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Heat Transfer and Entropy Generation in a Partial Porous Channel Using LTNE and Exothermicity/ Endothermicity Features

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Abstract**—**This work aims to provide a comprehensive study on the heat transfer and entropy generation rates of a horizontal channel partially filled with a porous medium which experiences internal heat generation or consumption due to exothermic or endothermic chemical reaction. The focus has been given to the local thermal non-equilibrium (LTNE) model. The LTNE approach helps us to deliver more accurate data regarding temperature distribution within the system and accordingly to provide more accurate Nusselt number and entropy generation rates. Darcy-Brinkman model is used for the momentum equations, and constant heat flux is assumed for boundary conditions for both upper and lower surfaces. Analytical solutions have been provided for both velocity and temperature fields. By incorporating the investigated velocity and temperature formulas into the provided fundamental equations for the entropy generation, both local and total entropy generation rates are plotted for a number of cases. Bifurcation phenomena regarding temperature distribution and interface heat flux ratio are observed. It has been found that the exothermicity or endothermicity characteristic of the channel does have a considerable impact on the temperature fields and entropy generation rates.

Keywords**—**Entropy generation, exothermicity, endothermicity, forced convection, local thermal non-equilibrium, analytical modeling.

I. Introduction

THERMAL systems can be analyzed with two different perspectives: the first law or the second law of thermodynamics. If the first law of thermodynamics is used for an optimization, the studied system neglects the quality of the process. This approach does not connect the heat flux of a system to its temperature and does not pay attention to the irreversibility of a considered thermal process [1]-[3]. Due to these negative points, the second law has been recently considered for optimization of a thermal process or system. The second law of thermodynamics gives researchers a powerful tool to monitor a system qualitatively. This method constructs fundamental formulas to calculate entropy generation within a system and accordingly realize the reversibility level of it. By employing the second law of thermodynamics, it is possible to optimize a thermal process, so that less entropy is generated and consequently less exergy is destructed.

Channels under forced convection are an important part of thermal systems. Recently, heat transfer and entropy generation analyses in horizontal channels which are completely or partially filled with porous media have attracted considerable attention. Although the local thermal equilibrium model [4] has been used a lot in this regard, the local thermal non-equilibrium (LTNE) model [5] is the focus of the research community these days. It is due to the fact that in many emerging fields such as MEMS, nuclear engineering, etc., the accuracy of the reported data is extremely important [6].

There have been a number of publications regarding the heat transfer within porous media under LTNE conditions. One of the pioneering works in this area has been done by Nield [5]. Following Nield's persuasive work, many scholars have tried to reexamine thermal porous systems from LTNE perspective [7]-[12]. Khashan et al. [7] reexamined the classical problem of steady state fluid flow and heat transfer in a porous pipe using SIMPLE algorithm under LTNE model. Effects of different values of Reynolds and Biot numbers on the temperature contours and other thermal characteristics of the system were investigated. Ouyang et al. [10] used three different fundamental LTNE models in a channel. By using the classical definition of the thermal entry length based on the Nusselt number, the dimensionless thermal entry length was predicted. Dehghan et al. [11], [12] considered both Darcy and Forchheimer terms into the momentum equation, without viscous dissipation in energy equations. Perturbation technique was used to tackle the governing equation due to the nonlinearity of the problem imposed by Forchheimer term. The semi-analytical solution for the temperature and accordingly Nusselt number was derived. Comparisons were also made to previous publications, and good agreement was observed.

Ochoa-Tapia and Whitaker [13] were the first scholars to consider a geometry partially filled with porous media. The partial filling is an effective avenue to circumvent significant pressure drop in a porous system [14]. By adopting this approach, one can use the positive effects of a porous medium within a thermal system, while maintaining the pumping power and consequently expenses within a plausible framework. Yang and Vafai [15], [16] provided three different main models for the interface condition together with their analytical solutions. They have discussed the limitations of each model and given illustrative figures regarding the variation of the Nusselt number in each model. Xu et al. [17], [18] analytically solved the flow field and energy equation for a parallel-plate channel [17] and a pipe [18] partially filled with porous media attached to the inner wall of each

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geometry. Convective boundary condition was considered at the porous fluid interface. They have illustrated the temperature distribution with different thermophysical parameters and showed that Nusselt number is decreased if the channel is fully filled with porous media. More recently, [19] and [20] have opted in favor of differences between resultant temperature distribution and Nusselt value for two different interface heat flux models, namely as models A and B.

Recently, [21] have accomplished a study in porous media using LTNE conditions. The hydrodynamic and thermal analyses between two parallel plates with a porous medium were investigated. The velocity and temperature fields were analytically investigated, and accordingly, local and total entropy generation rates were addressed. Torabi et al. [22] have opted in favor of heat transfer and entropy generation in a horizontal channel partially filled with porous media using LTNE model. They used asymmetric boundary conditions for the channel and incorporated the viscous dissipation term into the energy equations. For the first time, they have reported a bifurcation phenomenon for the total entropy generation rate.

This work is a comprehensive study on the heat transfer and entropy generation within a horizontal channel under forced convection which is partially filled with a porous medium. The Darcy-Brinkman model for the momentum equations are used, and internal heat generation or consumption is incorporated into the energy equation. These internal sources can model the exothermic or endothermic chemical reactions which occur in various processes [23], [24].

II. Mathematical Model

Consider a rectangular channel subjected to uniform and equal heat fluxes on the upper and lower surfaces. The channel is partially filled with a porous medium as it has been shown in Fig. 1. Constant thermophysical properties for both solid and fluid phases are assumed. This study assumes laminar flow, fully developed velocity, and temperature fields, Darcy-Brinkman model for momentum equation in the porous material, and homogeneous and isotropic characteristics for the porous structure. Bearing these assumptions in mind and considering LTNE models, the momentum, and energy equations are written as follow.

Momentum:

$$-\frac{\partial p}{\partial x}+\mu_f\frac{\partial^2 u_{f1}}{\partial y^2}=0 \qquad 0\le y\le h_c \tag{1a}$$

$$-\frac{\partial p}{\partial x}+\mu_{eff}\frac{\partial^2 u_{f2}}{\partial y^2}-\frac{\mu_f}{\kappa}u_{f2}=0 \qquad h_c\le y<h \tag{1b}$$

Energy:

$$\rho c_p u_{f1}\frac{\partial T_{f1}}{\partial x}=k_f\frac{\partial^2 T_{f1}}{\partial y^2}+s_f \qquad 0\le y\le h_c \tag{2a}$$

$$\rho c_p u_{f2}\frac{\partial T_{f2}}{\partial x}=k_{ef}\frac{\partial^2 T_{f2}}{\partial y^2}+h_{sf}a_{sf}\left(T_s-T_{f2}\right)+s_f \qquad h_c\le y<h \tag{2b}$$

$$0=k_{es}\frac{\partial^2 T_s}{\partial y^2}-h_{sf}a_{sf}\left(T_s-T_{f2}\right)+s_s \qquad h_c\le y<h \tag{2c}$$

where the $\mu_{eff}=\frac{\mu_f}{\varepsilon}$ is effective viscosity. The boundary conditions for the above system of equations are as follows:

$$y=0:\frac{\partial u_{f1}}{\partial y}=0,\frac{\partial T_{f1}}{\partial y}=0 \tag{3a}$$

$$y=h_c: u_{f1}=u_{f2}, \mu_f\frac{\partial u_{f1}}{\partial y}=\mu_{eff}\frac{\partial u_{f2}}{\partial y}, \tag{3b}$$

$$k_f\frac{\partial T_{f1}}{\partial y}=k_{ef}\frac{\partial T_{f2}}{\partial y}+k_{es}\frac{\partial T_s}{\partial y}, T_{f1}=T_{f2}=T_s$$

$$y=h: u_{f2}=0, T_{f2}=T_s=T_w, \tag{3c}$$

$$q_w=k_{ef}\frac{\partial T_{f2}}{\partial y}+k_{es}\frac{\partial T_s}{\partial y}$$

By performing some mathematical manipulations which have been elaborated in previous publications [20], [22] the following energy equations for the fluid flow regions is obtained:

$$u_{f1}\frac{q_w+\int_0^h s_f\mathrm{d}y+\int_{h_c}^h s_s\mathrm{d}y}{hu_m}=k_f\frac{\partial^2 T_{f1}}{\partial y^2}+s_f \quad 0\le y\le h_c \tag{4a}$$

$$u_{f2}\frac{q_w+\int_0^h s_f\mathrm{d}y+\int_{h_c}^h s_s\mathrm{d}y}{hu_m}$$

$$=k_{ef}\frac{\partial^2 T_{f2}}{\partial y^2}+h_{sf}a_{sf}\left(T_s-T_{f2}\right)+s_f \quad h_c\le y<h \tag{4b}$$

where

$$u_m=\frac{1}{h}\left(\int_0^{h_c}u_{f1}\mathrm{d}y+\int_{h_c}^h u_{f2}\mathrm{d}y\right) \tag{5}$$

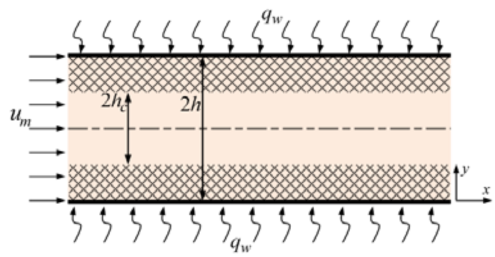

Fig. 1 Configuration of the Channel Partially Filled with Porous Media

Using the following dimensionless parameters:

$$U=\frac{u}{u_r},\theta=\frac{k_{es}\left(T-T_w\right)}{q_w h},k=\frac{k_{es}}{k_{ef}}=\frac{\left(1-\varepsilon\right)k_s}{\varepsilon k_f},$$
$$Bi=\frac{h_{sf}a_{sf}h^2}{k_{es}},Y=\frac{y}{h},X=\frac{x}{h},Y_c=\frac{h_c}{h}$$
$$Br=\frac{\mu_f u_r^2}{q_w h},Pe=\frac{\rho c_p u_r h}{k_{ef}},\gamma=\frac{q_{\text{int}}}{q_w},w_f=\frac{s_f h}{q_w}, \tag{6}$$
$$w_s=\frac{s_s h}{q_w},B=\frac{k_{ef}T_w}{q_w h},Da=\frac{\kappa}{h^2},u_r=-\frac{h^2}{\mu_f}\frac{\partial p}{\partial x}$$

into momentum and energy equations, and boundary equations, the studied thermal system using LTNE model, can be governed by the following non-dimensionalized equations.

Momentum:

$$1+\frac{\partial^2 U_{f1}}{\partial Y^2}=0 \qquad 0\le Y\le Y_c \tag{7a}$$

$$1+\frac{1}{\varepsilon}\frac{\partial^2 U_{f2}}{\partial Y^2}-\frac{U_{f2}}{Da}=0 \qquad Y_c<Y\le 1 \tag{7b}$$

Energy:

$$\frac{AU_{f1}}{U_m}=\frac{1}{\varepsilon k}\frac{\partial^2\theta_{f1}}{\partial Y^2}+w_f \quad 0\le Y\le Y_c \tag{8a}$$

$$\frac{AU_{f2}}{U_m}=\frac{1}{k}\frac{\partial^2\theta_{f2}}{\partial Y^2}+Bi\left(\theta_s-\theta_{f2}\right)+w_f \quad Y_c\le Y\le 1 \tag{8b}$$

$$0=\frac{\partial^2\theta_s}{\partial Y^2}-Bi\left(\theta_s-\theta_{f2}\right)+w_s \quad Y_c\le Y\le 1 \tag{8c}$$

Boundary conditions:

$$Y=0:\frac{\partial U_{f1}}{\partial Y}=0,\frac{\partial\theta_{f1}}{\partial Y}=0 \tag{9a}$$

$$Y=Y_c:U_{f1}=U_{f2},\frac{\partial U_{f1}}{\partial Y}=\frac{1}{\varepsilon}\frac{\partial U_{f2}}{\partial Y}, \tag{9b}$$
$$\gamma=\frac{1}{\varepsilon k}\frac{\partial\theta_{f1}}{\partial Y}=\frac{1}{k}\frac{\partial\theta_{f2}}{\partial Y}+\frac{\partial\theta_s}{\partial Y},\theta_{f1}=\theta_{f2}=\theta_s$$

$$Y=1:U_{f2}=0,1=\frac{1}{k}\frac{\partial\theta_{f2}}{\partial Y}+\frac{\partial\theta_s}{\partial Y},\theta_{f2}=\theta_s=0 \tag{9c}$$

where

$$A=1+\int_0^1 w_f\,\mathrm{d}Y+\int_{Y_c}^1 w_s\,\mathrm{d}Y \tag{10a}$$

$$U_m=\int_0^{Y_c}U_{f1}\mathrm{d}Y+\int_{Y_c}^1 U_{f2}\mathrm{d}Y \tag{10b}$$

By decoupling the energy equations of the fluid and solid phases in the porous medium, these two equations can be independently introduced as:

$$\frac{A}{U_m}\frac{\partial^2 U_{f2}}{\partial Y^2}=\frac{1}{k}\frac{\partial^4\theta_{f2}}{\partial Y^4}+\frac{\partial^2 w_f}{\partial Y^2} \tag{11a}$$
$$+Bi\left(\frac{AU_{f2}}{U_m}-\left(1+\frac{1}{k}\right)\frac{\partial^2\theta_{f2}}{\partial Y^2}-w_f-w_s\right)$$

$$0=\frac{\partial^4\theta_s}{\partial Y^4}+\frac{\partial^2 w_s}{\partial Y^2} \tag{11b}$$
$$-Bi\left(-\frac{kAU_{f2}}{U_m}+\left(1+k\right)\frac{\partial^2\theta_s}{\partial Y^2}+kw_f+kw_s\right)$$

III. Entropy Generation

Considering LTNE model, the following formulas are valid for the volumetric rate of local entropy generation within the fluid phase of the porous medium, the solid phase of the porous medium and fluid phase of the clear region, respectively [25]-[27]:

$$\dot{S}'''_{f1}=\frac{k_f}{T_{f1}^2}\left[\left(\frac{\partial T_{f1}}{\partial x}\right)^2+\left(\frac{\partial T_{f1}}{\partial y}\right)^2\right]+\frac{\mu_f}{T_{f1}}\left(\frac{\partial u_{f1}}{\partial y}\right)^2 \quad 0\le y\le h_c \tag{12a}$$

$$\dot{S}'''_{f2}=\frac{k_{ef}}{T_{f2}^2}\left[\left(\frac{\partial T_{f2}}{\partial x}\right)^2+\left(\frac{\partial T_{f2}}{\partial y}\right)^2\right]+\frac{\mu_f}{\kappa T_{f2}}u_{f2}^2+ \tag{12b}$$
$$\frac{h_{sf}a_{sf}\left(T_s-T_{f2}\right)^2}{T_sT_{f2}}+\frac{\mu_{eff}}{T_{f2}}\left(\frac{\partial u_{f2}}{\partial y}\right)^2 \quad h_c\le y<h$$

$$\dot{S}'''_s=\frac{k_{es}}{T_s^2}\left[\left(\frac{\partial T_s}{\partial x}\right)^2+\left(\frac{\partial T_s}{\partial y}\right)^2\right] \tag{12c}$$
$$+\frac{h_{sf}a_{sf}\left(T_s-T_{f2}\right)^2}{T_sT_{f2}} \quad h_c\le y<h$$

By employing the dimensionless parameters introduced in (6), the dimensionless local volumetric entropy generation rates are given by

$$N'''_{f1}=\frac{Br\left(\frac{\partial U_{f1}}{\partial Y}\right)^2}{\left(\theta_{f1}+B\right)}+\frac{\left[\left(\frac{1+\int_0^1 w_f\,\mathrm{d}Y+\int_0^{Y_c}w_s\,\mathrm{d}Y}{\left(Pe/k\right)U_m}\right)^2+\left(\frac{\partial\theta_{f1}}{\partial Y}\right)^2\right]}{\varepsilon k\left(\theta_{f1}+B\right)^2} \tag{13a}$$

$$N'''_{f2}=+\frac{Bi\left(\theta_s-\theta_{f2}\right)^2}{\left(\theta_s+B\right)\left(\theta_{f2}+B\right)}+\frac{BrU_{f2}^2}{Da\left(\theta_{f2}+B\right)}$$
$$\frac{\left[\left(\frac{1+\int_0^1 w_f\,\mathrm{d}Y+\int_0^{Y_c}w_s\,\mathrm{d}Y}{\left(Pe/k\right)U_m}\right)^2+\left(\frac{\partial\theta_{f2}}{\partial Y}\right)^2\right]}{k\left(\theta_{f2}+B\right)^2} \tag{13b}$$
$$+\frac{Br\left(\frac{\partial U_{f2}}{\partial Y}\right)^2}{\varepsilon\left(\theta_{f2}+B\right)}$$

$$N_s''' = \frac{\left[\left(\dfrac{1+\int_0^1 w_f \,\mathrm{d}Y + \int_0^{Y_c} w_s \,\mathrm{d}Y}{(Pe/k)U_m}\right)^2 + \left(\dfrac{\partial\theta_s}{\partial Y}\right)^2\right]}{(\theta_s + B)^2} + \frac{Bi\,(\theta_s - \theta_{f2})^2}{(\theta_s + B)(\theta_{f2} + B)} \tag{13c}$$

where the parameter $B = \dfrac{T_w k_{es}}{q_w h}$ and depends on the thermophysical properties of the channel. The dimensionless total entropy generation rate for the channel is given by:

$$N_t = \int_0^{Y_c} N_{f1}''' \mathrm{d}Y + \int_{Y_c}^1 \left(N_{f2}''' + N_s'''\right)\mathrm{d}Y \tag{14}$$

IV. Solution Procedure

The velocity fields within porous and clear regions are obtained as

$$U_{f1} = -\frac{1}{2}Y^2 + C_1 Y + C_2 \tag{15a}$$

$$U_{f2} = C_3 \sinh\left(\frac{Y}{\sqrt{Da/\varepsilon}}\right) + C_4 \cosh\left(\frac{Y}{\sqrt{Da/\varepsilon}}\right) + Da \tag{15b}$$

The temperature distributions using LTNE model within both porous and clear regions are obtained as:

$$\theta_{f1} = \frac{1}{2}\,\frac{k\,\varepsilon\left(\begin{array}{l}-\frac{1}{12}AY^4 + \frac{1}{3}AC_1Y^3 \\ +AC_2Y^2 - U_m w_f Y^2\end{array}\right)}{U_m} + D_1 Y + D_2 \tag{16a}$$

$$\theta_{f2} = \frac{-1}{(A_1 kDa + \varepsilon)}\left(\begin{array}{l}\dfrac{A_2 kDa^2}{\varepsilon}\cosh\left(\dfrac{Y}{\sqrt{Da/\varepsilon}}\right) \\ +\dfrac{A_3 kDa^2}{\varepsilon}\sinh\left(\dfrac{Y}{\sqrt{Da/\varepsilon}}\right)\end{array}\right) + \frac{D_3\cosh\left(\sqrt{-kA_1}Y\right)}{kA_1} + \frac{D_4 \sinh\left(\sqrt{-kA_1}Y\right)}{kA_1} - \frac{A_4}{2A_1}Y^2 + D_5 Y + D_6 \tag{16b}$$

$$\theta_s = \frac{-Da^2}{\varepsilon(B_1 Da + \varepsilon)}\left(\begin{array}{l}B_2\cosh\left(\dfrac{Y}{\sqrt{Da/\varepsilon}}\right) \\ +B_3 \sinh\left(\dfrac{Y}{\sqrt{Da/\varepsilon}}\right)\end{array}\right) - \frac{B_4}{2B_1}Y^2 - \frac{D_7}{B_1}\cosh\left(\sqrt{-A_1}Y\right) + \frac{D_8}{B_1}\sinh\left(\sqrt{-A_1}Y\right) + D_9 Y + D_{10} \tag{16c}$$

The ten unknown parameters $D_1 - D_{10}$ are obtained numerically using the **fsolve** command in Maple 17. The correctness of the solution procedure has repeatedly been verified in our previous work [22], [28].

V. Results and Discussion

Fig. 2 shows the effect of thermal conductivity parameter, i.e., k, on the temperature field within the channel. It has been observed that when the thermal conductivity ratio is equal to unity the difference between the temperatures of the solid and fluid phases of the porous section is very small. However, when the thermal conductivity ratios are smaller or bigger than unity, the temperature difference between two phases are larger. Moreover, with used parameter values within the system, it is seen that the temperature of the solid phase with the porous medium of the channel can be lower or higher than the temperature of the fluid phase. The bifurcation phenomenon is clearly seen from Fig. 3.

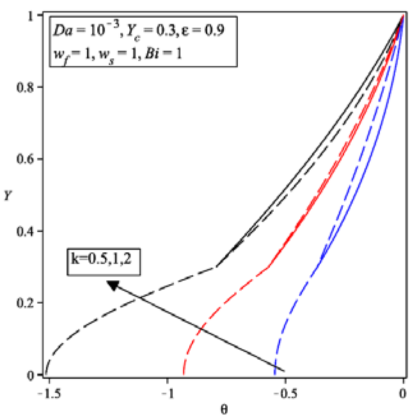

Fig. 2 Dimensionless temperature distribution with different values for the thermal conductivity ratio using $Da = 10^{-3}$

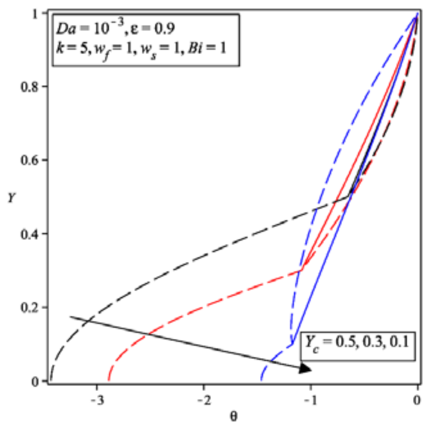

Fig. 3 Dimensionless temperature distribution with different values for the clear section thickness using $k = 5$

When the clear section has 0.1 height, the solid phase of the porous section has higher temperature compared with the fluid phase. However, when the clear section height is chosen 0.3 or 0.5, this behavior changes and the temperature of the fluid phase becomes higher than the temperature of the solid phase within the then porous medium. Effects of exothermicity or endothermicity on the temperature distribution have been plotted in Fig. 4. As expected, the less exothermic is the thermal system; the lower is the temperature distribution. Moreover, when the exothermicity of the studied system is lower, the difference between the temperature distributions of the solid phase and fluid phase within the porous section is lower, which can be seen clearly from Fig. 4.

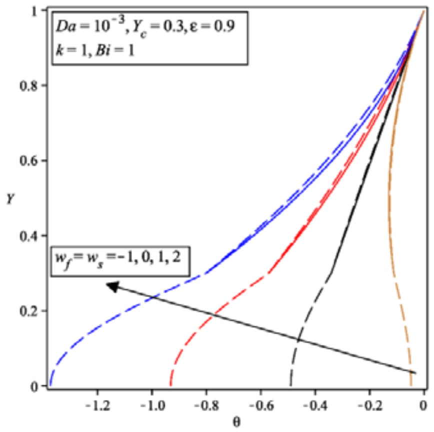

Fig. 4 Dimensionless temperature distribution with different values for the energy sources

Fig. 5 shows the effect of clear section thickness on the local entropy generation within the channel. As it can be seen from this figure, changing the clear section thickness from 0.3 to 0.4, decrease the local entropy generation rate in each section of the channel by four to fivefold. Fig. 6 compares the local entropy generation rate with two different Darcy numbers. As it can be seen from this figure, decreasing Darcy number increases the local entropy generation rate within the thermal system. It can be expected due to the fact that decreasing Darcy number increases the maximum temperature difference within the system, that has been shown in previous publications [22], which can increase the irreversibility and consequently the entropy generation of the system.

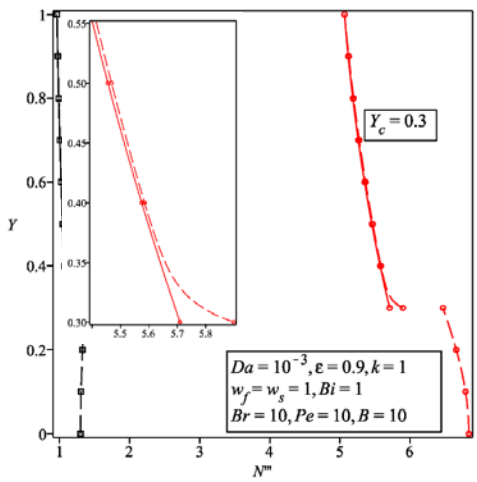

Fig. 5 Local entropy generation rate using $Y_c = 0.3$ and $Y_c = 0.4$

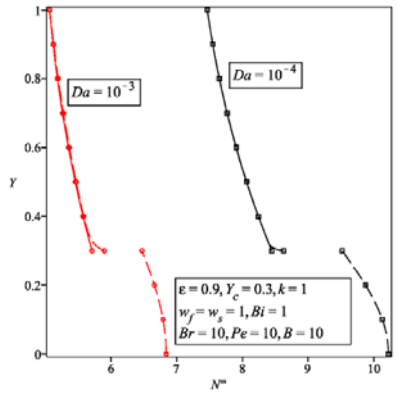

Fig. 6 Local entropy generation rate using two different Darcy numbers

Fig. 7 is a sample calculation regarding the total entropy generation rate within the system. The most important observation which can be seen from this figure is that the total entropy generation rate starts to decrease by increasing the clear section thickness sharply, its reaches a minimum value and then starts to increase by a slight amount. This figure proves that, within the studied system and with used parameter values, the total filling is the worst case from the second thermodynamics law of perspective. However, with partial filling, the total entropy generation rate can be minimized and the optimum value of the parameter Y_c, which has a direct connection with the porous thickness of the system, can be achieved. The total entropy generation rate increases with thermal conductivity ratio, as it can be seen from Fig. 7.

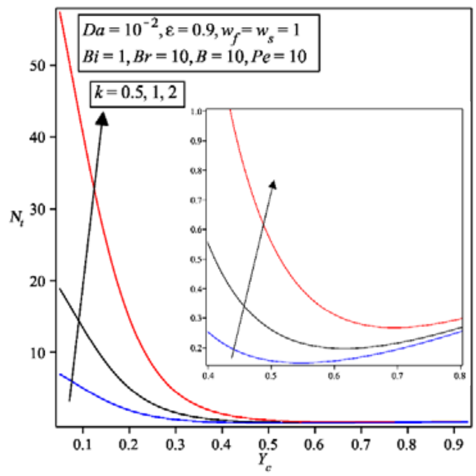

Fig. 7 Total entropy generation rate versus the clear section thickness with different thermal conductivity ratios

VI. Conclusions

A two-dimensional, axisymmetric channel with porous inserts attached to the walls and under constant wall heat flux, was considered. It was assumed that the solid and fluid phases could feature internal heat sources and the system is under LTNE. The problems of forced convection and entropy generation were investigated in this configuration. In keeping with the previous investigations, it was shown that the existence of internal heat sources could heavily affect the thermal equilibrium state. Analysis of the local generation of entropy revealed that this property of the system is heavily affected by the configuration of the channel. Considering the total entropy generation in the channel, an optimal value for the thickness of the porous insert was found, and the influences of pertinent parameters upon this optimal thickness were discussed. The results of this work provide a guide through the complex physical behavior of fluid conduits partially filled with porous media, which include internal heat sources.

REFERENCES

[1] T. L. Bergman, A. S. Lavine, F. P. Incropera, and D. P. DeWitt, *Introduction to Heat Transfer*, 6th ed. John Wiley and Sons, Inc., 2011.

[2] A. Bejan, *Entropy Generation Minimization: The Method of Thermodynamic Optimization of Finite-Size Systems and Finite-Time Processes*. CRC Press, 1995.

[3] A. Bejan, *Entropy Generation Through Heat and Fluid Flow*. New York: Wiley, 1982.

[4] D. A. Nield and A. Bejan, *Convection in Porous Media*, 4th edition. New York: Springer, 2013.

[5] D. Nield, "Effects of local thermal nonequilibrium in steady convective processes in a saturated porous medium: forced convection in a channel," *J. Porous Media*, vol. 1, pp. 181–186, 1998.

[6] B. Alazmi and K. Vafai, "Constant wall heat flux boundary conditions in porous media under local thermal non-equilibrium conditions," *Int. J. Heat Mass Transf.*, vol. 45, pp. 3071–3087, 2002.

[7] S. A. Khashan, A. M. Al-Amiri, and M. A. Al-Nimr, "Assessment of the local thermal non-equilibrium condition in developing forced convection flows through fluid-saturated porous tubes," *Appl. Therm. Eng.*, vol. 25, no. 10, pp. 1429–1445, Jul. 2005.

[8] G. M. Chen and C. P. Tso, "A two-equation model for thermally developing forced convection in a porous medium with viscous dissipation," *Int. J. Heat Mass Transf.*, vol. 54, no. 25–26, pp. 5406–5414, Dec. 2011.

[9] G. M. Chen and C. P. Tso, "Forced convection with viscous dissipation using a two-equation model in a channel filled with a porous medium," *Int. J. Heat Mass Transf.*, vol. 54, no. 9–10, pp. 1791–1804, Apr. 2011.

[10] X.-L. Ouyang, K. Vafai, and P.-X. Jiang, "Analysis of thermally developing flow in porous media under local thermal non-equilibrium conditions," *Int. J. Heat Mass Transf.*, vol. 67, pp. 768–775, Dec. 2013.

[11] M. Dehghan, M. S. Valipour, and S. Saedodin, "Perturbation analysis of the local thermal non-equilibrium condition in a fluid-saturated porous medium bounded by an isothermal channel," *Transp. Porous Media*, vol. 102, no. 2, pp. 139–152, Jan. 2014.

[12] M. Dehghan, M. T. Jamal-Abad, and S. Rashidi, "Analytical interpretation of the local thermal non-equilibrium condition of porous media embedded in tube heat exchangers," *Energy Convers. Manag.*, vol. 85, pp. 264–271, Sep. 2014.

[13] J. A. Ochoa-Tapia and S. Whitaker, "Heat transfer at the boundary between a porous medium and a homogeneous fluid," *Int. J. Heat Mass Transf.*, vol. 40, no. 11, pp. 2691–2707, 1997.

[14] F. Arpino, A. Carotenuto, N. Massarotti, and A. Mauro, "New solutions for axial flow convection in porous and partly porous cylindrical domains," *Int. J. Heat Mass Transf.*, vol. 57, no. 1, pp. 155–170, Jan. 2013.

[15] K. Yang and K. Vafai, "Restrictions on the validity of the thermal conditions at the porous fluid interface—An exact solution," *J. Heat Transfer*, vol. 133, no. 11, p. 112601, 2011.

[16] K. Yang and K. Vafai, "Analysis of heat flux bifurcation inside porous media incorporating inertial and dispersion effects – An exact solution," *Int. J. Heat Mass Transf.*, vol. 54, no. 25–26, pp. 5286–5297, Dec. 2011.

[17] H. J. Xu, Z. G. Qu, T. J. Lu, Y. L. He, and W. Q. Tao, "Thermal modeling of forced convection in a parallel-plate channel partially filled with metallic foams," *J. Heat Transfer*, vol. 133, no. 9, p. 092603, 2011.

[18] H. J. Xu, Z. G. Qu, and W. Q. Tao, "Analytical solution of forced convective heat transfer in tubes partially filled with metallic foam using the two-equation model," *Int. J. Heat Mass Transf.*, vol. 54, no. 17–18, pp. 3846–3855, Aug. 2011.

[19] Y. Mahmoudi and N. Karimi, "Numerical investigation of heat transfer enhancement in a pipe partially filled with a porous material under the local thermal non-equilibrium condition," *Int. J. Heat Mass Transf.*, vol. 68, pp. 161–173, Jan. 2014.

[20] Y. Mahmoudi, N. Karimi, and K. Mazaheri, "Analytical investigation of heat transfer enhancement in a channel partially filled with a porous material under the local thermal non-equilibrium condition: Effects of different thermal boundary conditions at the porous fluid interface," *Int. J. Heat Mass Transf.*, vol. 70, pp. 875–891, Mar. 2014.

[21] B. Buonomo, O. Manca, and G. Lauriat, "Forced convection in microchannels filled with porous media in local thermal non-equilibrium conditions," *Int. J. Therm. Sci.*, vol. 77, pp. 206–222, Mar. 2014.

[22] M. Torabi, K. Zhang, G. Yang, J. Wang, and P. Wu, "Heat transfer and entropy generation analyses in a channel partially filled with porous media using local thermal non-equilibrium model," *Energy*, vol. 82, pp. 922–938, 2015.

[23] T. M. Bandhauer, S. Garimella, and T. F. Fuller, "A critical review of thermal issues in lithium-ion batteries," *J. Electrochem. Soc.*, vol. 158, no. 3, pp. R1–R25, 2011.

[24] K. Zheng, Q. Sun, and M. Ni, "Local non-equilibrium thermal effects in solid oxide fuel cells with various fuels," *Energy Technol.*, vol. 1, no. 1, pp. 35–41, Jan. 2013.

[25] M. Torabi, K. Zhang, G. Yang, J. Wang, and P. Wu, "Temperature distribution, local and total entropy generation analyses in asymmetric cooling composite geometries with multiple nonlinearities: Effect of imperfect thermal contact," *Energy*, vol. 78, pp. 218–234, Dec. 2014.

[26] A. Aziz and W. A. Khan, "Entropy generation in an asymmetrically cooled slab with temperature-dependent internal heat generation," *Heat Transf. Res.*, vol. 41, no. 3, pp. 260–271, May 2012.

[27] A. Aziz and W. A. Khan, "Classical and minimum entropy generation analyses for steady state conduction with temperature-dependent thermal conductivity and asymmetric thermal boundary conditions: Regular and functionally graded materials," *Energy*, vol. 36, no. 10, pp. 6195–6207, Oct. 2011.

[28] M. Torabi and K. Zhang, "Temperature distribution, local and total entropy generation analyses in MHD porous channels with thick walls," *Energy*, vol. 87, pp. 540–554, 2015.

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