

A Fuzzy Mathematical Model for Order Acceptance and Scheduling Problem

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Abstract—The problem of Order Acceptance and Scheduling (OAS) is defined as a joint decision of which orders to accept for processing and how to schedule them. Any linear programming model representing real-world situation involves the parameters defined by the decision maker in an uncertain way or by means of language statement. Fuzzy data can be used to incorporate vagueness in the real-life situation. In this study, a fuzzy mathematical model is proposed for a single machine OAS problem, where the orders are defined by their fuzzy due dates, fuzzy processing times, and fuzzy sequence dependent setup times. The signed distance method, one of the fuzzy ranking methods, is used to handle the fuzzy constraints in the model.

Keywords—Fuzzy mathematical programming, fuzzy ranking, order acceptance, single machine scheduling.

I. INTRODUCTION AND LITERATURE REVIEW

OAS decisions have its source in the limited production capacity which entails a manufacturer to select among incoming orders. Because the manufacturing capacity is limited, the available capacity could be insufficient to meet promised or demanded delivery dates for all customer orders. So, the decision maker is faced with the decision which orders to accept and which orders to reject. While the manufacturer decides on a set of orders to be accepted and processed over a time frame, concurrently, how to process these orders should also be considered for efficient capacity utilization. Therefore, OAS is the joint decision of OAS [1].

A number of different versions of OAS problems with different settings and objectives have been studied over the last two decades. One type of OAS problems, which is related to this study, is to select and to schedule orders to maximizing the total revenue in a single machine environment. References [2], [3], and later [4] and [5] study an OAS problem in a single machine environment with the objective of maximizing the total profit. In all these studies, total profit is calculated as the sum of revenues minus total weighted tardiness. Reference [4] assumed that the revenue, the processing time, the due date, and the weights reflecting the importance of the customer/order are considered to be known for each order. They consider the situation in which customers receive a discount proportional to the time duration. The order is late if the orders are delivered after their due dates, and the early

delivery options are not taken into consideration. They then propose an optimal branch-and-bound method for simultaneous order acceptance-sequencing problems. Alternative heuristic methods are also developed.

Reference [5] proposed a genetic algorithm for the same problem. Reference [6] provided two branch- and-bound algorithms and several heuristics for the OAS problem without release dates, deadlines, and sequence dependent setup times in a single machine environment. Reference [7] focuses on the order selection and scheduling problem in a preemptive single machine environment to maximize the profit, which is defined as revenue minus manufacturing, holding, and tardiness costs. The authors assume deterministic demand and neglect the setup times and setup costs in the problem. In this problem, the manufacturer selects the lead time for each order. The objective of their study is defined as maximizing the profit (revenue minus manufacturing, holding, and tardiness costs). The objective is pursued by coordinating the order selection, scheduling and lead time decisions. They suggest a time-indexed Mixed Integer Programming (MIP) formulation for their preemptive problem. Reference [8] includes sequence-dependent setup times in a model that maximizes profit. They define profit as total revenues of accepted orders minus total weighted tardiness penalties. Due date is considered while calculating total weighted tardiness penalties. A weighted tardiness penalty is incurred if the product completion time exceeds due date. If any order would be completed after deadline, it should not be accepted. This work adds the strict deadline and sequence- dependent setup times to the problem studied by [4]. They present a Mixed Integer Linear Programming (MILP) model and solved optimally for ten and some fifteen-order problems. Reference [9] considers order selection when there are planned orders as well as potential orders. They present two MILP procedures and two B&B algorithms, which include features from [4], [5]. They conducted a computational study that compares the performance of the procedures under different scenarios.

Unlike the studies in the literature, in the present study, fuzzy OAS problem is defined, and a fuzzy mathematical model is developed for this problem. The signed distance methods are used to handle fuzzy constraints in the model. An example problem containing ten orders is solved using GAMS solver, and the results are given.

II. PROBLEM DESCRIPTION

The system under consideration of this study is a make-to-order system with single machine, as many other studies mentioned before, where products are sophisticated and

specially designed for each customer. It is very difficult to define such a complex system exactly using precise terms, like in most of the real-world problems. Fuzzy set theory gives an opportunity to handle vagueness in such situations.

Sources of uncertainty in OAS problem can be classified into two groups: processing times and sequence dependent setup times, and due dates of orders. In MTO environment, firms offer more customized and unique products, so many of the products can be novel for the firms. In such a situation, it is very hard to define crisp processing and sequence dependent setup times. Also, uncertainty in the processing and setup times can be result of the poor reliability in production process on account of, e.g. machine hold ups and man-made factors. In addition, it may be more appropriate to consider fuzzy due date that tolerates a certain amount of early delivery and delay in the due date. Under these circumstances, in this paper, OAS problem incorporating fuzzy processing and setup time and fuzzy due date is formulated in order to more properly model actual situation. In this problem setting, a single machine environment where the production capacity is limited is considered. Also, it is assumed that a set of incoming orders is available at time zero, and the schedule is non-pre-emptive, meaning that once an order starts to be processed on the machine, the process cannot be interrupted before its completion. In fuzzy OAS problem considered in this paper, the fuzzy processing times, fuzzy sequence dependent setup times, and fuzzy due dates are represented by triangular fuzzy numbers (TFN).

A fuzzy number \tilde{A} is called a TFN if its membership function $\mu_{\tilde{A}}$ is defined in [1]:

$$\mu_{\tilde{A}} = \begin{cases} 0, & x < a_1, x > a_3 \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2 \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x < a_3 \\ 0, & x > a_3 \end{cases} \quad (1)$$

Left and right function of a TFN is linear, and \tilde{A} is denoted by the triplet $\tilde{A} = \{a_1, a_2, a_3\}$ with a_1 and a_3 which are the lower and upper bounds of \tilde{A} , respectively. A TFN shape is given in Fig. 1.

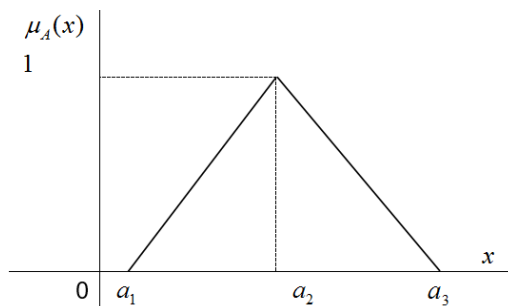


Fig. 1 Triangular fuzzy number (TFN)

The basis of proposed the fuzzy mixed integer mathematical model is MILP model proposed by [8]. It is assumed that a set of independent orders named as SO is given at the beginning

of the planning period. For each order, we have fuzzy data of processing time, sequence dependent setup time, and deadline. Sets of indices and parameters for the model are given in Table I.

There are two sets of binary variables, $Y_{ij} = \begin{cases} 1 & \text{if order } i \text{ precedes order } j, i \neq j, i, j \in SO \\ 0 & \text{otherwise} \end{cases}$ and $I_i = \begin{cases} 1 & \text{if order } i \text{ is selected, } i \in SO \\ 0 & \text{otherwise} \end{cases}$

TABLE I
 SETS OF INDICES AND PARAMETERS

Set of indices
i : set of orders ($i=1,2,\dots,n, i \in SO$)
j : set of orders ($j=0,1,\dots,n+1, j \in SO$)
Parameters (fuzzy parameters are shown with a tilde : ~)
r_i : revenue of order i
\tilde{s}_{ij} : fuzzy sequence dependent setup times
\tilde{p}_j : fuzzy processing time of order i
Decision Variables
C_i : completion time of order i

The fuzzy mixed integer model for fuzzy OAS problem is formulated in [2]-[10]:

$$\text{Maximize } \sum_{i=1}^n r_i * x_i \quad (2)$$

Subject to:

$$\sum_{j=1, j \neq i}^{n+1} y_{ij} = I_i \quad \forall i = 0, \dots, n \quad (3)$$

$$\sum_{j=0}^n y_{ji} = I_i \quad \forall i = 1, \dots, n+1 \quad (4)$$

$$C_i + (\tilde{s}_{ij} + \tilde{p}_j)y_{ij} + \tilde{d}_i * (y_{ij} - 1) \leq C_j \quad \forall i = 0, \dots, n, j = 1, \dots, n+1, i \neq j \quad (5)$$

$$C_i \leq \tilde{d}_i * x_i \quad \forall i = 1, \dots, n+1 \quad (6)$$

$$C_0 = 0, \quad (7)$$

$$I_0 = 1, \quad (8)$$

$$I_{n+1} = 1 \quad (9)$$

$$I_i \in \{0,1\}, y_{ij} \in \{0,1\} \quad \forall i = 0, \dots, n, \quad (10)$$

In this model, as in the study of [8], dummy orders, order 0 and order $n+1$, are defined. Order 0 is assigned to first position, and order $n+1$ is assigned to last position in the schedule. Dummy orders are available at time zero, with $r_0, r_{n+1}, \tilde{p}_0, \tilde{p}_{n+1}, \tilde{d}_0$ being zero (fuzzy $\tilde{p}_0, \tilde{p}_{n+1}, \tilde{d}_0$ values are defined as $(0,0,0)$). \tilde{d}_{n+1} is taken equal to the maximum of deadline of all orders. The objective function (2) maximizes the total revenue from selected order. Equations (3) and (4) ensure that, if an order is accepted, this order precedes only one order and is succeeded by only one order [8]. Equation (5) implies that, if order j is preceded by order i , then completion time of order j should be greater than completion time of order

i plus fuzzy sequence dependent setup time between order i and order j , plus fuzzy processing time of order j . If order i does not precede order j , the only constraint is that completion time of order j must be equal or greater than zero ($C_j \geq 0$). Equation (6) ensures that any order completed after fuzzy deadline cannot be accepted. Equation (7) is useful to set completion time of beginning dummy order 0. Equations (8) and (9) guarantee that dummy order positioned first and dummy order positioned last are selected.

III. SOLUTION METHODOLOGY AND AN ILLUSTRATIVE EXAMPLE

The fuzzy OAS model is transformed into a crisp model using the signed distance method [10]. The signed distance of a TFN $\tilde{A} = \{a_1, a_2, a_3\}$ is defined as [11], $d(\tilde{A}, 0) = \frac{1}{4}(2a_2 + a_1 + a_3)$.

Using this definition, the fuzzy constraints (5), (6) are converted into crisp constraints (11), (12).

$$C_i + \left(\frac{1}{4}(s1_{ij} + 2s2_{ij} + s3_{ij}) + \frac{1}{4}(p1_j + 2p2_j + p3_j) \right) y_{ij} + \frac{1}{4}(d1_i + 2d2_i + d3_i) * (y_{ij} - 1) \leq C_j \quad \forall i = 0, \dots, n \quad \forall j = 1, \dots, n + 1 \quad i \neq j \quad (11)$$

$$C_i \leq \frac{1}{4}(d1_i + 2d2_i + d3_i) * x_i \quad \forall i = 1, \dots, n + 1 \quad (12)$$

TABLE II
INPUT DATA

Order No	r_i	\tilde{p}_j	\tilde{d}_{ij}
1	5	(16,17,18)	(107,109,111)
2	19	(18,19,20)	(113,115,117)
3	16	(12,13,14)	(104,106,108)
4	10	(3,4,5)	(102,104,106)
5	12	(12,13,14)	(105,107,109)
6	19	(11,12,13)	(107,109,111)
7	3	(11,12,13)	(99,101,103)
8	16	(1,2,3)	(104,106,108)
9	19	(5,6,7)	(101,103,105)
10	5	(2,3,4)	(113,115,117)

The other non-fuzzy constraints (3), (4), (7)-(10), and the objective function (2) have also to be included in the model as in the original way. An example, inspired from data of OAS problem generated by [8], is solved using defuzzified model. The input data of the problem are given in Tables II-IV.

The resultant crisp model is solved by using the GAMS solver. The obtained solution is $x_1 = 1, x_3 = 1, x_4 = 1, x_5 = 1, x_6 = 1, x_7 = 1, x_9 = 1, x_{10} = 1, x_{11} = 1, x_{12} = 1$. So, except order 2 and order 8, all of the other orders are accepted and the corresponding objective value is 116. The other decision variable values are given in Table V.

IV. CONCLUSION

In this study, Fuzzy OAS problem is defined. Fuzzy model

can represent better real process due to the sources of uncertainty inherent in MTO production environment. Firstly, Fuzzy MILP model is proposed and then converted into the equivalent crisp model using the signed distance method. The given example is solved by using GAMS solver.

TABLE III
SEQUENCE DEPENDENT SETUP TIMES-1-

Order No	2	3	4	5	6
1	(6,7,8)	(9,10,11)	(2,3,4)	(1,2,3)	(3,4,5)
2	-	(9,10,11)	(8,9,10)	(5,6,7)	(3,4,5)
3	(1,2,3)	-	(5,6,7)	(8,9,10)	(6,7,8)
4	(8,9,10)	(1,2,3)	-	(6,7,8)	(5,6,7)
5	(9,10,11)	(1,2,3)	(1,2,3)	-	(9,10,11)
6	(5,6,7)	(5,6,7)	(7,8,9)	(5,6,7)	-
7	(9,10,11)	(6,7,8)	(5,6,7)	(6,7,8)	(2,3,4)
8	(3,4,5)	(4,5,6)	(9,10,11)	(2,3,4)	(9,10,11)
9	(7,8,9)	(5,6,7)	(8,9,10)	(6,7,8)	(4,5,6)
10	(5,6,7)	(1,2,3)	(2,3,4)	(9,10,11)	(1,2,3)
11	(8,9,10)	(6,7,8)	(2,3,4)	(3,4,5)	(6,7,8)

TABLE IV
SEQUENCE DEPENDENT SETUP TIMES -2-

Order No	7	8	9	10	11
1	(2,3,4)	(1,2,3)	(3,4,5)	(5,6,7)	(6,7,8)
2	(6,7,8)	(2,3,4)	(5,6,7)	(6,7,8)	(1,2,3)
3	(6,7,8)	(4,5,6)	(9,10,11)	(1,2,3)	(8,9,10)
4	(4,5,6)	(3,4,5)	(8,9,10)	(6,7,8)	(2,3,4)
5	(6,7,8)	(4,5,6)	(9,10,11)	(4,5,6)	(5,6,7)
6	(4,5,6)	(6,7,8)	(8,9,10)	(9,10,11)	(8,9,10)
7	-	(7,8,9)	(2,3,4)	(7,8,9)	(10,11,12)
8	(7,8,9)	-	(9,10,11)	(3,4,5)	(4,5,6)
9	(3,4,5)	(3,4,5)	-	(6,7,8)	(4,5,6)
10	(8,9,10)	(2,3,4)	(9,10,11)	-	(6,7,8)
11	(6,7,8)	(1,2,3)	(2,3,4)	(1,2,3)	-

TABLE V
SOLUTION USING THE SIGNED DISTANCE METHOD

Accepted Order No	C_i	Immediate Predecessor Number	Y_{ij}
1	0	-	-
3	92	9	$Y_{93} = 1$
4	57	5	$Y_{54} = 1$
5	42	6	$Y_{65} = 1$
6	30	7	$Y_{76} = 1$
7	15	1	$Y_{17} = 1$
9	68	11	$Y_{119} = 1$
10	100	3	$Y_{310} = 1$
11	63	4	$Y_{411} = 1$
12	100	10	$Y_{1012} = 1$

It is proved that OAS problem is strongly NP-hard [12], so the problem identified in this study is also strongly NP-hard. Since exact solutions can be obtained for only instances with a limited number of orders, developing a metaheuristic solution method for the fuzzy OAS problem can be further exploration. Also, fuzzy OAS problem can be solved directly without transforming model to crisp equivalent via any metaheuristic method that uses ranking methods.

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