutrosophic soft set.

II. Preliminaries

Definition 1: [1] Let U be a universe of discourse, and A a set included in U. An element x from U is defined as $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle, x \in X$ where $T, I, F : X \to [0, 1]$ and $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$, where T, I and F represents truth value, indeterministic value and the false value respectively.

Definition 2: [1] Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ are neutrosophic sets. Then A is a subset of B if $\forall x \in X$
$$T_A(x) \leq T_B(x), I_A(x) \leq I_B(x)), F_A(x) \geq F_B(x))$$

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Definition 3: [1] Let X be a non empty set, and $A = \langle x, T_A(x), I_A(x), F_A(x) \rangle$, $B = \langle x, T_B(x), I_B(x), F_B(x) \rangle$ are neutrosophic sets.Then
$A \cup B = \langle x, max(T_A(x), T_B(x)), max(I_A(x), I_B(x)),$
$min(F_A(x), F_B(x)) \rangle$
$A \cap B = \langle x, min(T_A(x), T_B(x)), min(I_A(x), I_B(x)),$
$max(F_A(x), F_B(x)) \rangle$

Definition 4: [1] A collection (F,A) is called a Neutrosophic soft set iff $F : A \to P(U)$, where $P(U)$ is the collection of all neutrosophic sets on the universal set U and A is a non-empty subset of the parameter set E.

Definition 5: [1] A void neutrosophic soft set (F,A) over the universe U with respect to the parameter A is defined as $T_{F(e)} = 0$, $I_{F(e)} = 0, F_{F(e)} = 1, \forall$ x $\in$ U, $\forall$ e $\in$ A and is denoted by $\widetilde{0}$.

Definition 6: [1] A neutrosophic soft set (F,A) over the universe U is said to be absolute neutrosophic soft set with respect to the parameter A if $T_{F(e)} = 1$, $I_{F(e)} = 1, F_{F(e)} = 0$, $\forall$ x $\in$ U, $\forall$ e $\in$ A. It is denoted by $\widetilde{1}$.

Definition 7: [1] A neutrosophic soft set (F,A) is said to be a subset of neutrosophic soft set (G,B) if A$\subseteq$ B and F(e)$\subseteq$ G(e) $\forall$ e $\in$ E, u $\in$ U. We denote it by (F,A)$\widetilde{\subseteq}$ (G,B).

Definition 8: [1] The complement of neutrosophic soft set (F,A) denoted by $(F, A)^c$ and is defined as $(F, A)^c = (F^c, \neg A)$ where $F^c : \neg A \longrightarrow P(U)$ is a mapping given by
$F^c(\alpha) = \langle x, T_{F^c}(x) = F_F(x), I_{F^c}(x) = 1 - I_F(x), F_{F^c}(x) = T_F(x) \rangle$

Definition 9: [1] The union of two neutrosophic soft sets (F,A) and (G,B) over (U,E) is neutrosophic soft set where C = A$\cup$B, $\forall e \in C$
$$H(e) = \begin{cases} F(e) & \text{if e} \in A - B \\ G(e) & \text{if e} \in B - A \\ F(e) \cup G(e) & \text{if e} \in A \cap B \end{cases}$$
and is written as (F,A)$\widetilde{\cup}$(G,B)= (H,C).

Definition 10: [1] The intersection of two neutrosophic soft sets (F,A) and (G,B) over (U,E) is neutrosophic soft set where C = A$\cap$B, $\forall e \in C$ $H(e) = F(e) \cap G(e)$ and is written as (F,A)$\widetilde{\cap}$(G,B)= (H,C).

III. Entropy and Distance Measure of Neutrosophic Soft Set and Its Application

Definition 11: Let H: NSS(U)$\to$ [0,1] be a mapping, where NSS(U) denotes the set of all neutrosophic soft sets on U. For (F,E)$\in$ NSS(U), H(F,E) is called the entropy of (F,E) if it satisfies the following conditions:

1) H(F,E)= 0 $\Leftrightarrow$ $\forall$ e $\in$E, x $\in$U, $T_{F(e)}(x) = 0$, $I_{F(e)}(x) = 0$ and $F_{F(e)}(x) = 1$ or $T_{F(e)}(x) = 1$, $I_{F(e)}(x) = 1$ and $F_{F(e)}(x) = 0$

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2) H(F,E)=1 $\forall$ e $\in$E, x $\in$U, $T_{F(e)}(x) = I_{F(e)}(x) = F_{F(e)}(x) = 0.5$

3) H(F,E)= $H(F,E)^c$

4) $\forall$ e $\in$E, x $\in$U when (F,E)$\subseteq$ (G,E), and $T_{G(e)}(x) \leq F_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$ if $I_{G(e)}(x) \leq 0.5$ or (F,E)$\supseteq$ (G,E), and $T_{G(e)}(x) \geq F_{G(e)}(x)$, $I_{F(e)}(x) \geq I_{G(e)}(x)$ if $I_{G(e)}(x) \geq 0.5$ then H(F,E)$\leq$ H(G,E)

Definition 12: Let d:NSS(U)$\times$ NSS(U)$\rightarrow$ [0,1] be a mapping for (F,E), (G,E)$\in$ NSS(U), $d((F,E),(G,E))$ is called the degree of distance between (F,E) and (G,E) if it satisfies the following conditions:

1) d((F,E),(G,E))=d((G,E),(F,E)).

2) d((F,E),(G,E)) $\in$[0,1]

3) d((F,E),(G,E))= 1 $\Leftrightarrow$ $\forall$ e $\in$E, x $\in$U, $T_{F(e)}(x) = 0$, $I_{F(e)}(x) = 0$, $F_{F(e)}(x) = 1$ and $T_{G(e)}(x) = 1$, $I_{G(e)}(x) = 1$, $F_{G(e)}(x) = 0$ or $T_{F(e)}(x) = 1$, $I_{F(e)}(x) = 1$ and $F_{F(e)}(x) = 0$ or $T_{G(e)}(x) = 0$, $I_{G(e)}(x) = 0$ and $F_{G(e)}(x) = 1$

4) d((F,E),(G,E))=0 $\Leftrightarrow$ (F,E)=(G,E)

5) (F,E)$\subseteq$(G,E)$\subseteq$ (P,E)$\Rightarrow$ d((F,E), (P,E))$\geq$ max(d((F,E),(G,E)), d((G,E),(P,E)), (P,E)$\in$ NSS(U).

Definition 13: Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters, then we have

$$H(F,E) = \frac{1}{m} \sum_{i=1}^{n} H_i(F,E)$$

where $H_i(F,E) = \dfrac{1}{n} \dfrac{max \sum_j count((F,E) \cap (F,E)^c)}{max \sum_j count((F,E) \cup (F,E)^c)}$

$max \sum_j count((F,E) \cap (F,E)^c))$

$= \sum_{j=1}^{n} \left(T_{((F,E)\cap(F,E)^c)(e_i)}(x_j) + I_{((F,E)\cap(F,E)^c)(e_i)}(x_j) \right)$

$max \sum_j count((F,E) \cup (F,E)^c))$

$= \sum_{j=1}^{n} \left(T_{((F,E)\cup(F,E)^c)(e_i)}(x_j) + I_{((F,E)\cup(F,E)^c)(e_i)}(x_j) \right)$

is the entropy of neutrosophic soft sets.

Definition 14: Let U= $\{x_1, x_2, ...x_n\}$ be the universal set of elements and E = $\{e_1, e_2, ...e_m\}$ be the universal set of parameters, then we define normalized Euclidean distance based on Hausdorff metric as

$d_1((F,E),(G,E)) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} max(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|,$
$|I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|, |F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)$

and normalized Hamming distance based on Hausdorff metric as

$d_2((F,E),(G,E)) =$
$\{\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} max((T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j))^2,$
$(I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j))^2, (F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j))^2)\}^{\frac{1}{2}}$

Theorem 1: Let (F,E), (G,E) and (H,E) be Neutrosophic soft set over U, then distance measure $d_i((F,E),(G,E))$ for i = 1,2 between (F,E) and (G,E) satisfies the following properties.

(i) $0 \leq d_i((F,E),(G,E)) \leq 1$

(ii) $d_i((F,E),(G,E)) = 0$ if and only if $(F,E) = (G,E)$

(iii) $d_i((F,E),(G,E)) = d_i((G,E),(F,E))$

(iv) If $F \subseteq G \subseteq H$, then $d_i((F,E),(G,E)) \leq d_i((F,E),(H,E))$ and $d_i((G,E),(H,E)) \leq d_i((F,E),(H,E))$

Proof:

(i) As truth-membership, indeterminacy-membership and falsity-membership functions lies between 0 and 1, the distance measure based on these function also lies between 0 to 1.

(ii) If $d_i((F,E),(G,E)) = 0$ implies
$|T_F(e_i)(u_j) - T_G(e_i)(u_j)| = 0$
$|I_F(e_i)(u_j) - I_G(e_i)(u_j)| = 0$
$|F_F(e_i)(u_j) - F_G(e_i)(u_j)| = 0$ implies
$T_F(e_i)(u_j) = T_G(e_i)(u_j)$,
$I_F(e_i)(u_j) = I_G(e_i)(u_j)$, $F_F(e_i)(u_j) = F_G(e_i)(u_j)$
i.e., (F,E) = (G,E).
Conversely, Let (F,E) = (G,E), implies
$T_F(e_i)(u_j) = T_G(e_i)(u_j)$, $I_F(e_i)(u_j) = I_G(e_i)(u_j)$, $F_F(e_i)(u_j) = F_G(e_i)(u_j)$, implies
$|T_F(e_i)(u_j) - T_G(e_i)(u_j)| = |I_F(e_i)(u_j) - I_G(e_i)(u_j)| = |F_F(e_i)(u_j) - F_G(e_i)(u_j)| = 0$
ie., $d_i((F,E),(G,E)) = 0$.

(iii) Clearly $d((F,A),(G,B)) = d((G,B),(F,A))$.

(iv) Let $D((F,E),(G,E)) = max(|T_{F(e_i)}(x_j) - T_{G(e_i)}(x_j)|, |I_{F(e_i)}(x_j) - I_{G(e_i)}(x_j)|, |F_{F(e_i)}(x_j) - F_{G(e_i)}(x_j)|)$.
$F \subseteq G \subseteq H$ implies
$T_F(e_i)(u_j) \leq T_G(e_i)(u_j) \leq T_H(e_i)(u_j)$
$I_F(e_i)(u_j) \leq I_G(e_i)(u_j) \leq I_H(e_i)(u_j)$
$F_F(e_i)(u_j) \geq F_G(e_i)(u_j) \geq F_H(e_i)(u_j)$
To Prove: $d_1((F,E),(G,E)) \leq d_1((F,E),(H,E))$ and $d_1((G,E),(H,E)) \leq d_1((F,E),(H,E))$
Case: 1 If $|T_F(e_i)(u_j) - T_H(e_i)(u_j)| \geq |I_F(e_i)(u_j) - I_H(e_i)(u_j)| \geq |F_F(e_i)(u_j) - F_H(e_i)(u_j)|$ then $D((F,E),(G,E)) = |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$

(i) $|I_F(e_i)(u_j) - I_G(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$, $\forall$ i and j
$|F_F(e_i)(u_j) - F_G(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$, $\forall$ i and j

(ii) $|I_G(e_i)(u_j) - I_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$, $\forall$ i and j
$|F_G(e_i)(u_j) - F_H(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$, $\forall$ i and j

(iii) $|T_F(e_i)(u_j) - T_G(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$,
$|T_G(e_i)(u_j) - T_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)|$,
$\forall$ i and j.

Combining (i) - (iii) we have $D((F,E),(G,E)) \leq D((F,E),(H,E))$ and $D((G,E),(H,E)) \leq D((F,E),(H,E))$. Therefore $d_1((F,E),(G,E)) \leq d_1((F,E),(H,E))$ and $d_1((G,E),(H,E)) \leq d_1((F,E),(H,E))$

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Case: 2 If $|T_F(e_i)(u_j) - T_H(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$ then $D((F,E),(G,E)) = |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$

(i) $|T_F(e_i)(u_j) - T_G(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$, $\forall$ i and j
$|F_F(e_i)(u_j) - F_G(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$, $\forall$ i and j

(ii) $|T_G(e_i)(u_j) - T_H(e_i)(u_j)| \leq |T_F(e_i)(u_j) - T_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$, $\forall$ i and j
$|F_G(e_i)(u_j) - F_H(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$, $\forall$ i and j

(iii) $|I_F(e_i)(u_j) - I_G(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$,
$|I_G(e_i)(u_j) - I_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)|$, $\forall$ i and j.

Combining (i)-(iii) we have $D((F,E),(G,E)) \leq D((F,E),(H,E))$ and $D((G,E),(H,E)) \leq D((F,E),(H,E))$. Therefore $d_1((F,E),(G,E)) \leq d_1((F,E),(H,E))$ and $d_1((G,E),(H,E)) \leq d_1((F,E),(H,E))$

Case: 3 If $|T_F(e_i)(u_j) - T_H(e_i)(u_j)| \leq |I_F(e_i)(u_j) - I_H(e_i)(u_j)| \leq |F_F(e_i)(u_j) - F_H(e_i)(u_j)|$ then $D((F,E),(G,E)) = |F_F(e_i)(u_j) - F_H(e_i)(u_j)|$
Proof is similar to Case 1 and Case 2.
Hence from Case 1 Case 2 and Case 3 we have if $F \subseteq G \subseteq H$, then $d_1((F,E),(G,E)) \leq d_1((F,E),(H,E))$ and $d_1((G,E),(H,E)) \leq d_1((F,E),(H,E))$.
Similarly we can prove that if $F \subseteq G \subseteq H$, then $d_2((F,E),(G,E)) \leq d_2((F,E),(H,E))$ and $d_2((G,E),(H,E)) \leq d_2((F,E),(H,E))$.

Theorem 2: Let d((F,E),(G,E)) be the distance measure between two neutrosophic soft sets (F,E) and (G,E). Define $H(F,E) = \dfrac{1 - d((F,E),(F,E)^c)}{1 + d((F,E),(F,E)^c)}$ then H(F,E) is an entropy of neutrosophic soft sets.
Proof:

(1) H(F,E)= $0 \Leftrightarrow 1 - d((F,E),(F,E)^c) = 0 \Leftrightarrow d((F,E),(F,E)^c) = 1 \Leftrightarrow \forall$ e $\in$ E, x $\in$U, $T_{F(e)}(x) = 0$, $I_{F(e)}(x) = 0$ and $F_{F(e)}(x) = 1$ or $T_{F(e)}(x) = 1$, $I_{F(e)}(x) = 1$ and $F_{F(e)}(x) = 0$

(2) H(F,E)= $1 \Leftrightarrow 1 - d((F,E),(F,E)^c) = 1 + d((F,E),(F,E)^c) \Leftrightarrow d((F,E),(F,E)^c) = 0 \Leftrightarrow (F,E) = (F,E)^c \Leftrightarrow \forall$e $\in$ E, x $\in$U, $T_{F(e)}(x) = I_{F(e)}(x) = F_{F(e)}(x) = 0.5$

(3) $H(F,E)^c = \dfrac{1 - d((F,E)^c, ((F,E)^c)^c)}{1 + d((F,E)^c, ((F,E)^c)^c)}$
$= \dfrac{1 - d((F,E)^c, ((F,E))}{1 + d((F,E)^c, ((F,E))}$ = H(F,E)

(4) $\forall$ e $\in$E, x $\in$U when (F,E)$\subseteq$ (G,E), and $T_{G(e)}(x) \leq$

$F_{G(e)}(x)$, $I_{F(e)}(x) \leq I_{G(e)}(x)$ if $I_{G(e)}(x) \leq 0.5$ implies $T_{F(e)} \leq T_{G(e)} \leq F_{G(e)} \leq F_{F(e)}$; also $I_{F(e)} \leq I_{G(e)} \leq 1 - I_{G(e)} \leq 1 - I_{F(e)}$ hence $|T_{F(e)}(x) - F_{F(e)}(x)| \geq |T_{G(e)}(x) - F_{G(e)}(x)|$ and $|I_{F(e)}(x) - (1 - I_{F(e)}(x))| \geq |I_{G(e)}(x) - (1 - I_{G(e)}(x))|$.
Therefore $d((F,E),(F,E)^c) \geq d((G,E),(G,E)^c)$.
Also $f(x) = \dfrac{1 - x}{1 + x}$ is monotone decreasing. So we have $H(F,E) \leq H(G,E)$. Similarly we can prove in the other case.

Definition 15: Let $\alpha_i^+ = (1, 1, 0)$ (i =1,2,3...m) be the largest neutrosophic number and we call $A^+ = (\alpha_1^+, \alpha_2^+, ... \alpha_m^+)$ as neutrosophic ideal solution.

IV. APPLICATION OF DISTANCE AND ENTROPY MEASURES OF NEUTROSOPHIC SOFT SET

A. Application Using Entropy Measure

In order to obtain an efficient risk management in the field of construction, certain risks are classified along with some parameters and these risks are evaluated by the team of experts. Assume that there is a set of 3 experts evaluating the five different kinds of risks namely construction risk, design risk, physical risk, financial and economic risk and natural risk with the set of parameters unclear detail design or specification, inadequate or insufficient site information, material and equipment quality, shortage of labour, material and equipment, labour injuries, funding shortage and natural disasters.
Let U denote the set of risks $U = \{x_1, x_2, x_3, x_4, x_5\}$
Let E denote the set of parameters $E = \{e_1, e_2, e_3, e_4, e_5\}$
The neutrosophic soft set (F,E) describes the evaluation of expert A.
$F(e_1)(x_1) = (0.7,0.2,0.1)$, $F(e_2)(x_1) = (0.5,0.5,0.5)$,
$F(e_3)(x_1) = (0.7,0.2,0.1)$, $F(e_4)(x_1) = (0.6,0.7,0.8)$,
$F(e_5)(x_1) = (0.7,0.5,0.4)$

$F(e_1)(x_2) = (0.8,0.3,0.4)$, $F(e_2)(x_2) = (0.7,0.3,0.2)$,
$F(e_3)(x_2) = (0.6,0.8,0.9)$, $F(e_4)(x_2) = (0.8,0.1,0.9)$,
$F(e_5)(x_2) = (0.8,0.2,0.1)$

$F(e_1)(x_3) = (0.4,0.6,0.2)$, $F(e_2)(x_3) = (0.3,0.7,0.2)$,
$F(e_3)(x_3) = (0.2,0.9,0.2)$, $F(e_4)(x_3) = (0.4,0.6,0.5)$,
$F(e_5)(x_3) = (0.3,0.8,0.7)$

$F(e_1)(x_4) = (0.4,0.5,0.3)$, $F(e_2)(x_4) = (0.3,0.6,0.5)$,
$F(e_3)(x_4) = (0.3,0.5,0.4)$, $F(e_4)(x_4) = (0.3,0.4,0.5)$,
$F(e_5)(x_4) = (0.4,0.8,0.3)$

$F(e_1)(x_5) = (0.3,0.2,0.7)$, $F(e_2)(x_5) = (0.2,0.7,0.1)$,
$F(e_3)(x_5) = (0.3,0.5,0.4)$, $F(e_4)(x_5) = (0.3,0.6,0.9)$,
$F(e_5)(x_5) = (0.4, 0.2,0.1)$

The neutrosophic soft set (G,E) describes the evaluation of expert B.
$G(e_1)(x_1) = (0.4 0.3,0.2)$, $G(e_2)(x_1) = (0.3,0.1,0.1)$,
$G(e_3)(x_1) = (0.5,0.5,0.9)$, $G(e_4)(x_1) = (0.4,0.5,0.4)$,
$G(e_5)(x_1) = (0.4,0.2,0.1)$

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$G(e_1)(x_2) = (0.3,0.4,0.5)$, $G(e_2)(x_2) = (0.3,0.4,0.2)$,
$G(e_3)(x_2) = (0.3,0.6,0.7)$, $G(e_4)(x_2) = (0.4,0.9,0.1)$,
$G(e_5)(x_2) = (0.3,0.2,0.1)$

$G(e_1)(x_3) = (0.4,0.6,0.5)$, $G(e_2)(x_3) = (0.3,0.9,0.8)$,
$G(e_3)(x_3) = (0.3,0.8,0.1)$, $G(e_4)(x_3) = (0.4,0.7,0.8)$,
$G(e_5)(x_3) = (0.2,0.4,0.6)$

$G(e_1)(x_4) = (0.3,0.2,0.1)$, $G(e_2)(x_4) = (0.3,0.1,0.2)$,
$G(e_3)(x_4) = (0.3,0.2,0.5)$, $G(e_4)(x_4) = (0.5,0.3,0.4)$,
$G(e_5)(x_4) = (0.4,0.6,0.8)$

$G(e_1)(x_5) = (0.4,0.6,0.8)$, $G(e_2)(x_5) = (0.3,0.7,0.8)$,
$G(e_3)(x_5) = (0.4,0.2,0.6)$, $G(e_4)(x_5) = (0.4,0.2,0.1)$,
$G(e_5)(x_5) = (0.3,0.4,0.8)$

The neutrosophic soft set (P,E) describes the evaluation of expert C.
$P(e_1)(x_1) = (0.4,0.5,0.1)$, $P(e_2)(x_1) = (0.3,0.1,0.1)$,
$P(e_3)(x_1) = (0.3,0.2,0.3)$, $P(e_4)(x_1) = (0.3,0.4,0.7)$,
$P(e_5)(x_1) = (0.5,0.5,0.1)$

$P(e_1)(x_2) = (0.3,0.4,0.5)$, $P(e_2)(x_2) = (0.2,0.1,0.3)$,
$P(e_3)(x_2) = (0.3,0.2,0.1)$, $P(e_4)(x_2) = (0.3,0.5,0.6)$,
$P(e_5)(x_2) = (0.5,0.4,0.1)$

$P(e_1)(x_3) = (0.3,0.7,0.9)$, $P(e_2)(x_3) = (0.3,0.1,0.5)$,
$P(e_3)(x_3) = (0.3,0.5,0.4)$, $P(e_4)(x_3) = (0.3,0.9,0.8)$,
$P(e_5)(x_3) = (0.3,0.1,0.7)$

$P(e_1)(x_4) = (0.3,0.2,0.6)$, $P(e_2)(x_4) = (0.3,0.3,0.2)$,
$P(e_3)(x_4) = (0.3,0.5,0.6)$, $P(e_4)(x_4) = (0.3,0.8,0.9)$,
$P(e_5)(x_4) = (0.3,0.1,0.8)$

$P(e_1)(x_5) = (0.4,0.5,0.9)$, $P(e_2)(x_5) = (0.4,0.5,0.9)$,
$P(e_3)(x_5) = (0.4,0.5,0.7)$, $P(e_4)(x_5) = (0.3,0.7,0.8)$,
$P(e_5)(x_5) = (0.3,0.2,0.6)$

Using the definition 12 we have
H_1(F,E)=0.0906, H_2(F,E)=0.1148, H_3(F,E)=0.0984,
H_4(F,E)=0.1143, H_5(F,E)=0.0746

H_1(G,E)=0.1069, H_2(G,E)=0.0646, H_3(G,E)=0.0923,
H_4(G,E)=0.0918, H_5(G,E)=0.0857

H_1(P,E)=0.1031, H_2(P,E)=0.0742, H_3(P,E)=0.1222,
H_4(P,E)=0.0822, H_5(P,E)=0.0706

Therefore, H(F,E)=0.0985, H(G,E)=0.0883, H(P,E)= 0.0905.
Entropy is an important notion for measuring uncertain information. The less uncertainty information has the larger possibility to select the optimal. From the computation we have H(G,E) $\leq$ H(P,E)$\leq$ H(F,E). Therefore, the expert B has larger possibility to make the decision on risk management than expert A and C. According to expert B H_1(G,E)=0.1069 has the largest entropy value between the risks. Hence, construction risk has to be minimized to have an efficient risk management system, this in turn points out that the parameters related with the construction risk namely, material and equipment quality and shortage of labour has to be given a proper attention for a qualitative and quantitative risk management system.

B. Application Using Distance Measure

For a Multi attribute decision making problem of the evaluation of university faculty for tenure and promotion. There are six faculty candidates (alternatives) A_j (j =1, 2, ... 6) to be evaluated, the criteria (attributes) used at some universities are e_1:teaching, e_2: research, and e_3: service. Let U denote the tenure and promotion $U = \{x_1\}$
Let E denote the set of parameters $E = \{e_1, e_2, e_3\}$

The neutrosophic soft sets (F_1, A_1), (F_2, A_2),.....(F_6, A_6) describes the evaluation of university faculty $A_1.A_2,.....A_6$ respectively.
$F_1(e_1)(x_1) = (0.4,0.5,0.3)$, $F_1(e_2)(x_1) = (0.6,0.4,0.1)$,
$F_1(e_3)(x_1) = (0.5,0.6,0.4)$.

$F_2(e_1)(x_1) = (0.5,0.4,0.2)$, $F_2(e_2)(x_1) = (0.3,0.6,0.4)$,
$F_2(e_3)(x_1) = (0.8,0.2,0.1)$.

$F_3(e_1)(x_1) = (0.7,0.4,0.2)$, $F_3(e_2)(x_1) = (0.3,0.6,0.7)$,
$F_3(e_3)(x_1) = (0.6,0.1,0.2)$.

$F_4(e_1)(x_1) = (0.4,0.6,0.3)$, $F_4(e_2)(x_1) = (0.6,0.4,0.2)$,
$F_4(e_3)(x_1) = (0.7,0.4,0.1)$.

$F_5(e_1)(x_1) = (0.6,0.8,0.2)$, $F_5(e_2)(x_1) = (0.5,0.7,0.1)$,
$F_5(e_3)(x_1) = (0.4,0.7,0.6)$.

$F_6(e_1)(x_1) = (0.6,0.1,0.3)$, $F_6(e_2)(x_1) = (0.7,0.5,0.2)$,
$F_6(e_3)(x_1) = (0.5,0.6,0.4)$.

The neutrosophic soft ideal solution (F, A^+) is given by
$F(e_1)(x_1) = (1,1,0)$, $F(e_2)(x_1) = (1,1,0)$,$F(e_3)(x_1) = (1,1,0)$

Using definition 12 we have
$d_1((F_1, A_1),(F, A^+))= 0.5667$, $d_1((F_2, A_2),(F, A^+))= 0.7$,
$d_1((F_3, A_3),(F, A^+))= 0.7333$, $d_1((F_4, A_4),(F, A^+))= 0.6$,
$d_1((F_5, A_5),(F, A^+))= 0.5$, $d_1((F_6, A_6),(F, A^+))= 0.6333$.

Since
$d_1((F_5, A_5),(F, A^+)) < d_1((F_1, A_1),(F, A^+)) < d_1((F_4, A_4),(F, A^+)) < d_1((F_6, A_6),(F, A^+)) < d_1((F_2, A_2),(F, A^+)) < d_1((F_3, A_3),(F, A^+))$
then
$(F_5, A_5) > (F_1, A_1) > (F_4, A_4) > (F_6, A_6) > (F_2, A_2) > (F_3, A_3)$
Hence the most desirable alternative is A_5

V. CONCLUSION

An efficient risk management system saves time, labour and economy, if the risks are given proper attention, according to their order of severity. The identification and analysis demonstrates the application and efficiency of the entropy measure in decision making.Finally we have developed the

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model that utilizes the neutrosophic ideal solution and the distance measures to find the best alternative, based on which some practical procedures have been established to determine the ranking of all alternatives. In future work we can extend the model using interval valued neutrosophic soft set.

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