Mathematical Modeling of Human Cardiovascular System: A Lumped Parameter Approach and Simulation

Ketan Naik, P. H. Bhathawala

***Abstract*—**The purpose of this work is to develop a mathematical model of Human Cardiovascular System using lumped parameter method. The model is divided in three parts: Systemic Circulation, Pulmonary Circulation and the Heart. The established mathematical model has been simulated by MATLAB software. The innovation of this study is in describing the system based on the vessel diameters and simulating mathematical equations with active electrical elements. Terminology of human physical body and required physical data like vessel's radius, thickness etc., which are required to calculate circuit parameters like resistance, inductance and capacitance, are proceeds from well-known medical books. The developed model is useful to understand the anatomic of human cardiovascular system and related syndromes. The model is deal with vessel's pressure and blood flow at certain time.

***Keywords*—**Cardiovascular system, lumped parameter method, mathematical modeling, simulation.

NOMENCLATURE

μ - Fluid Viscosity
R - Resistivity of Blood Vessel (Resistance)
C - Compliance of Blood Vessel (Capacitor)
T - Duration of Cardiac Cycle
V - Volume of Blood in Vessel
T_{as} - Time of Atrial contraction
T_{av} - Time between the one set of atrial and Ventricle systole
la - Left Atrium
ra - Right Atrium
B_{mv} - Resistance in Mitral Valve
B_{av} - Resistance in Aortic Valve
$E_{ch}(t)$ - Elastance function for Heart Chamber
$e_{ch}(t)$ - Normalized Time varying function
a_{chD} - Minimum diastolic elastance of Chamber
PT - Pulmonary Trunk
$Rcap$ - Right Capillary
RPA - Right Pulmonary Artery
$Rpvenous$ - Right Pulmonary Venous
E - Young's Modulus of Elasticity
h - Wall Thickness of Blood Vessel
L - Blood Inertance (Inductor)
Q - Flow in Blood Vessel
P - Pressure in Blood Vessel
t_c - Elapsed time during each cardiac cycle
T_{vs} - Time of Ventricles contraction
lv - Left Ventricle
rv - Right Ventricle
B_{pv} - Resistance in Pulmonary Valve
B_{tv} - Resistance in Tricuspid Valve
$V_{d,ch}$ - Death Blood Volume of Heart Chamber
a_{chS} - Maximum systolic elastance of Chamber
Sv - Vena Cava Vein (Superior and Inferior)
Pv - Pulmonary Vein
$Rart$ - Right Artery
LPA - Left Pulmonary Artery
V_{un} - Unstressed Volume
$Lpvenous$ - Left Pulmonary Venous

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I. INTRODUCTION

THIS work is about the mathematical modeling of cardiovascular system using Lumped parameter model and simulation of these models using MATLAB software.

The cardiovascular system, base of our study, is fully analogous to the electrical circuits. In fact, for every closed fluid system, there is an electrical circuit whose behavior is alike (up to conversion factors). Rideout et al. [15]-[19] used a lumped parameter model approach in learning of various parts of cardiovascular systems in their study. While emerging a model of the human systemic arterial tree, Snyder et al. [20] used an equal volume modeling feature in the simulation. According to the equal volume feature, the arterial system has been separated into segments in which length and cross sectional area was in reverse proportional [20].

Commonly, models created on lumped representations were employed to accomplish this job [8], Liang and Liu [22]; Formaggia, et al. [9]; incorporating 0D models to simulate flow in the larger arteries, veins and cardiac circulation. Avolio had created multi-branch model of the human arterial system based on the functional branching structure of arterial tree [2]. Olfusen et al. developed a lumped parameter model for systemic arteries of human cardiovascular system using the fluid dynamic equations [6], [13]. The model for arterial system with 42 section was derived by Hassani et al. [1], [4].

II. METHOD

Arteries and veins may be assumed to be made up of cylindrical vessels with linearly elastic walls, and for a good approximation, the blood flows in arteries and veins may be regarded as an incompressible fluid with simple Newtonian characteristics [5]. Let us initially assume that vessel walls are rigid, the input and output pressures are P_i and P_o and the flow is f. Let the internal radius of vessel be r, with corresponding cross-sectional area $A = \pi r^2$. Also let vessel length be $\Delta z = l$. The force moving the blood in the segment is given by the product of the pressure difference between its ends and the area.

$$(P_i - P_o) * A$$

Here we assume that the pressure is uniform across the diameter of the vessel. This force is balanced by fluid flow resistance and of fluid mass acceleration. The resistance to flow is approximated by Poiseuille steady-state formula [5].

$$R = (8 * \pi * \mu * \ \Delta z)/A\text{^}2 \ = (8 * \mu * \ \Delta z)/\pi * r\text{^}4 \quad (1)$$

where μ is fluid viscosity. If it can be assumed that the flow is uniform across the diameter of the vessel, the viscous resistance part of the pressure drop is given by

$$(P_i - P_o)_{vis} = f * R \quad (2)$$

To find the pressure drop due to acceleration of the mass of the blood in segment, we determine the mass as

$$M = \rho * A * \Delta z \quad (3)$$

where ρ is blood density. If we assume that the blood flow is of uniform velocity v across the vessel radius, then the total flow is $f = v * A$. The force needed to balance acceleration of blood in the vessel will be, given by Newton's Second law:

$$M \frac{dv}{dt} = (\rho * A * \Delta z) * \frac{d\left(\frac{f}{A}\right)}{dt} = (\rho * \Delta z) * \frac{df}{dt} \quad (4)$$

This acceleration force must equal the acceleration part of the pressure difference between the ends of the vessel times its cross-sectional area, $(P_i - P_o) * A$. Using this on the left side of (4) and dividing through by A, we get the acceleration part of the pressure drop:

$$(P_i - P_o)_{accel} = (\rho * \Delta z/A) * \frac{df}{dt} \quad (5)$$

The coefficient of the flow derivative in this equation is called inertance L (Inductance in electrical circuit) [5]

$$L = (\rho * \Delta z/A) \quad (6)$$

The sum of the viscous resistance and mass acceleration pressure drops given by (2) and (5) gives the total pressure drop

$$(P_i - P_o) = (R * f) + (L * \frac{df}{dt}) \quad (7)$$

where L and R are given by (1) and (6).

A correction for the fact that the velocity is low near the walls of the vessel, with an overall parabolic cross section of flow velocities (in steady state), gives a slightly better value for the inertance L based on a two radial segment approximation [20]

$$L = \frac{9 * \rho * \Delta z}{4 * A} = \frac{9 * \rho * \Delta z}{4 * \pi r^2} \quad (8)$$

So far, we have neglected the elasticity of the vessel walls. It can be shown that the compliance C of the cylindrical vessel of radius r, length Δz, wall thickness h and Young's bulk modulus of elasticity E:

$$C = \frac{3 * \pi * r^3 * \Delta z}{2 * E * h} \quad (9)$$

III. Mathematical Modeling

In this section, we represent modified and extended models for pulmonary circulation and systemic circulation. Modeling explanations are mainly followed from [6], [3] and [1]. Fig. 1 shows hemodynamics of cardiovascular system.

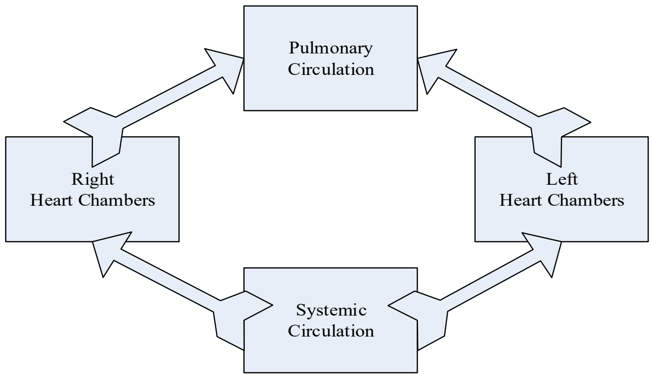

Fig. 1 Hemodynamics of Cardiovascular System

Assumptions for the Models

I. The blood flows in parallel layers with no disruption of layers i.e. laminar flow.
II. The model is not dealing with any kind of control mechanism like baroreceptors and central nervous systems.
III. Body tissues have not any membrane association, means there is no dispersion throughout membrane.
IV. Blood vessels are cylindrical and walls of vessels are flexible.
V. Vessel's curve is ignored (there is no turbulence flow).
VI. The unidirectional blood flow is represented by diode.
VII. All the blood vessels have not any physiological bifurcation. Although, the model contains loops presenting blood vessels from various part of human body.
VIII. Effect of gravitational force on human body is not considered.

A. Mathematical Modeling of Heart

We have divided heart in four chambers: a) Left Atrium, b) Left Ventricle, c) Right Atrium, d) Right Ventricle.

Pressure–Flow in Left Atrium

From Fig. 2 we have

$$\frac{dQ_{la}}{dt} = \frac{1}{L_{la}}(P_{la} - P_{lv}) - \frac{R_{la}Q_{la}}{L_{la}} - \frac{Sgn(Q_{la})B_{mv}Q_{la}}{L_{la}} \quad (10)$$

When $P_{la} - P_{lv} > 0$, then valve is open, and $Q_{la} > 0$, which implies $(Q_{la}) = 1$. If $P_{la} - P_{lv} < 0$, then $Q_{la} = 0$. If $P_{la} - P_{lv} = 0$ then and only then, back flow is possible. In this case $Q_{la} < 0$ and $Sgn(Q_{la}) = -1$. Now let assume first $P_{la} - P_{lv} > 0$ then equation becomes

$$\therefore \frac{dQ_{la}}{dt} = \frac{1}{L_{la}}\Big(E_{la}(t)(V_{la} - V_{d,la}) - E_{lv}(t)(V_{lv} - V_{d,lv})\Big) - \frac{R_{la}Q_{la}}{L_{la}} - \frac{B_{mv}Q_{la}}{L_{la}} \quad (11)$$

$$\therefore \frac{dQ_{la}}{dt} = \frac{1}{L_{la}}\Big(a_{las}e_{la}(t) + a_{laD})(V_{la} - V_{d,la}) - (a_{lvs}e_{lv}(t) + a_{lvD})(V_{lv} - V_{d,lv})\Big) - \frac{R_{la}Q_{la}}{L_{la}} - \frac{B_{mv}Q_{la}}{L_{la}} \quad (12)$$

also,

$$\frac{dV_{la}}{dt} = Q_{pv} - Q_{la} \quad (13)$$

where Q_{pv} is average flow in Pulmonary Veins.

$$a_{las} = 93.31\ \frac{dyne}{cm^5} \qquad \text{[From Table I]}$$

$$= 93.31 * 0.00075\ \frac{mmHg}{ml} \quad = 0.0699\ \frac{mmHg}{ml}$$

$$V_{la} - V_{d,la} = 42 - 4 = 38\ ml\ [20] \quad (14)$$

$$V_{lv} - V_{d,lv} = 83 - 40 = 43\ \text{ml}\ [7] \quad (15)$$

TABLE I
FOR CONVERSATION IN MEDICAL UNIT [23]

	SI system (kg m s)	Cgs System(g semi s)	Medical Units
Area Compliance C_A	$1\ m^2/Pa = m^4/N$	$10^4\ cm^4/dyn$	$1.33 \times 10^6\ cm^2/mmHg$
Compliance C	$1\ m^3/Pa = 1\ m^5/N$	$10^5\ cm^5/dyn$	$1.33 \times 10^8\ ml/mmHg$
Bulk Modulus	$1Pa = 1\frac{N}{m^2}$	$10\frac{dyn}{cm^2}$	$7.5\times 10^{-3}\ mmHg$
Diameter Compliance C_D	$1\frac{m}{Pa} = 1\ m^3/N$	$10^3\ cm^4/dyn$	$1.33 \times 10^4\ \frac{cm}{mmhg}$
Elastance E	$1\frac{Pa}{m^3} = 1\frac{N}{m^5}$	$10^{-3}\frac{dyn}{cm^5}$	$7.5 \times 10^{-9}\frac{mmHg}{ml}$
Flow Q	$1\frac{m^3}{s}$	$10^6\frac{cm^3}{s} = 10^6\frac{ml}{s}$	$1\frac{l}{min} = 16.66\frac{ml}{s}$
Frequency F	$Hz = s^{-1}$	$Hz = s^{-1}$	$min^{-1}(60bpm = 1Hz)$
Inertance L	$1\frac{pas^2}{m^3} = \frac{Ns^2}{m^3}$	$10^{-5}\frac{dyn\ s^2}{cm^5}$	$7.5 \times 10^{-9}\frac{mmHg}{ml}$
Young Modules	$1Pa = 1\frac{N}{m^2}$	$10\frac{dyn}{cm^2}$	$7.5\times 10^{-3}\ mmHg$
Pressure P	$1Pa = 1\frac{N}{m^2}$	$10\frac{dyn}{cm^2}$	$7.5\times 10^{-3}\ mmHg$
Resistance R	$1\frac{Pa.s}{m^3} = 1\frac{N.s}{m^5}$	$10^{-5}\frac{dyn.s}{cm^5}$	$7.5 \times 10^{-9}\frac{mmHg.s}{ml}$

Pressure–Flow in Left Ventricle

Left ventricle is the source of pulse waves in the cardiovascular system. Now let us assume first $P_{lv} - P_{as} > 0$ and $P_{as} = 91.6$, which represent average pressure in ascending order. [3] From Fig. 2, for Left Ventricle we obtained equations as

$$\frac{dQ_{lv}}{dt} = \frac{1}{L_{lv}}(P_{lv} - P_{as}) - \frac{R_{lv}Q_{lv}}{L_{lv}} - \frac{Sgn(Q_{lv})B_{av}Q_{lv}}{L_{lv}} \quad (16)$$

In similar fashion, from Fig. 3, we have equations for Right Atrium and Right Ventricle.

$$\frac{dQ_{ra}}{dt} = \frac{1}{L_{ra}}(P_{ra} - P_{rv}) - \frac{R_{ra}Q_{ra}}{L_{ra}} - \frac{Sgn(Q_{ra})B_{tv}Q_{ra}}{L_{ra}} \quad (17)$$

$$\frac{dQ_{rv}}{dt} = \frac{1}{L_{rv}}(P_{rv} - P_{ap}) - \frac{R_{rv}Q_{rv}}{L_{rv}} - \frac{Sgn(Q_{rv})B_{av}Q_{rv}}{L_{rv}} \quad (18)$$

also,

$$\frac{dV_{rv}}{dt} = Q_{plt} - Q_{lv} \quad (19)$$

where Q_{plt} represents flow in pulmonary trunk.

$$a_{ras} = 79.98\ \frac{dyne}{cm^5} \qquad \text{[From Table I]}$$

$$= 93.31 * 0.00075\ \frac{mmHg}{ml} \quad = 0.0599\ \frac{mmHg}{ml}$$

Similarly, we can convert remaining parameters in $\frac{mmHg}{ml}$

$$V_{ra} - V_{d,ra} = 42 - 4 = 38\ ml[20] \quad (20)$$

where, V_{ra} is Stress Volume of right Atrium in “ml” and $V_{d,ra}$ is Un-stress Volume of right Atrium in “ml”

$$V_{rv} - V_{d,rv} = 93 - 40 = 53\ \text{ml} \quad [7] \quad (21)$$

Pressure in each chamber, denoted by P_{ch}, is given by

$$P_{ch} = E_{ch}(t)(V_{ch} - V_{d,ch}) \quad (22)$$

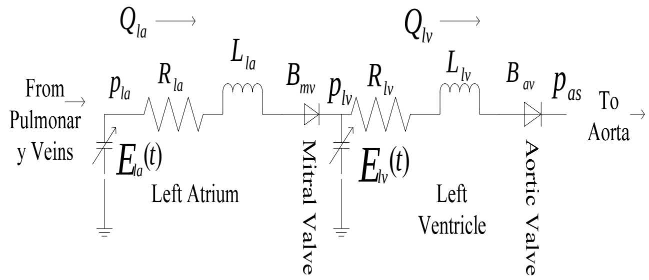

Fig. 2 Left Heart Chamber

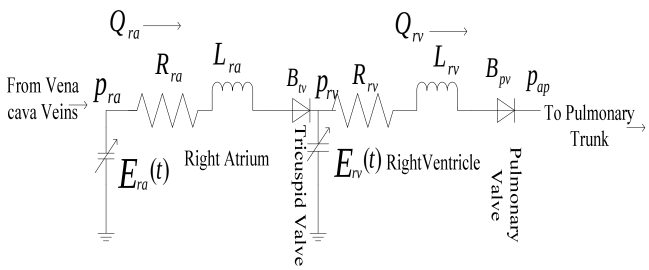

Fig. 3 Right Heart Chamber

where $V_{d,ch}$ represents the volume of chamber "ch'', V_{ch} represents current chamber volume and $E_{ch}(t)$ is Elastance function, which is given by

$$E_{ch}(t) = a_{chS} e_{ch}(t) + a_{chD} \tag{23}$$

where a_{chS} is maximum Systolic Elastance of chamber, a_{chD} is minimum Diastolic Elastance of chamber. $e_{ch}(t)$ is normalized time varying function that represents the forcing source for the model, which for atria is

$$e_{(ch),a}(t) = \begin{cases} \sin\left(\frac{\pi\, t_c}{T_{as}}\right) & 0 < t_c \leq T_{as} \\ 0 & T_{as} < t_c \leq T \end{cases} \tag{24}$$

and for ventricles is

$$e_{(ch),v}(t) = \begin{cases} \sin\left(\frac{\pi\,(t_c - T_{av})}{T_{vs}}\right) & T_{av} < t_c < T_{av} + T_{vs} \\ 0 & T_{av} + T_{vs} \leq \ t_c \leq T \end{cases} \tag{25}$$

Here, the subscript a denotes the atria v denotes ventricles.

B. Mathematical Modeling of Pulmonary Circulation

In pulmonary circulation, oxygen-poor blood is circulated from right ventricle to lungs and oxygen-rich blood is circulated from lung to left atrium.

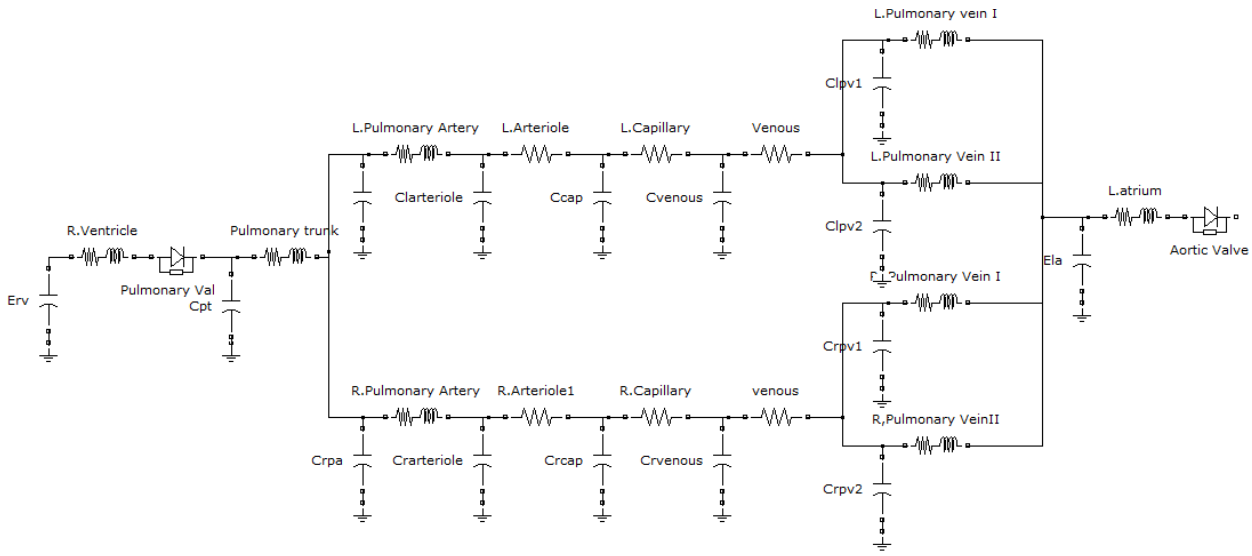

Fig. 4 Lumped Parameter Model for Pulmonary Circulation

Pulmonary Arterial System

First Arterial Section

$$\frac{dQ_{PT}}{dt} = \frac{1}{L_{PT}}(p_{PA} - p_{PT}) - \frac{R_{PT}}{L_{PT}} Q_{PT} \tag{26}$$

$$\frac{dV_{PT}}{dt} = Q_{rv} - Q_{PT} \tag{27}$$

$$p_{PT} = \frac{1}{C_{PT}}(V_{PT} - V_{un,PT}) \tag{28}$$

Second Arterial Section

Left Pulmonary Artery

$$\frac{dQ_{LPA}}{dt} = \frac{1}{L_{LPA}}(p_{Lart} - p_{PA}) - \frac{R_{LPA}}{L_{LPA}} Q_{LPA} \tag{29}$$

$$\frac{dV_{LPA}}{dt} = Q_{PT} - Q_{LPA} \tag{30}$$

Right Pulmonary Artery

$$\frac{dQ_{RPA}}{dt} = \frac{1}{L_{RPA}}(p_{Rart} - p_{PA}) - \frac{R_{RPA}}{L_{RPA}} Q_{RPA} \tag{31}$$

$$\frac{dV_{RPA}}{dt} = Q_{PT} - Q_{RPA} \tag{32}$$

Arteriole Section

$$\frac{dV_{Rart}}{dt} = Q_{RPA} - Q_{Rart} \tag{33}$$

$$p_{Rcap} - p_{Rart} = R_{Rart}Q_{Rart} \tag{34}$$

Similarly for Arteriole in left lung, we have

$$\frac{dV_{Lart}}{dt} = Q_{LPA} - Q_{Lart} \tag{35}$$

$$p_{Lcap} - p_{Lart} = R_{Lart}Q_{Lart} \tag{36}$$

Capillary Section

$$\frac{dV_{Rcap}}{dt} = Q_{Rart} - Q_{Rcap} \tag{37}$$

$$p_{Rpvenous} - p_{Rcap} = R_{Rcap}Q_{RCap} \tag{38}$$

Similarly for Capillary in left lung, we have

$$\frac{dV_{Lcap}}{dt} = Q_{Lart} - Q_{Lcap} \tag{39}$$

$$p_{Lpvenous} - p_{Lcap} = R_{Lcap}Q_{LCap} \tag{40}$$

Pulmonary Veins System

First Venous Section

$$\frac{dV_{Rpvenous}}{dt} = Q_{cap} - Q_{Rpvenous} \tag{41}$$

$$p_{RPV} - p_{Rpvenous} = R_{Rpvenous}Q_{Rpvenous} \tag{42}$$

Similarly, we can define for venous in left lung.

Second Section

For all four Pulmonary veins:

$$\frac{dQ_{Pv}}{dt} = \frac{1}{L_{Pv}}(p_{Pv} - p_{la}) - \frac{R_{Pv}}{L_{Pv}}Q_{Pv} \tag{43}$$

$$\frac{dV_{Pv}}{dt} = Q_{venous} - Q_{Pv} \tag{44}$$

$$p_{Pv} = \frac{1}{C_{Pv}}(V_{Pv} - V_{un,Pv}) \tag{45}$$

C. Mathematical Modeling of Systemic Circulation

In systemic circulation, oxygen-rich blood is circulated from left ventricle to upper body parts and lower body part, and oxygen poor blood from different body part is circulated to right atrium.

Systemic Arterial System

First Arterial Section (Ascending Aorta)

$$\frac{dQ_{As}}{dt} = \frac{1}{L_{As}}(p_{As} - p_{AArchI}) - \frac{R_{As}}{L_{As}}Q_{As} \tag{46}$$

$$\frac{dV_{As}}{dt} = Q_{lv} - Q_{As} \tag{47}$$

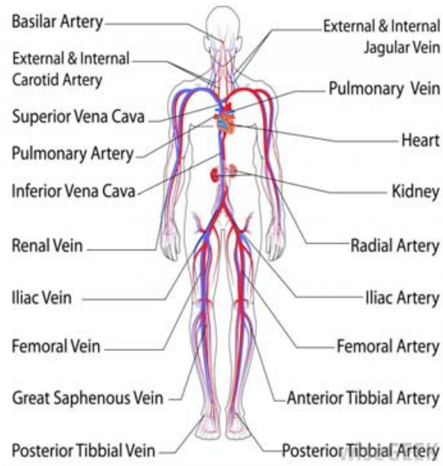

Fig. 5 Systemic Circulation

$$p_{As} = \frac{1}{C_{As}}(V_{As} - V_{un,As}) \tag{48}$$

where As represents Ascending Aorta and $AArchI$ represents Aortic Arch I. Similarly for other systemic arteries, we can defined flow, pressure relationship.

Systemic Capillary system (Upper and Lower Body)

$$\frac{dV_{Cap}}{dt} = Q_{Arteriole} - Q_{Cap} \tag{49}$$

$$p_{Svenouse} - p_{Cap} = R_{Cap}Q_{Cap} \tag{50}$$

Systemic Veins (Superior Vena Cava & Inferior Vena Cava)

$$\frac{dQ_{Sv}}{dt} = \frac{1}{L_{Sv}}(p_{Sv} - p_{ra}) - \frac{R_{Sv}}{L_{Sv}}Q_{Sv} \tag{51}$$

$$\frac{dV_{Sv}}{dt} = Q_{venous} - Q_{Sv} \qquad (49)$$

$$p_{Sv} = \frac{1}{C_{Sv}}(V_{Sv} - V_{un,Sv}) \tag{52}$$

where Sv represents superior and inferior vena cava vein, which is the longest vein.

IV. Simulation

The parameter values are either taken from Medical and Research literature or estimated using the formula given in previous research papers. The whole cardiovascular system model is simulated in MATLAB, Simulink using ODE45 [10]-[14]. Simulation results are compared with standard results given in medical literature. We take simulation time period as H- Heart Rate =60/0.8=75 beats/min =1.25 Hz [3] and when required, we have done unit conversation using Table I.

In our simulation model, we have taken variable compliance for left ventricle, right ventricle, left atrium and right atrium. The data for simulation are taken for a man with average 70 kg weight and in sleeping positions.

V. Explanation and Conclusion

The research carried out here has been mainly keen to develop techniques and to perceive practical procedures for numerical simulation of biophysical phenomena in the cardiovascular system. In this work, our cardiovascular system model contains much more detail than in [1] and [3]. This model has expressed a deeper study on cardiovascular system hemodynamics. We have outlined the use of the model as a tool to better understand the physiology of the system.

We have successfully developed and implemented a model of cardiovascular system, by first dividing whole system in three parts, The heart, pulmonary circulation and systemic circulation and then combine them as one unit to make close loop circuit as shown in Fig. 6, for zero-dimensional lumped parameter model. We have taken variable compliance for all four Heart chambers, i.e. for Left Ventricle, Right Ventricle, Left Atrium and Right Atrium. We have defined elastance function for each four chambers and then pressure as function of elastance and difference of stressed and unstressed blood volume of that chamber. We have replaced each artery and vein by R, L, C element, each arteriole, capillary and venous by R, C element to obtain equivalent electrical circuit of cardiovascular system in Simulink (MATLAB) [21]. We simulate whole circuit for 0.8 second using ODE45 method and have obtained different curves of pressure, volume, flow. We have compared our obtained graphs with standard graphs given in medical books. Our calculated graphs are approximately similar to all above standard graphs. We have not considered the control mechanism of cardiovascular system, therefore our obtained graphs are not exactly coinciding but they are reasonably closed to standard graphs. It is a very complicated task to consider all control nerves and built a model because it leads to very complex system [3], [5].

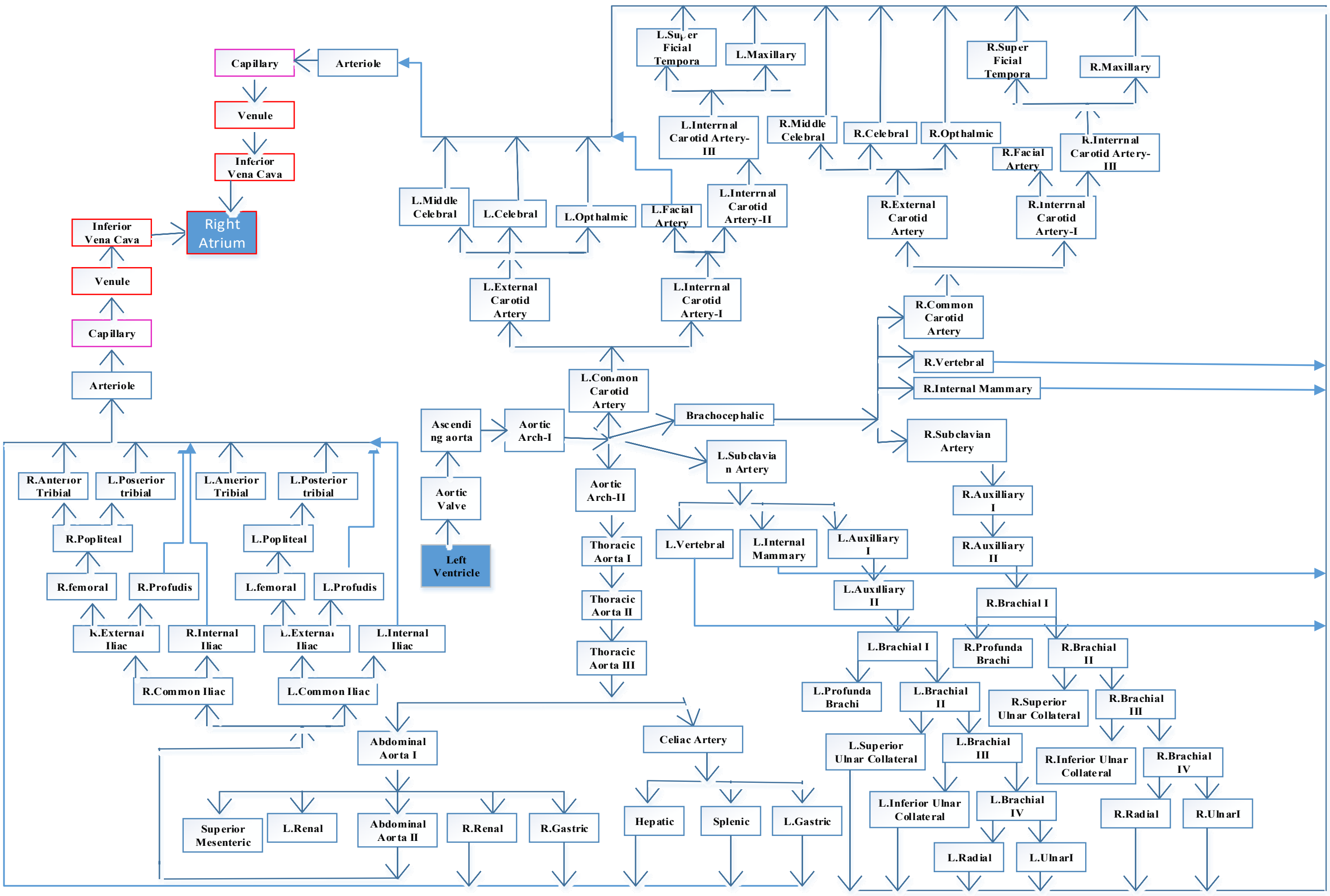

Fig. 6 Schematically Diagram of Systemic Circulation

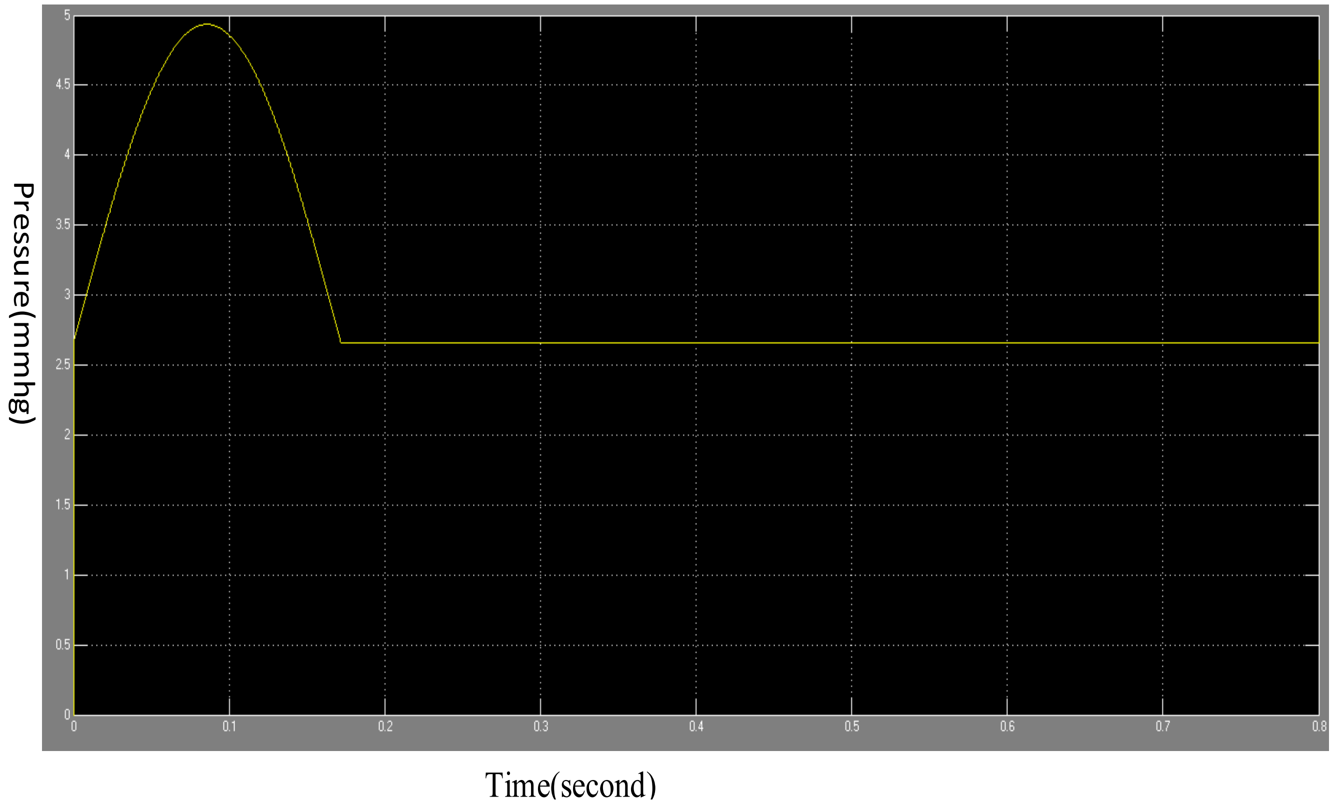

Fig. 7 Left Atrium Time vs Pressure curve

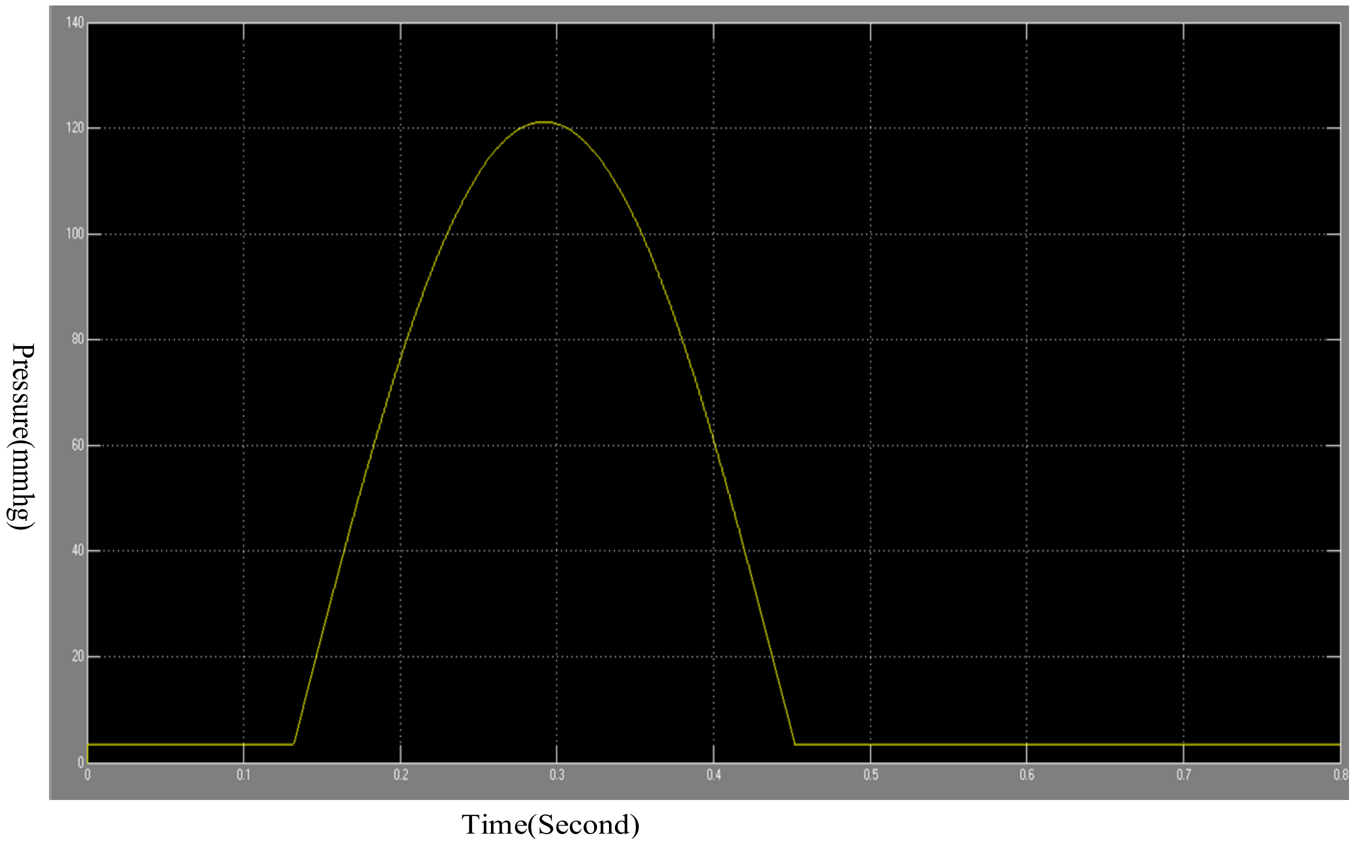

Fig. 8 Left Ventricle Time vs Pressure curve

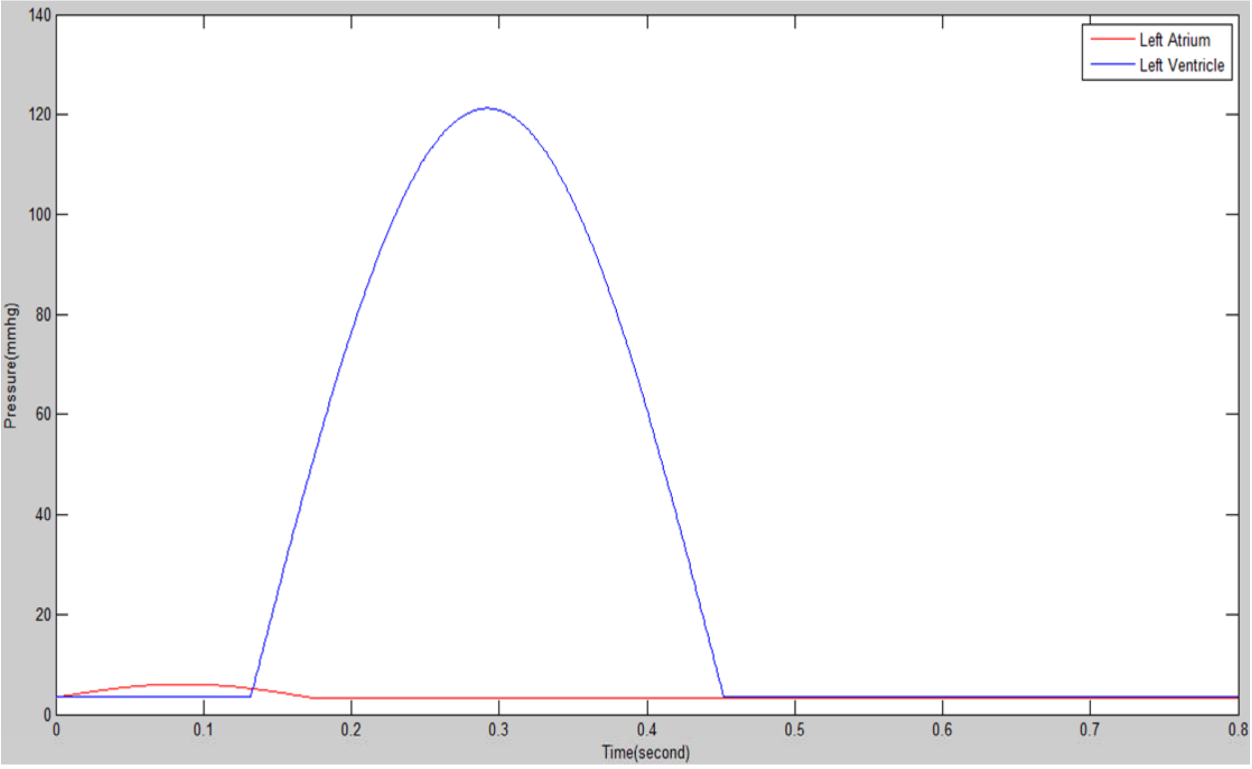

Fig. 9 Left Atrium & Left Ventricle Time vs Pressure curve

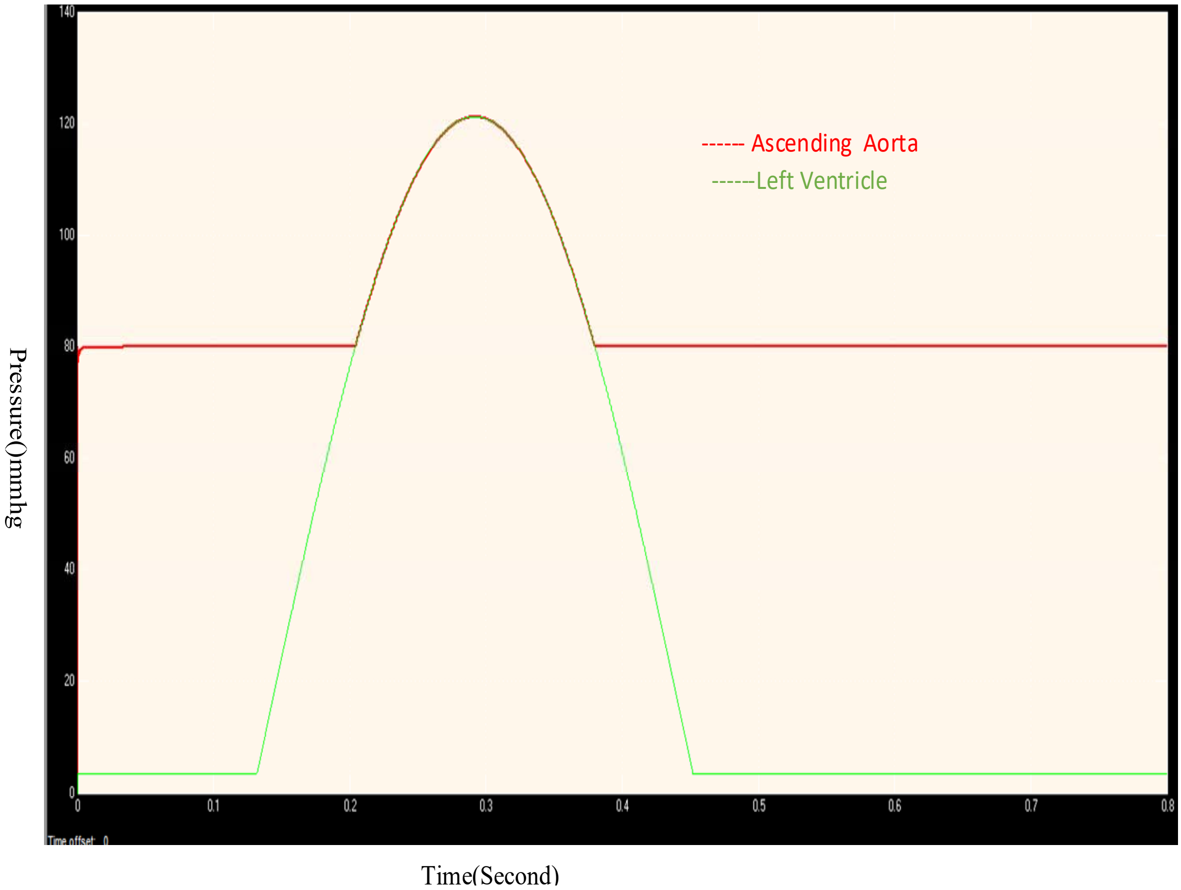

Fig. 10 Time vs Pressure Curve of Left Ventricle and Ascending Aorta

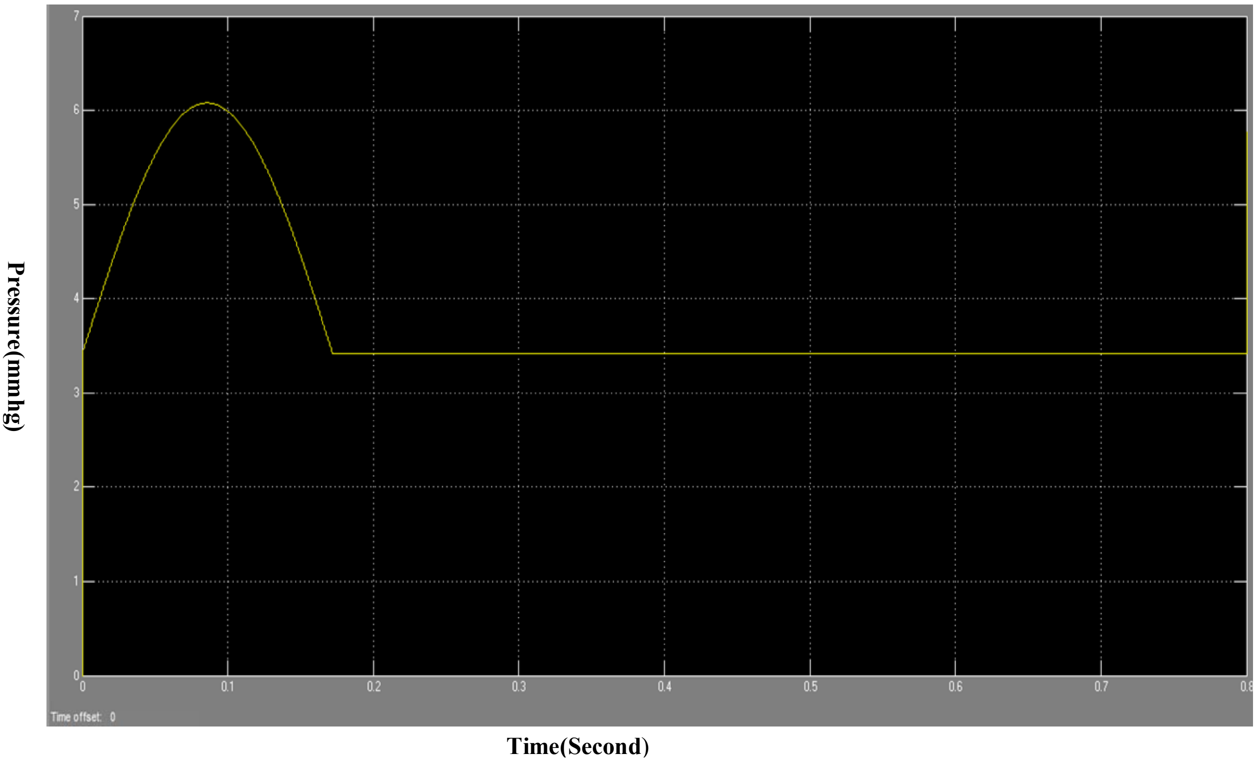

Fig. 11 Pressure vs Time curve for Right Atrium

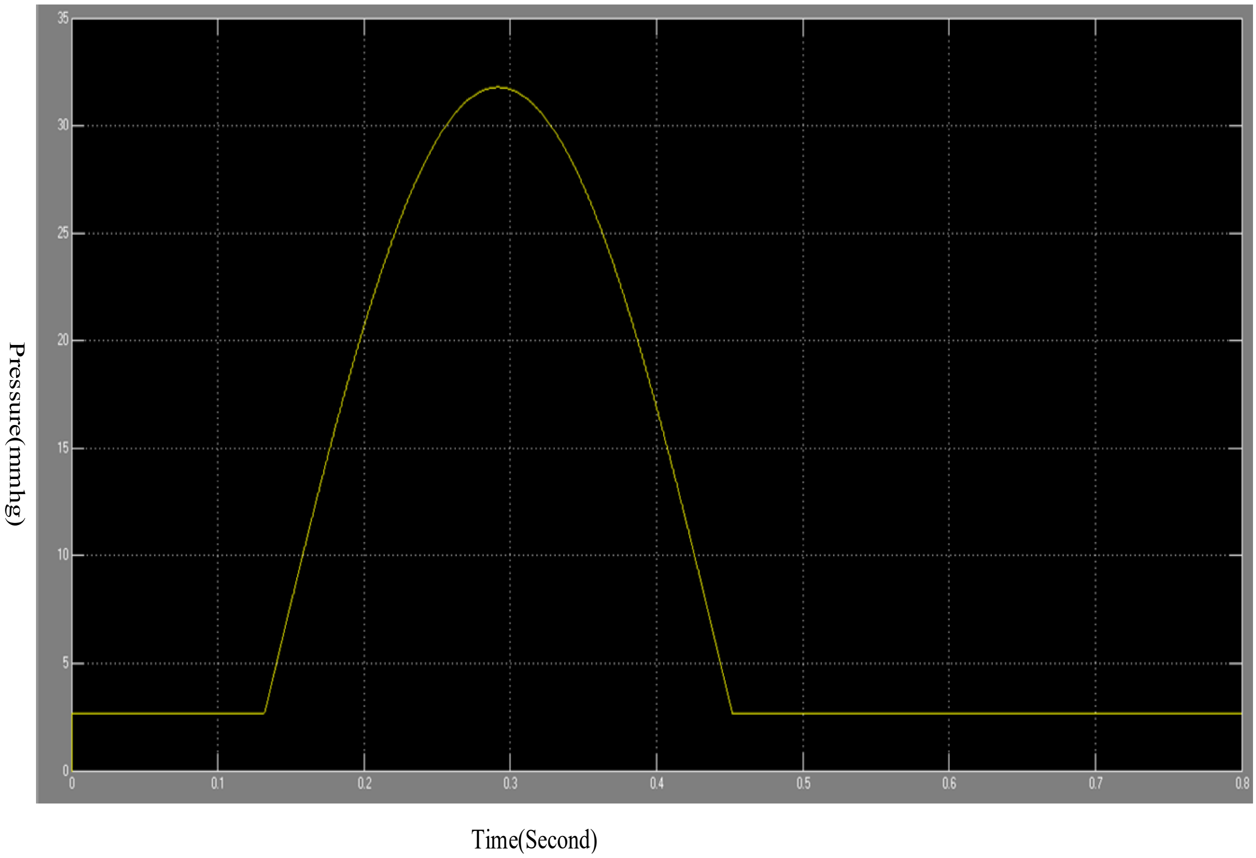

Fig. 12 Time vs Pressure Curve for Right Ventricle

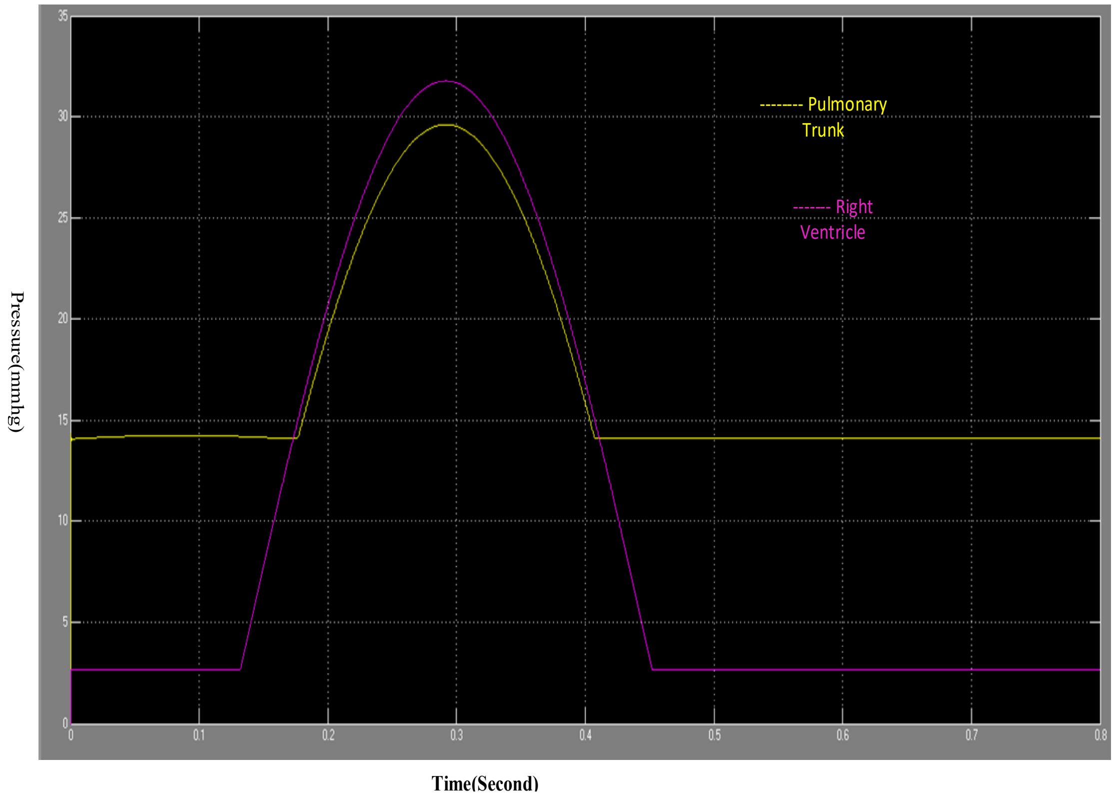

Fig. 13 Time vs Pressure Curve for Right Ventricle and Pulmonary Artery (Pulmonary Trunk)

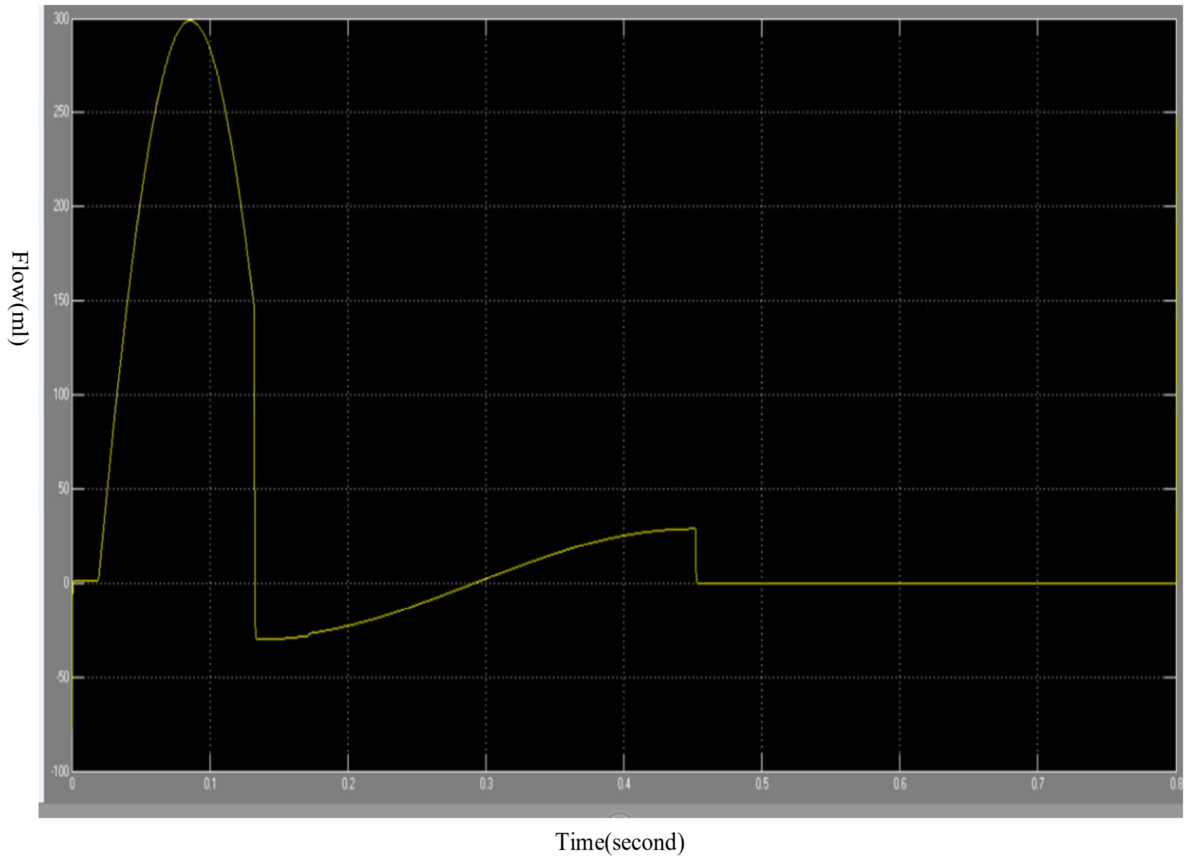

Fig. 14 Flow vs Time Curve for Left Atrium

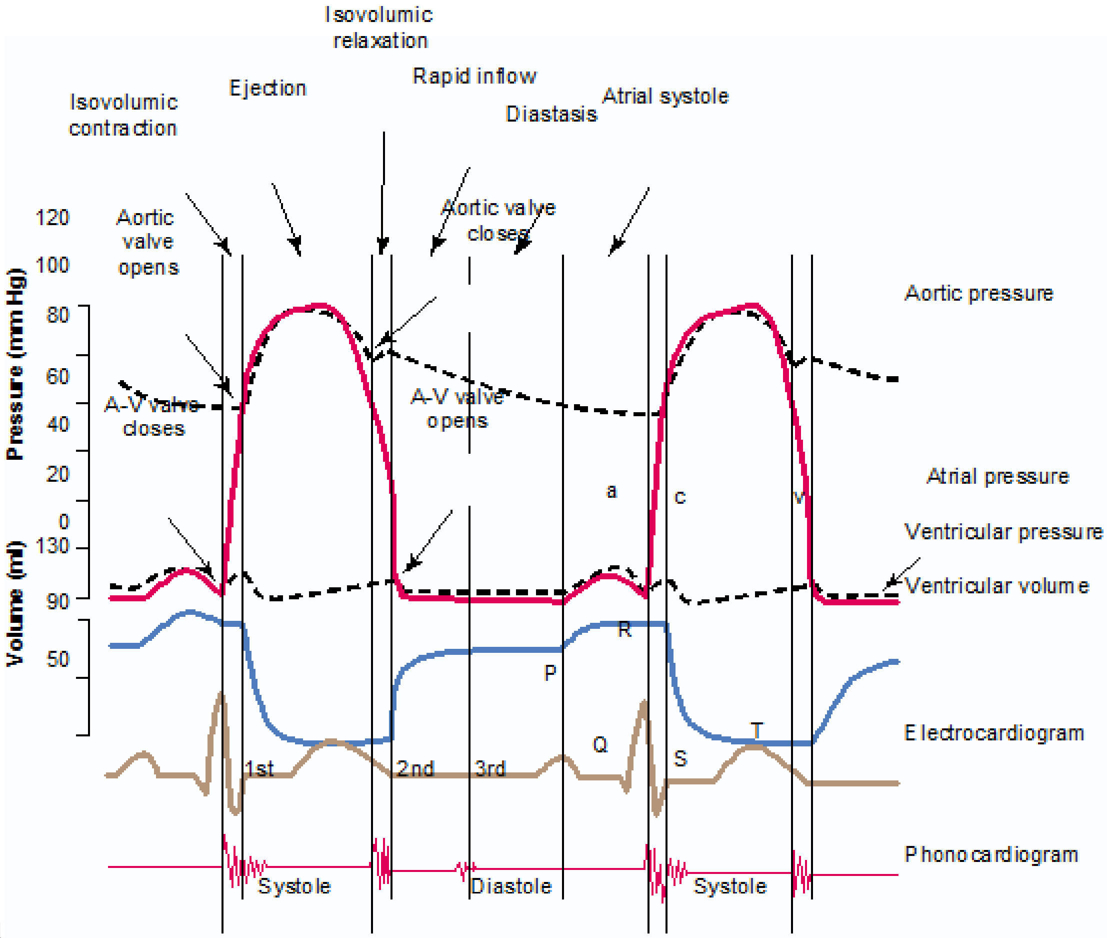

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Fig. 15 Events of the cardiac cycle for left ventricular function, showing changes in left atrial pressure, left ventricular pressure, aortic pressure ventricular volume, the electrocardiogram, and the phonocardiogram in [7]

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