

Myths and Strategies for Teaching Calculus in English for Taiwanese Students: A Report Based on Three-Years of Practice

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Abstract—This paper reviews the crucial situation in higher education in Taiwan due to the rapid decline of the birth rate in the past three decades, and how the government and local colleges/universities work to face the challenge. Recruiting international students is one of the possible ways to resolve the problem, but offering enough courses in English is one of the main obstacles when the majority of learners are still Taiwanese students. In the academic year of 2012, Chung Yuan Christian University determined to make its campus international and began to enforce two required courses for freshmen taught in English. It failed in the beginning, but succeeded in the following academic year of 2013. Using the teaching evaluations accumulated in the past three years, this paper aims to clarify the myths which had been bothering most faculties. It also offers some suggestions for college/university teachers interested in giving lectures in English to English as Second Language (ESL) learners. A conclusion is presented at the end of the paper, in which the author explained why Taiwanese students could learn their profession in English.

Keywords—Calculus, English, teaching evaluation, teaching strategy, vocabulary.

I. INTRODUCTION

HIGHER education in Taiwan has been facing tough challenges due to the country's decreasing birth rate. According to the population statistics from the Ministry of the Interior, Republic of China, published in 2009, the number of newborns and the birth rate for Taiwan in 1981 was 414,096 and 2.30%, while in 2008 it was 198,773 and 0.86%, respectively [1]. Table I, gives the corresponding figures in recent years.

It can be seen from Table I that the total number of newborns in 2008 is less than the half of that of 1981. This situation lead to a substantial drop of senior high school graduates in 2016, and hence, inevitable low registration rates at colleges and universities. More precisely, there are 159 colleges/universities [2], but there are roughly only 200,000 students. Administrators of Taiwanese higher education institutions have been aware of the problem of ever decreasing birth rate for years. To resolve the problem, the government encourages consolidation between schools with complementary departments, and urges traditional departments or sections to develop new directions with more applications or with closer connections with industries. At the same time, schools are motivated to recruit

more international students to study in Taiwan, or to set up overseas branches to release the pressure of too many employees.

TABLE I
NUMBER OF NEWBORNS AND BIRTH RATE OF TAIWAN [1]

Year	Number of newborns	Birth rate (%/permillage)
1981	414,096	23.0
1986	309,230	15.9
1991	321,932	15.7
1996	325,545	15.2
2001	260,354	11.7
2005	205,854	9.1
2006	204,459	8.96
2007	204,414	8.92
2008	198,773	8.64

Chung Yuan Christian University, abbreviated as CYCU in the sequel, is a 61-year-old, private, comprehensive university located in the northern part of Taiwan. Currently, the student population is around 16,000 (See [3] for more information). Throughout the past decades, it has been highly ranked and well-known for offering holistic education. For example, in 2015, according to the UK-based Times Higher Education Emerging Economies Rankings, CYCU was ranked among the top-100 universities in Asia, and was first among all private comprehensive universities in Taiwan. In addition, among all private universities in Taiwan, CYCU was awarded the largest funding from the Ministry of Education of Taiwan for the past consecutive three years (2013-2015). Such information and other honors are announced in [3]. To overcome the difficulty of the low birth rate in Taiwan, the office of Academic Affairs of CYCU in the school year of 2012 determined to transform the campus into a truly international campus. The first preparation for such a vision was to provide courses in English. Even though at most universities in Taiwan, courses are taught with textbooks in English and students are used to reading professional books in English, we still faced various objections. In the beginning, some faculty members were not convinced at all. Seeing that local students were the majority, they doubted the fairness of teaching core courses in English and considered such a policy a "lose-lose" situation in the sense that instructors would suffer from teaching subjects in a second language and that most students would end up sacrificing their professional knowledge and ability. In the fall semester of 2012, we offered two required courses in English, Calculus and General Physics for freshmen, and each for one class only. The capacity of each

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class is 60 students. That is, at most 120 students, among the total of nearly 3,000 first-year students, could enroll in these two classes. However, this attempt failed. The number of students enrolled in Calculus in English was two and no one desired to take General Physics in English. This failure forced the realization of teaching professional subjects entirely in English at CYCU to be postponed for one more year.

In the fall semester of 2013, the office of Academic Affairs of CYCU decided to take the initiative for this vision in order to enforce the internationalization of the campus, which became a more urgent task as time went by. They screened all freshmen to select 60 students with preferable English performances in their entrance exams. These 60 students were somehow labeled as “elite”, and must take the required Calculus and General Physics in English. The author of this paper has been the instructor of Calculus in English since the fall of 2013. This class is still continuing, and more and more courses are now offered entirely in English at CYCU. Based on the author’s experiences during the past three years, this paper aims to clarify some myths and to share the teaching strategies which make this unique class successful. A brief summary and our overall opinion about this policy are given in the conclusion.

II. SOME MYTHS ABOUT TEACHING CORE MATERIALS IN ENGLISH AND CLARIFICATIONS

In CYCU, every semester, each teacher must be evaluated by their students for each of the classes they teach. In the questionnaire, students are asked to grade the instructor according to the following nine aspects:

- i. he/she is well prepared for the class;
- ii. he/she follows the course outline;
- iii. he/she has good presentation skills;
- iv. his/her way of teaching is inspiring;
- v. he/she is happy to communicate with students;
- vi. he/she teaches with passion;
- vii. the way he/she scores the students is objective and fair;
- viii. he/she attends the class on time regularly;
- ix. overall, I am satisfied with his/her teaching [4].

The full score for each question is five, where five means “fully agree”, four means “agree”, three means “so-so”, two means “disagree”, and one means “totally disagree”. The instructor will be given the list of their score on each item, the average score, and any comments provided anonymous by students. Moreover, the system automatically shows the average of all the evaluations in the same semester at CYCU, in the specific college, in the given department, and where the scores are ranked (as the percentage) compared with all the other evaluations belonging to the same category.

Just as in many universities in Taiwan, the faculties of CYCU had many questions and mistrust about teaching the required courses in English, especially for a class with so many local students and only a few international students. Their major concerns were related to the outcome of the teaching evaluations. These myths will be discussed below.

- Myth 1. For the same subject, if we offer classes in both Chinese and English, then the one taught in English must be evaluated as the worst. If not the worst, it must be far

below the average.

- Myth 2. For the same instructor, if he/she teaches the same subject in both Chinese and English, then the one in English must be evaluated as the worst.
- Myth 3. For the same instructor, if he/she teaches any elective course in Chinese and any required course in English, then the evaluation for the former must be better than the latter.
- Myth 4. For the same subject, students taking it in English will perform worse than those taking it in Chinese.

To clarify Myth 1, the author compares the evaluation of her class, Calculus in English, with that of all other Calculus classes in Chinese. In CYCU, all classes for first-year Calculus are taught by faculty members from the department of Applied Mathematics, and for each semester, we offer 35 classes for students from different colleges all over the school. The results are presented in Table II. The symbol F ’13 (respectively S ’13) denotes the fall semester (represents spring semester) of the academic year of 2013 and similarly for the other years. The third row on Table II lists the average of the evaluations of all required courses of the College of Science on the same semester. The last row shows the ranking of the author’s class compared with all other required courses in the College of Science. Notice that the required courses offered by the College of Science include the other 34 classes of Calculus taught in Chinese. From these figures, it is observed that Myth 1 comes from the teachers’ psychological fears more than from fact.

TABLE II
 EVALUATIONS OF CALCULUS IN ENGLISH IN THE PAST THREE ACADEMIC YEARS

Semester	F ’13	S ’13	F ’14	S ’14	F ’15	S ’15
Evaluation of Calculus in English	4.88	4.63	4.81	4.80	4.84	4.83
Average of College of Science	4.35	4.39	4.40	4.42	4.40	4.42
Ranking (%)	0.85	15.79	2.5	4.24	0.83	4.03

TABLE III
 EVALUATION OF CALCULUS IN CHINESE BY THE AUTHOR IN THE SCHOOL YEAR OF 2012

Semester	F ’12	S ’12
Evaluation of Calculus in Chinese	4.81	4.86
Average of College of Science	4.30	4.28
Ranking (%)	5.31	1.85

To investigate whether the statement in Myth 2 is true, the author lists her evaluation of Calculus in F ’12 and S ’12 in Table III. Note that in the academic year of 2012, we intended to teach Calculus in English but failed. Thus, all Calculus classes offered were in Chinese in that year. A comparison between Tables III and II shows no difference in the evaluations between teaching Calculus in English and in Chinese. Many faculty members were concerned that students could not understand advanced mathematics or sciences in college/university level, and therefore, would provide negative feedback. In fact, such a scenario did not happen at all. Even though English is a second language for both the instructor and the students, the comments offered by the students suggests that

they sense almost no obstacles to learning this course.

Myth 3 seems very reasonable. Many of our colleagues insisted on teaching elective courses because they believe that students enrolling in the class should be more engaged. Such students usually show more appreciation to the endeavor of the instructor, and will offer more positive feedback at the end of the course. For faculty members with such a belief, teaching Calculus in English is something like a disaster due to the following two reasons. First of all, students are forced to take Calculus. Secondly, they are forced to study a hard subject in a foreign language. To realize whether the claim of Myth 3 is true, the author lists the corresponding evaluations of other elective courses during the past three years in Table IV. By comparing Tables IV and II, it is possible to see a completely opposite result. That is, the evaluations obtained from Calculus in English are in general much higher than those from other elective courses in Chinese.

How can such a surprising outcome occur? Maybe the following two points can explain something. The first is that students in the author's class are selected for their high English performance, not for their mathematics performance, in the entrance exam. However, in Taiwan, it appears that young people with better English communication abilities are from urban families with superior social and financial statuses. They not only have more resources available for developing their language skills, but also have better overall support in learning sciences and taking part in other extra-curriculum activities. The second is that once students are assigned to the "honors program", where first-year Calculus and General Physics are taught in pure English, they feel a sense of pride in themselves, which enhances their confidence and encourages them to study hard. The positive attitude from each student could be the most important factor in making this special class succeed.

TABLE IV
EVALUATIONS OF OTHER ELECTIVE COURSES IN CHINESE BY THE AUTHOR IN THE PAST THREE YEARS

Semester	Other elective course, *, taught by the author	Evaluation of *	Average of CYCU
F '13	Numerical Analysis	4.49	4.40
S '13	Creative Problem Solving	4.90	4.42
F '14	Science and Ethics	4.60	4.44
S '14	None		
F '15	Science and Ethics	4.69	4.44
S '15	Creative Problem Solving	4.67	4.43

Myth 4 is the one the author finds very difficult to verify. In CYCU, all instructors teaching Calculus must use the same textbook. We have the same syllabus, follow the same course outline, and give four "united examinations" in every semester. Given the fact that the students in the author's class are highly self-motivated, comparing the scores among classes and showing that learning Calculus in English would be a better choice is not convincing at all. One can only say that studying a professional subject in a foreign language does not imply a worse performance. It is still possible that students can learn the materials well regardless of the language.

III. SOME TIPS FOR REDUCING THE BURDEN OF STUDENTS' LANGUAGE BARRIER

After three-years of practice, the author has become much more confident in teaching Calculus in English and started enjoying the class. The following four strategies are considered the most essential whenever the author has an opportunity to advise anyone interested in teaching college/university students a mathematical course in English.

Tip 1. Help students with building up the vocabularies. A Taiwanese student usually starts learning English in third grade at elementary school, which means they have learned English for nine years before attending college/university. However, students might not be familiar with corresponding English terms for specific mathematical terminology, because their previous studies in mathematics were always in Chinese until they take the class taught by the author for the first time. Since it is required in CYCU that no spoken-Chinese be allowed in a pure English class, the author prepares handouts for students to build up their vocabulary for this course. An example of the handout, which is called "Vocabulary for today", is given in Fig. 1. Since the Chinese translation is printed right beside the English term, there is no need to provide an explanation, but we do need to pronounce each of them. Usually the author would ask her students to "repeat after me" so that they themselves learn the pronunciation of each new term. Such a preparation is especially important for the first month. The class always begins with reading vocabulary, which takes only five minutes, before teaching new material.

Calculus MA107A		S. Kao	
Vocabulary for today		Sep. 2016	
• variable	n. 變數	• polynomial	n. 多項式
		• coefficient	n. 係數
		• degree	n. 次數
• interval	n. 區間	• function	n. 函數
		• even function	偶函數
		• odd function	奇函數
• domain	n. 定義域	• range	n. 值域
• greater than	大於 (>)	• less than	小於 (<)
• fraction	n. 分數	• infinite	adj. 無窮的
• numerator	n. 分子	• infinity	n. 無窮大、無限
• denominator	n. 分母	• finite	adj. 有限的 n. 有限
• real numbers	實數	• addition	加 → sum 和
• rational numbers	有理數	• subtraction	減 → difference 差
• irrational numbers	無理數	• multiplication	乘 → product 乘積
• integers	整數	• division	除 → quotient 商, remainder 餘
• natural numbers	自然數	• power	冪、次方
		• root	開方根

Fig. 1 An example of the handout of 'Vocabulary for Today' for teaching Calculus in the first month

Tip 2. Help students with building up their professional language. In many cases, a direct translation of a Chinese expression for a mathematical term does not imply the correct way of saying the same thing in English. For example, in Chinese, x^n is called " x n 次", which is " x , n , time, square" in English. Thus, when the author presents to students today's vocabulary, she shows them how the associated terms are applied/communicated in English. For example, students know that "a power function" is a function of the form $f(x) = x^n$, but

they do not know the correct way of saying “ x^n ” in English. Thus the instructor could list x^n with different n 's and show them how we communicate with each other in English. Fig. 2 gives an illustration. As for Tip 1, such a step cannot be skipped. And the author suggests that instructors replenish the students' vocabularies and professional language skills whenever necessary. In fact, a broader interpretation for “professional languages” in mathematics includes the rigorous way of proving or disproving a given statement. Students must be able to write down their reasoning using the formal mathematical language and style. Take the mathematical induction for instance. However, that is not the main focus of this paper.

x^2 : x square	\sqrt{x} : the square root of x
x^3 : x cube	$\sqrt[3]{x}$: the cubic root of x
x to the power of three	the third root of x
x^4 : x to the power of four	$\sqrt[n]{x}$: the n th root of x
x^n : x to the power of n	

Fig. 2 An example of showing students how these power functions and root functions are communicated verbally

Tip 3. Illustrate your verbal expressions with body language, pictures or multi-medias. No matter if it is in Chinese or in English, the use of body language, pictures or multi-media is always a very efficient way of communication. With an exaggerative but clear gesture or facial expression, students catch the message quickly and correctly, which narrows the gap for those students with a weaker listening ability. When three-dimensional surfaces are involved, pictures and multi-media are very useful. It explains many things clearly, and often far better than verbal language. Simple pictures illustrating abstract theorems or concepts are very useful. When the author talks about a theorem with an “if..., then...” statement, she would use a diagram showing students the relationship between the sufficient condition (the “if” part) and its result (the “then” part). For example, a well-known theorem in Calculus is stated as follows:

Theorem. Let $f(x)$ be a function defined on $[a, b]$. $f(x)$ is continuous on $[a, b]$ and differentiable on (a, b) .

If c is a point in $[a, b]$ such that $f(c)$ attains the extreme value, then c must be a critical point of $f(x)$.

The above theorem can be found in Chapter 3 in [5] and [6], and other standard calculus textbooks. Many students misunderstand the theorem and think that critical points imply the existence of extreme values of $f(x)$. Namely, they think if they locate the critical points of $f(x)$, then they find the places where $f(x)$ attains its maximum or minimum. A figure indicating the relationship between the two sets will help students to clarify the concept. See Fig. 3. It can be observed that if $f(c)$ is the maximum or minimum of $f(x)$, then c belongs to the set of critical points of $f(x)$, but the converse is not true. Take $f(x) = x^3$ for instance. On the real line, $f(x)$ has only one critical point, zero. But $f(0) = 0$ is neither the maximum nor the minimum of $f(x)$.

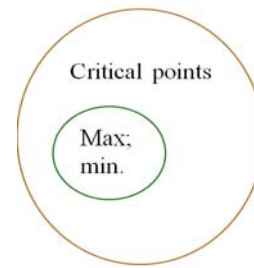


Fig. 3 A diagram indicating the relationship between the sufficient condition and the necessary condition of the theorem stated in Tip 3

Tip 4. Simplify your verbal expressions. Instructors must understand that our speaking cannot simply copy what we are reading. For students, reading textbooks in English is easier because it allows them to check dictionaries, to reread the same content several times, and to stop anytime when they feel frustrated. However, listening to any subject in English requires an immediate response. They need to stay focused and concentrate all the time, and they cannot miss a sentence from the instructor or they may get lost for the rest of the class. Unless a student raises a hand and asks questions, they have no right to stop the lecture. For each of the students in the author's class, the Calculus in English lesson is the first class they have taken in which speaking their native Chinese is forbidden. Thus, the author tries to simplify her language, and to avoid complicated language structures or descriptions with vocabularies which are seldom used. The main purpose for doing so is to ensure that all students understand all the content in each lecture. Here we give an example of how one can transform a sentence into an equivalent version with much simpler language. The original paragraph in the book is “*In the 5th century BC, the Greek philosopher Zeno of Elea posed four problems, now known as Zeno's paradoxes that were intended to challenge some of the ideas concerning space and time that were held in his say. Zeno's second paradox concerns a race between the Greek hero Achilles and a tortoise that has been given a head start. Zeno argued, as follows, that Achilles could never pass the tortoise...*” [5] (p. 5). It is alright in the written form, but students would miss several terms, such as “Elea” and “tortoise”, when spoke by the teacher during a lecture. To orally express the same idea, the following changes are implemented. Beginning with an explanation of *paradox*, neglect *Elea*, portray *tortoise* with *turtle* and describe a *head start*. The more suitable expression is suggested by the author as follows. “*By paradox, we mean something that looks wrong but is indeed correct, or something on the surface you think it makes sense but is total nonsense. In the 5th century BC, the Greek philosopher Zeno posed four problems. We call them 'Zeno's paradoxes' now. They were intended to challenge some of the ideas about space and time back in Zeno's day. The second paradox talks about a race between the Greek hero Achilles and a tortoise, which is a turtle on land. At the beginning of the race, the tortoise starts one meter ahead of Achilles. Zeno argued that Achilles could never pass the tortoise. His argument is...*” .With such a transformation and the assistance of the previous three tips, the percentage of

students succeeding in studying Calculus in English is very high. Note that the information about “Elea” is gone and the definition of “tortoise” may not be precise; however, this is a course in Calculus, not in English, and thus, such a compromise should be allowed.

IV. CONCLUSION

In this paper, the author’s experience of teaching Calculus in English in CYCU is presented as the journey of a four-year experiment. Due to the ever more crucial problem of the low birth rate in Taiwan, some sections/departments in colleges/universities have been forced to close in the fall semester of 2016; this is expected to continue to be a serious issue. Taiwanese colleges/universities need to become more international in order to face this challenge. We have good reasons to claim that teaching professional subjects in English is one way, based on the present outcome with good evaluations, which have been accumulated for six semesters. However, the questions about “why we offer courses in English for local students” and the impression that “teaching core subjects in a foreign language must compromise the profession” have not disappeared. The latter suspicions cannot be clarified with enough evidence due to the special backgrounds of students in the class taught by the author, while the former question can now be answered with confidence. Teaching professional courses in English are not for international students only, the also provides for local students ready to challenge themselves. When the instructor provides students with the scaffold of learning, students can build up their vocabulary bank and professional language with much less difficulty. That tears down the English barrier. Not to mention that many formulas and equations are expressed in a universal language, which consists of purely mathematics symbols. The complements of body languages, diagrams and multi-media resources are very helpful. Furthermore, trying to simplify the sentences in the oral presentation in English reduces the burden for students with a weaker listening ability. In fact, students with the experience of studying in pure English tend to be more confident in communicating with foreigners and be more ambitious to study or work abroad. The internationalization and globalization of Taiwanese higher education is an inevitable trend. It is hoped that the studies and experiences in this paper encourage more teachers interested in teaching subjects in any foreign language, and give insights for administrators in higher education everywhere.

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