

# Unbalanced Distribution Optimal Power Flow to Minimize Losses with Distributed Photovoltaic Plants

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**Abstract**—Electric power systems are likely to operate with minimum losses and voltage meeting international standards. This is made possible generally by control actions provide by automatic voltage regulators, capacitors and transformers with on-load tap changer (OLTC). With the development of photovoltaic (PV) systems technology, their integration on distribution networks has increased over the last years to the extent of replacing the above mentioned techniques. The conventional analysis and simulation tools used for electrical networks are no longer able to take into account control actions necessary for studying distributed PV generation impact. This paper presents an unbalanced optimal power flow (OPF) model that minimizes losses with association of active power generation and reactive power control of single-phase and three-phase PV systems. Reactive power can be generated or absorbed using the available capacity and the adjustable power factor of the inverter. The unbalance OPF is formulated by current balance equations and solved by primal-dual interior point method. Several simulation cases have been carried out varying the size and location of PV systems and the results show a detailed view of the impact of PV distributed generation on distribution systems.

**Keywords**—Distribution system, losses, photovoltaic generation, primal-dual interior point method, reactive power control.

## I. INTRODUCTION

THE rapid development of photovoltaic technologies around the world and their integration into distribution systems may present numerous challenges that need to be studied [1]. Some challenges related to the proliferation of large-scale PV installations in distribution system can be listed as the reverse power flow, voltage increase, power losses [2], [3]. In most applications of grid-connected systems, PV plants are installed near the consumers. This consideration helps not only to improve the system's reliability but also the quality of energy delivered to the load. However, there are still few computational tools to accurately assess the impact of distributed PV generation. One reason is the approximate representation of the PV generation and another is the use of single-phase equivalents for the distribution system. Tools based on single-phase equivalents may not provide the best operating solutions for three-phase systems, especially when circuits and/or loads are considerably unbalanced [4]. In order to analyze the impact of three-phase representation of distribution network on the power quality, this paper presents

a TOPF (three-phase optimal power flow) model in which single-phase and three-phase PV plant are allocated together and modeled via an equivalent circuit.

The three-phase representation of the distribution system was first made in power flow programs that use the backward/forward sweeping method to obtain system's voltages [5]. However, in [6] a three-phase current injection method was proposed that presented better convergence properties than the conventional method. The same formulation was used later in the TOPF [7].

Some researches on the TOPF have been published in recent years. In [8], a solution for an unbalanced FPO via the Quasi-Newton method has been presented; [9] considers discrete control operations such as capacitor switching and taps adjustment of OLTCs; [10] is based on semidefinite programming; [11] extends the TOPF based on current injections for optimization of  $n$ -conductors systems; [12] achieves optimal adjustment of capacitor banks and voltage regulators to minimize losses and [13] is based on locational marginal price concept applied to distribution systems. Some papers also present studies on the impact of distributed PV generation in the system [7], [14], [15].

In studies on steady-state operation, PV plants are often represented by active power injections of values equal to their generation capacities. However, a more realistic view of the operating conditions of the system is obtained if the PV generation is calculated from measurements of solar irradiation and temperature. For this, the PV module can be represented by the five-parameter model [16], [17]. This representation was used in the power flow problem [18] and, more recently, in the single-phase OPF [19] and three-phase optimal [20] problems. This work describes a TOPF model based on the proposed formulation in [7] in which the PV plants are represented by the five-parameter model. The TOPF allows the injection of reactive power by the PV plant, which contributes to improve voltage level and reduce the losses.

The next section describes the formulation of the TOPF problem with PV plants modeling. Section III analyzes the results obtained and, finally, Section IV summarizes the main conclusions of the study.

## II. TOPF

The TOPF problem is formulated from current injections [7]. In the problem, the active power supplied by the PV plants is not controllable. However, these power plants can generate reactive power since the inverters can operate with power

factors (PF) less than 1.

### A. Modeling of PV Plant in the TOPF

The three-phase PV power plants are connected to the grid via a transformer. Depending on the capacity of the PV plant, the connection can be done without a transformer. In this study, the single-phase PVs are connected in the grid without transformer. The AC power supplied by the PV plant is:

$$P_{PV,\omega} = P_{PV,inv,\omega} N_{inv} \quad (1)$$

where  $N_{inv}$  is the number of inverters and  $P_{PV,inv,\omega}$ , the inverter output power calculated according to [20].

If the PV plant operates with adjustable power factor within the inverter limits, its reactive generation,  $Q_{PV,\omega}$  must respect the nominal inverter capacity,  $S_{inv}^{max}$ , and its power factor,  $PF_{inv}$ , as presented in (2):

$$|Q_{PV}| \leq \min \left\{ \sqrt{\left(\frac{P_{PV,\omega}}{PF_{inv}}\right)^2 - (P_{PV,\omega})^2}, \sqrt{(S_{inv}^{max} \times N_{inv})^2 - (P_{PV,\omega})^2} \right\} \quad (2)$$

### B. TOPF Problem Formulation

#### 1) Objective Function

The network performance index considered in this study is the minimum transmission losses expressed as:

$$F = \min \sum_{k=1}^n \sum_{i=1}^3 (P_{gk}^i - P_{dk}^i) \quad (3)$$

where  $n$  the number of bus,  $i$  the phasor index and  $P_{gk}^i$  &  $P_{dk}^i$ , active power generation and load of bus  $k$ .

#### 2) Equality Constraints

The current balance in a given bus  $k$  of the system is made by adding the current injections per phase of the elements connected to this bus:

$$I_{g,k}^{abc} - I_{d,k}^{abc} - I_k^{abc} = 0 \quad (4)$$

where  $I_{g,k}^{abc}$  are the contributions of generators,  $I_{d,k}^{abc}$ , the contributions of loads and  $I_k^{abc}$  the lines and transformers contributions connected to the phases  $a$ ,  $b$  and  $c$  of the bus  $k$ .

Separating the real and imaginary parts of (4), the current balance equations of TOPF are obtained:

$$\begin{aligned} I_{Re,g,k}^{abc} - I_{Re,d,k}^{abc} - I_{Re,k}^{abc} &= 0 \\ I_{Im,g,k}^{abc} - I_{Im,d,k}^{abc} - I_{Im,k}^{abc} &= 0 \end{aligned} \quad (5)$$

The currents injected by the generator are expressed as:

$$I_{g,k}^{abc} = I_{Re,g,k}^{abc} + jI_{Im,g,k}^{abc} = \left( \frac{P_{g,k}^{abc} + jQ_{g,k}^{abc}}{V_k^{abc}} \right)^* \quad (6)$$

where  $P_{gk}^{abc}$  and  $Q_{gk}^{abc}$  are active and reactive powers generated. On the other hand, the currents consumed by the loads are:

$$I_{d,k}^{abc} = I_{Re,d,k}^{abc} + jI_{Im,d,k}^{abc} = \left( \frac{P_{d,k}^{abc} + jQ_{d,k}^{abc}}{V_k^{abc}} \right)^* \quad (7)$$

where  $P_{dk}^{abc}$  and  $Q_{dk}^{abc}$  are active and reactive powers consumed.

The current contributions of the lines are calculated by the network equations, expressed in matrix form as:

$$\begin{bmatrix} I_{Re,k}^{abc} \\ I_{Im,k}^{abc} \\ \vdots \\ I_{Re,m}^{abc} \\ I_{Im,m}^{abc} \end{bmatrix} = \begin{bmatrix} G^{abc} & -(B^{abc})^t & \dots & -G^{abc} & B^{abc} \\ (B^{abc})^t & G^{abc} & \dots & -B^{abc} & -G^{abc} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -G^{abc} & B^{abc} & \dots & G^{abc} & -(B^{abc})^t \\ -B^{abc} & -G^{abc} & \dots & (B^{abc})^t & G^{abc} \end{bmatrix} \begin{bmatrix} V_{Re,k}^{abc} \\ V_{Im,k}^{abc} \\ \vdots \\ V_{Re,m}^{abc} \\ V_{Im,m}^{abc} \end{bmatrix} \quad (8)$$

In (8),  $G^{abc}$  and  $B^{abc}$  are  $3 \times 3$  matrix composed of the real and imaginary parts of the elements of the admittance matrix of the system,  $I_{Re,k}^{abc}$ ,  $I_{Re,k}^{abc}$ ,  $I_{Im,k}^{abc}$  and  $I_{Im,m}^{abc}$  are vectors  $3 \times 1$  and  $t$  indicates transposed matrix.

The transformers are considered fixed and represented by the  $\pi$ -equivalent circuit [21]. So that the voltages in the reference bus (*ref*) to be lagged of  $120^\circ$ , (9) is introduced into the formulation of the problem:

$$\begin{aligned} V_{Im,ref}^a &= 0 \\ V_{Im,ref}^b - V_{Re,ref}^b \tan(-2\pi/3) &= 0 \\ V_{Im,ref}^c - V_{Re,ref}^c \tan(2\pi/3) &= 0 \end{aligned} \quad (9)$$

Finally, in order to keep the voltage magnitude of the reference bus equal in the three-phase, (10) can be included in the TOPF model:

$$\begin{aligned} V_{Re,ref,a}^2 + V_{Im,ref,a}^2 - V_{Re,ref,b}^2 - V_{Im,ref,b}^2 &= 0 \\ V_{Re,ref,a}^2 + V_{Im,ref,a}^2 - V_{Re,ref,c}^2 - V_{Im,ref,c}^2 &= 0 \end{aligned} \quad (10)$$

#### 3) Inequality Constraints

They represent operating limits and/or security aspects of the system. For each bus  $k$  and phases  $a$ ,  $b$  and  $c$ , these are expressed by (11):

$$\begin{aligned} V_{min,k}^{abc} \leq V_{Re,k}^{abc} + V_{Im,k}^{abc} \leq V_{max,k}^{abc} \\ P_{gmin,k}^{abc} \leq P_{g,k}^{abc} \leq P_{gmax,k}^{abc} \\ Q_{gmin,k}^{abc} \leq Q_{g,k}^{abc} \leq Q_{gmax,k}^{abc} \end{aligned} \quad (11)$$

At the bus  $k$ , if there is a three-phase PV plant, in (11),  $P_{gmin,k}^{abc} = P_{gmax,k}^{abc} = P_{PV,\omega}$  and  $Q_{gk} = Q_{PV,\omega}$  with limits given in (2).

The TOPF problem was solved by the primal-dual interior point method [22].

### III. RESULTS

The TOPF methodology was implemented in MATLAB. The barrier parameter and the tolerance adopted for all simulations were  $10^{-10}$  and  $10^{-6}$  respectively.

The studies were performed with the IEEE13 and IEEE34 bus system [23] in which the mutual impedances are not considered and the loads are modeled as constant power. In IEEE13 system, a voltage regulator is disconnected. Solar irradiation and temperature data were obtained at the INMET station in Santa Marta (SC) in January 2014. Data from the Hanwha SF220-30-1P240L (240 Wp) panel and the SUNNY TRIPOWER 12000TL-US (12 kVA,  $PF \geq 0,8$ ) inverter are used.

Fig. 1 shows the output power of the inverter on 23/01/2014, affected by the efficiency of the inverter, which varies from about 80% to about 97% depending on the irradiation and temperature. The maximum power point is achieved at 13 h.

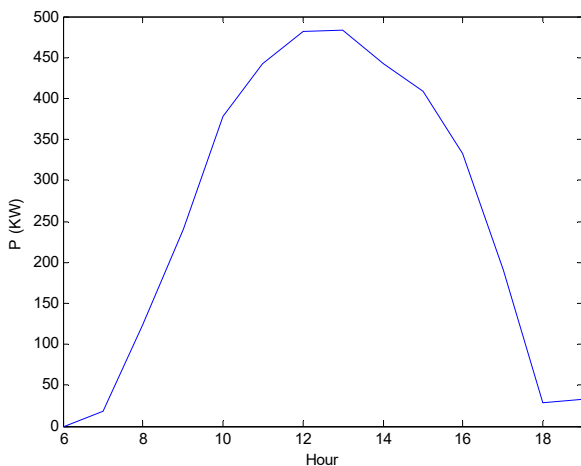


Fig. 1 Inverter Output Power on the day 23/01/2014

#### A. Formation of PV Plants

The PV plants were formed by connecting 48 SF220-30-1P240L panels per inverter, 24 of which were connected in series ( $N_{PV,S} = 24$  e  $N_{Str} = 2$ ). The number of inverters was chosen according to the capacity of the plant. Single-Phase PVs have a capacity of 12 kW and the three-phase PV a capacity of 500 kW. One three-phase PV and 10 single-phase PVs were used for IEEE13-bus system and one three-phase PV and 15 single-phase PVs for IEEE34-bus system. Single-Phase PVs are equally distributed among the three phases of the systems and connected at load's buses. The location of three-phase PVs in the grid is randomly selected.

#### B. Results for a Specific Scenario of Irradiation and Temperature

The results were obtained with the PV generation operating at maximum power on 23/01/2014. Table I indicates the TOPF solutions for (i) without PV systems (Base), (ii) only single-

phase PVs operating with power factor adjustable ( $PF \neq 1$ ) and (iii) association of single-phase and three-phase PV operating with  $PF \neq 1$ . The insertion of PV plants reduces the losses substantially. The operating mode of single-phase and three-phase PVs association reduces the losses of 37% for IEEE13-bus system and 67% for IEEE34-bus system.

TABLE I  
IMPACT OF PV PLANTS ON LOSSES

Systems	Base	Single-Phase PV	Single-Phase & Three-Phase PV
IEEE13	55,62	53,33	35,12
IEEE34	100,02	81,40	32,67

Tables II and III present the power generations at the substation bus and by the PV plants for each system. Note that in the base case, the substation supplies on average 1173,87 kW and 443,08 kvar per phase for IEEE13-bus system and 623 kW and 126,77 kvar per phase for IEEE34-bus system. These values dropped to 970 kW and 233 kvar for IEEE13 and, 126,77 kW and 26,98 kvar for IEEE34 with the allocation of PV plants. Fig. 5 shows that single-phase PVs provide or absorb reactive power depending of their location in the grid. The resulting reactive power of the single-phase PVs in Table II is 48 kvar and 8,31 kvar in Table III. According to Fig. 4, the reactive power is mainly provided by the three-phase PV plant, which generates almost 67% of the total reactive power generation of the IEEE34-bus system.

Fig. 2 shows a contrary situation in which the great part of the system's reactive power generation is provided by the substation bus and Fig. 3 presents the reactive power providing by single-phase PVs, which is close to their maximum limit (4,8 kvar).

TABLE II  
ACTIVE AND REACTIVE POWER INJECTIONS OF IEEE13

Base case	$P_{grefa}$	$P_{grefb}$	$P_{grefc}$	$Q_{grefa}$	$Q_{grefb}$	$Q_{grefc}$
	1195	1051,1	1275,5	357,51	387,51	584,22
Three-phase & Single-phase PV	$P_{grefa}$	$P_{grefb}$	$P_{grefc}$	$Q_{grefa}$	$Q_{grefb}$	$Q_{grefc}$
	994	852,3	1062	149,80	182,84	365,62
	$P_{PVa}$	$P_{PVb}$	$P_{PVc}$	$Q_{PVa}$	$Q_{PVb}$	$Q_{PVc}$
	161	161	161	60,80	60,80	60,80
	$P_{single PV}$			$Q_{single PV}$		
	110			48		

TABLE III  
ACTIVE AND REACTIVE POWER INJECTIONS OF IEEE34

Base case	$P_{grefa}$	$P_{grefb}$	$P_{grefc}$	$Q_{grefa}$	$Q_{grefb}$	$Q_{grefc}$
	640,02	615,46	613,54	138,01	119,93	122,36
Three-phase & Single-phase PV	$P_{grefa}$	$P_{grefb}$	$P_{grefc}$	$Q_{grefa}$	$Q_{grefb}$	$Q_{grefc}$
	401,63	378,55	373,79	30,83	21,19	28,92
	$P_{PVa}$	$P_{PVb}$	$P_{PVc}$	$Q_{PVa}$	$Q_{PVb}$	$Q_{PVc}$
	161	161	161	58,08	58,08	58,08
	$P_{single PV}$			$Q_{single PV}$		
	165			8,31		

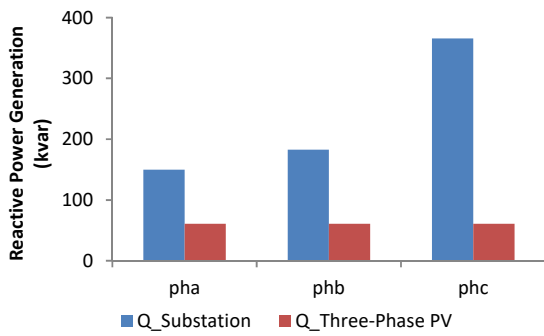


Fig. 2 Reactive power generations of substation bus and three-phase PV, IEEE13

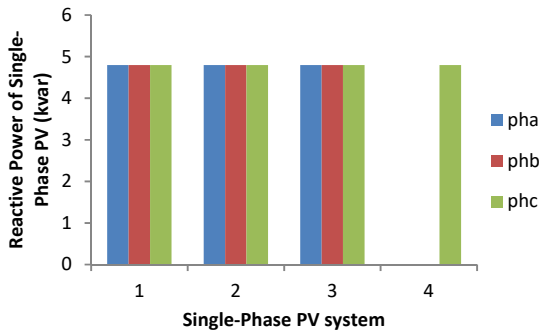


Fig. 3 Reactive power generations of single-phase PVs, IEEE13

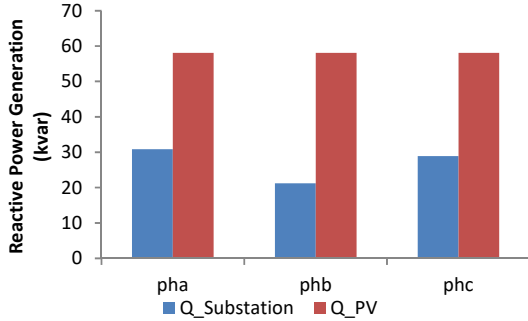


Fig. 4 Reactive power generations of substation bus and three-phase PV, IEEE34

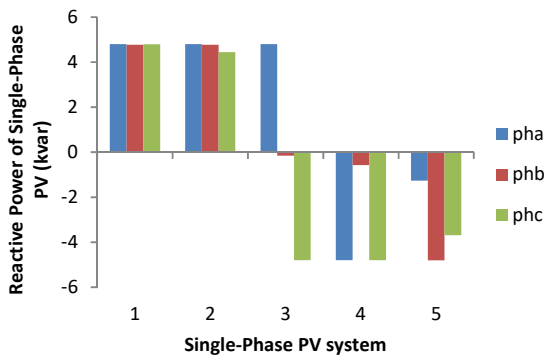


Fig. 5 Reactive power generation of single-phase PVs, IEEE34

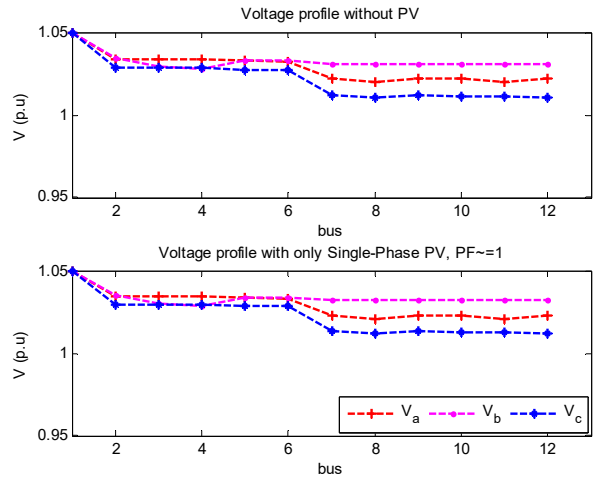


Fig. 6 Voltage profile with single-phase PVs, IEEE13

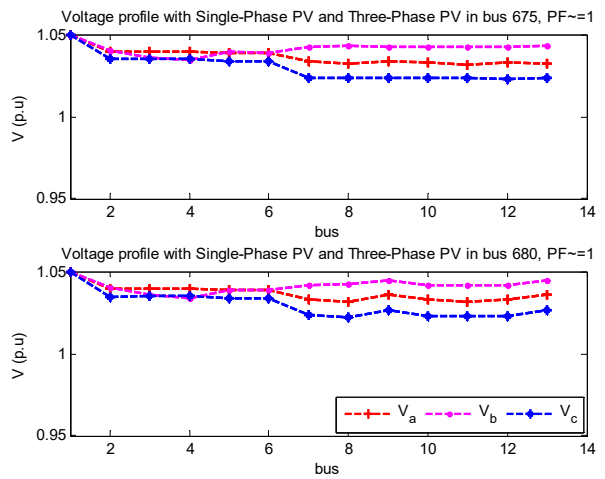


Fig. 7 Voltage profile with single-phase and three-phase PVs, IEEE13

Fig. 6 presents almost the same voltage profile for IEEE13-bus system in both the base case and when only single-phase PV is allocated. This may be justified by the very low PV systems size compared to the system's loading, and Table I shows low values of the system's loss when only single-phase PV is allocated in the network. Fig. 8 shows that when single-phase and three-phase PV plants are allocated together, voltage profile increases and the losses are reduced in the circuit as mentioned in Table I.

Regarding the location of the PV systems, Fig. 7 presents two locations (buses 675 and 680) where the three-phase PV plant is connected in IEEE13-bus system. Note that voltage profile changes when modifying PV plant location. This aspect is also observed in Fig. 9 where (a) 15 single-phase PVs are allocated only in the phase C and (b) 5 single-phase PVs are connected in the three phases of IEEE34-bus system. Simulation results show that the system is more unbalanced when compared its state with the base case (Fig. 8). The system's losses are evaluated at 85 kW and 81 kW respectively for (a) and (b). This aspect shows the detail view

of the allocation of only single-phase PV plants in unbalanced systems.

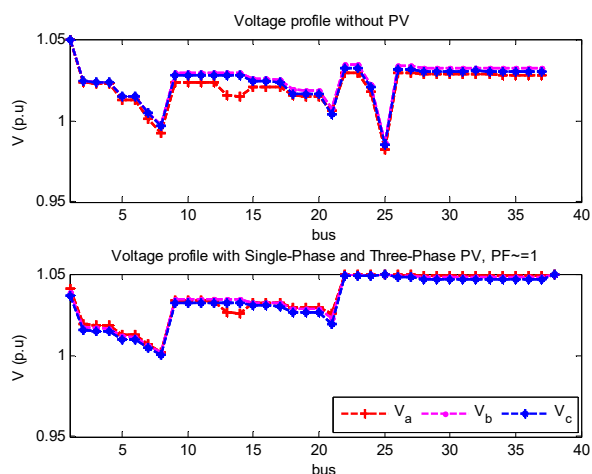


Fig. 8 Voltage profile with single-phase and three-phase PVs, IEEE34

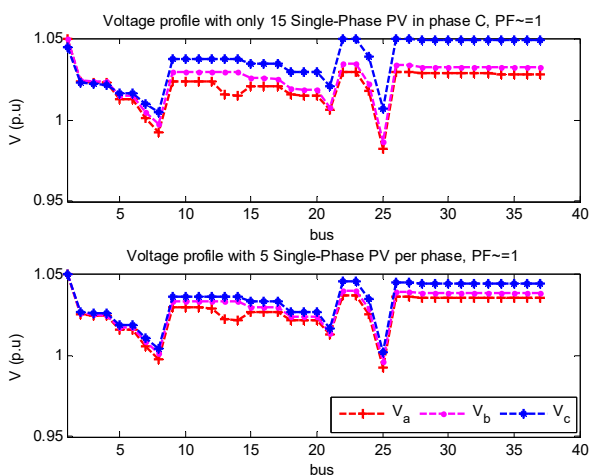


Fig. 9 Voltage profile with only single-phase PVs, IEEE34

#### IV. CONCLUSION

The modeling of single-phase and three-phase PV power plants association in the TOPF problem has showed a detailed view of the impact of PV systems in distribution networks. The simulation results confirm the variability of this generation and show its importance in improving the voltage, which contribute to reduce losses and the unbalance between the phases. However, a poor allocation of single-phase PVs in the network substantially increases the voltage unbalance and could even aggravate the initial state of system's losses depending of their size. The methodology used to obtain the PV generation is still approximate and the TOPF program needs to be improved to represent control equipment of the distribution systems. The improvement of these tools is the next step of the research being developed.

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