

Simulation of the Large Hadrons Collisions Using Monte Carlo Tools

E. Al Daoud

Abstract—In many cases, theoretical treatments are available for models for which there is no perfect physical realization. In this situation, the only possible test for an approximate theoretical solution is to compare with data generated from a computer simulation. In this paper, Monte Carlo tools are used to study and compare the elementary particles models. All the experiments are implemented using 10000 events, and the simulated energy is 13 TeV. The mean and the curves of several variables are calculated for each model using MadAnalysis 5. Anomalies in the results can be seen in the muons masses of the minimal supersymmetric standard model and the two Higgs doublet model.

Keywords—Feynman rules, hadrons, Lagrangian, Monte Carlo, simulation.

I. INTRODUCTION

LARGE hadrons collider (LHC) enable us to probe the fundamental nature of matter by observing exotic particles produced by high-energy collisions. Typically, hundreds of particles are produced, and in most processes of interest, their momentum ranges over many orders of magnitude. All the particle species of the standard model (SM), and maybe some beyond, are involved [1]. A full analytical treatment is out of reach. There are two reasons for this: first, there is a limited understanding of the non-perturbative phase of quantum chromodynamics (QCD), or, in the other words, of how colorless hadrons are built from the colored quarks and gluons. Second, due to the large number of the Feynman diagrams even at the tree-level, it is very difficult to find all amplitudes of the involved processes, the number of the Feynman diagrams increases exponentially with the number of the particles that are involved in the experiment. Therefore, the simulation of signal and background processes plays a major role in searching and finding new phenomena at the LHC. Monte Carlo event simulation is the most powerful tool has been used to simulate the processes and the results of the high-energy collisions. Although, the most of the produced particles are undetectable due to its life time (decay time) is very short, but, by comparing the stable particles that are detected in the colliders with the simulated results, the properties of unstable particles can be reconstructed. This approach is used to confirm the existence of the undetectable and unstable particles such as Higgs boson [2]. Monte Carlo is very important in almost any particle physics experiment and are used to prototype the experiment without spending many

millions of dollars. They can be used to show that the proposed physics signal will be detectable among the many known physics effects and determine what kind of computation resources will be needed to manage the data. Moreover, there are tens of the elementary particles models, Monte Carlo simulation can be used to study and compare the distributions of the masses, energy and directions of the particles that are generated from each model. Any significant anomalies between the real data and the simulated data indicate that a new phenomenon and a new physics should be considered and a new model could be suggested. Although Monte Carlo has slow convergence in few dimensions, but it is much more efficient in computer time than the numerical integrations necessary over the innumerable functions entering the problems, detector, and theory. Once a Monte Carlo events sample is generated, it can be used "as if it is data" over and over again to get plots not thought up beforehand. Fig. 1 shows the relation between the real data and the generated data by using the Monte Carlo tools [3].

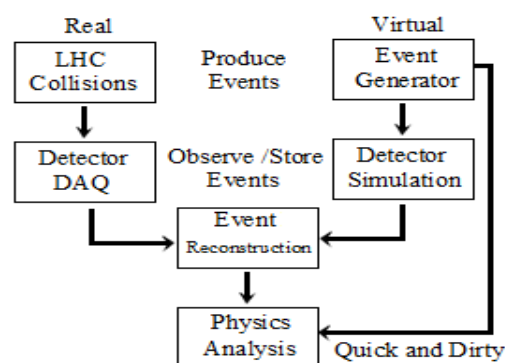


Fig. 1 The relation between real and virtual data

II. LAGRANGIAN

Technically, a particle physics model can be written in several different formulations, but, despite appearances, the Lagrangian is one of the easiest and most compact ways of presenting the theory. The Lagrangian can be derived from the difference of kinetic and potential energy. It captures all the dynamics of the system and allows us to determine many useful properties such as averages and dynamic behavior [4]. Lagrangian of any elementary particles model has several properties:

- (1) Invariant under the Poincaré group and certain symmetry groups,
- (2) Naturalness: all the symmetry terms can be a part from the Lagrangian,

Essam Al-Daoud is with the Faculty of Information Technology, Computer Science Department, Zarqa University, Jordan (phone: +96279668005, e-mail: essamdz@zu.edu.jo).

- (3) Translational invariance, therefore, it only depends on the fields and their derivatives,
(4) The total probability is conserved [5].

Equations (1) - (11) describe the Lagrangian of the standard model (SM), it is a mathematical description of four types of interactions: The strong interactions, the electromagnetic interactions, the weak interactions, and the Yukawa interactions. The first three types of interactions are mediated by vector-boson (spin-1) force carriers: eight massless gluons mediate the strong interactions, one massless photon mediates the electromagnetic interactions, and the three massive W^+ , W^- , and Z bosons mediate the weak interactions.

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermioni} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa} \quad (1)$$

$$\mathcal{L}_{gauge} = -\frac{1}{4}\bar{W}_{\mu\nu}\bar{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \quad (2)$$

$$\mathcal{L}_{fermioni} = \mathcal{L}_{lept} + \mathcal{L}_{quark} \quad (3)$$

$$\mathcal{L}_{Higgs} = \frac{1}{2}(D_\mu\phi)^2 - V(\phi) \quad (4)$$

$$\mathcal{L}_{Yukawa} = -G_i(\bar{L}\phi R + \bar{R}\phi^{T*}L) \quad (5)$$

$$V = a\phi^2 + b\phi^4 \quad (6)$$

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad (7)$$

$$\mathcal{L}_{lept} = \sum_{l=e,\mu,\tau}(\bar{\nu}_l, \bar{\ell})_L \gamma D_\mu \begin{pmatrix} \nu_l \\ \ell \end{pmatrix}_L + \bar{\ell}_R \gamma_\mu G_\mu \ell_R \quad (8)$$

$$\mathcal{L}_{quark} = \sum_{d=u,c,t} \sum_{d=d,s,b}(\bar{u}, \bar{d})_L \gamma D_\mu \begin{pmatrix} u \\ d \end{pmatrix}_L + \bar{u}_R \gamma_\mu G_\mu u_R + \bar{d}_R \gamma_\mu G_\mu d_R \quad (9)$$

$$D_\mu = \partial_\mu + i\frac{g}{2}\bar{\sigma}\bar{W} - i\hat{g}\frac{Y}{2}B_\mu \quad (10)$$

$$G_\mu = \partial_\mu - i\hat{g}\frac{Y}{2}B_\mu \quad (11)$$

Lagrangian can be used to obtain the type of the motion for a system by applying variational calculus to the action S which is the integral of the Lagrangian. For problems with expected symmetries or conserved quantities, we require that the Lagrangian remains invariant under the corresponding transformations. When a given transformation leaves the form of the Lagrangian invariant, we call that transformation a symmetry [6]. Noether's theorem allows us to derive conservation laws, including conserved charges and currents from symmetries in the Lagrangian. Despite the success of the standard model, it is incomplete theory. There are several phenomena, theoretical and experimental results are not explained by the standard model, such as: gravity, dark matter, dark energy, neutrino masses, proton radius puzzle, and hierarchy problem [7]. Thus, new models that would modify the standard model in ways subtle enough to be consistent with existing data are proposed. New models such as: Minimal Supersymmetric Standard Model (MSSM), two-Higgs-Doublet Model (2HDM), Higgs Effective Field Theory (HEFT), and Minimal Walking Technicolor (MWT).

III. FEYNMAN RULES

Lagrangian is used to deduce Feynman rules and calculate scattering amplitudes, and the input of the particle physics simulators are the Feynman rules and the related parameters. Therefore, FeynRules package has been used to extract the Feynman rules from any given Lagrangian, it provides all the necessary ingredients to generate the model file for several Monte Carlo tools. FeynRules package needs four main steps to generate Feynman rules: define the particles, describe the parameters, write the Lagrangian of a model and output all the information in the format required by the Monte Carlo code [8]. All the particles (symbols) that appear inside the Lagrangian must be properly declared. For example, physical particles W -boson and the upper quarks must be declared in the scope "M\$ClassesDescription" as following:

```
M$ClassesDescription = {
V[3] == {
  ClassName -> W,
  SelfConjugate -> False,
  Mass -> {MW, Internal},
  Width -> {WW, 2.085},
  ParticleName -> "W+",
  AntiParticleName -> "W-",
  QuantumNumbers -> {Q -> 1},
  PDG -> 24,
  PropagatorLabel -> "W",
  PropagatorType -> Sine,
  PropagatorArrow -> Forward,
  FullName -> "W"
},
F[3] == {
  ClassName -> uq,
  ClassMembers -> {u, c, t},
  Indices -> {Index[Generation], Index[Colour]},
  FlavorIndex -> Generation,
  SelfConjugate -> False,
  Mass -> {Mu, {MU, 2.55*^-3}, {MC, 1.27}},
  Width -> {0, 0, {WT, 1.50833649}},
  QuantumNumbers -> {Q -> 2/3},
  PropagatorLabel -> {"uq", "u", "c", "t"},
  PropagatorType -> Straight,
  PropagatorArrow -> Forward,
  PDG -> {2, 4, 6},
  ParticleName -> {"u", "c", "t"},
  AntiParticleName -> {"u~", "c~", "t~"},
  FullName -> {"u-quark", "c-quark", "t-quark"}
},
....
};
```

Each particle has several options, for example the decay rate (width) for w -boson is 2.085 and the quantum number (QuantumNumbers) is 1. The full description of the particles options is available in FeynRules manual [9]. The Lagrangian parameters can be declared in the class $M\$Parameters$. For example, the electric coupling constant and the Cabibbo angle can be declared as following:

```
M$Parameters = {
```

```

ee == {
ParameterType -> Internal,
Value -> Sqrt[4 Pi aEW],
InteractionOrder -> {QED,1},
TeX -> e,
Description -> "Electric coupling constant"
},

cabi == {
ParameterType -> External,
BlockName -> CKMBLOCK,
OrderBlock -> 1,
Value -> 0.227736,
TeX -> Subscript[φ, c],
Description -> "Cabibbo angle"
},
.....
};
    
```

The option Parameter Type can be internal or external; if the value is internal (as the electric coupling constant), it means that the constant depends on other parameters (in this case, it depends on the electroweak constant). The third step is the Lagrangian of the model, and it can be written using Mathematica and FeynRules functions. For example, The kinetic terms for the new scalars:

$$\frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 - \frac{1}{2} m_1^2 \phi_1^2 \quad (12)$$

$$i \bar{U} \gamma^\mu D_\mu U - M_U \bar{U} U \quad (13)$$

which can be written as

$$1/2 \text{del}[p1, mu] \text{del}[p1, mu] - 1/2 M M1^2 p1^2$$

$$I \text{uvbar}.Ga[mu].DC[uv, mu] - Muvuvbar.uv$$

where $\text{del}[..., mu]$ is the space-time derivative, $Ga[mu]$ is the FeynRules symbol for the Dirac matrix (γ^μ), DC is the covariant derivative and bar is added to the anti-particles. The Feynman rules of the new sector can be obtained by the function FeynmanRules[LNew]. The last step exports a given model directly to various matrix element generators by using the FeynRules interfaces such as WriteUFO[LNew] for UFO format [10].

TABLE I
 THE NUMBER OF PROCESSES FOR EACH COLLISION

Collision	HEFT	SM	2HDM	MSSM
pp>all all	75	75	935	304
pp> j jj	97	97	293	97
pp> a a	5	4	4	4
pp>leptlept~	20	20	44	20
pp > l+ l-	8	8	16	8
pp>l+ l+ l- l-	15	12	72	12
p> all all	41	39	143	152

IV. SIMULATION RESULTS

In this section, four models are studied and compared: The standard model (SM), Higgs effective field theory (HEFT), two-Higgs doublet model (2HDM), and minimal supersymmetric standard model (MSSM). MadGraph 5 is used to mimic what happens in the detectors, while MadAnalysis 5 is used to draw the curves and interpret the results [11]-[13]. Table I and II show the number of processes and diagrams generated by the four models.

TABLE II
 THE NUMBER OF DIAGRAMS FOR EACH COLLISION

Collision	HEFT	SM	2HDM	MSSM
pp>all all	126	126	4058	501
pp> j jj	781	781	7565	781
pp> a a	9	8	8	8
pp>leptlept~	28	28	124	28
pp > l+ l-	16	16	112	16
pp>l+ l+ l- l-	490	480	34512	480
p> all all	41	39	155	152

TABLE III
 THE MEAN FOR EACH MEASUREMENT

Variable	HEFT	SM	2HDM	MSSM
PT (I[1])	37.0	37.0	36.5	36.4
ETA (I[2])	0.007	0.007	0.026	0.001
M(e+ e-)	47.2	47.2	47.3	46.2
M (mu+ mu-)	15.1	15.1	4.0	7.4
Cross Section	843.6	843.6	852.4	851.9

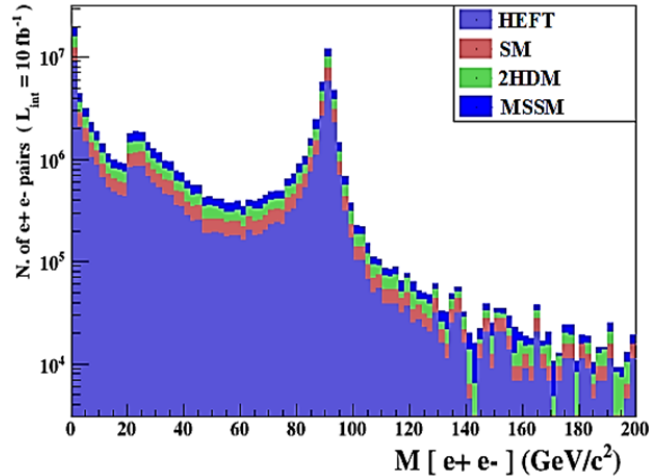


Fig. 2 The distribution of two electrons invariant mass

To compare the models, 10000 events are generated for each experiment with center-of-mass energy 13 TeV. Table III shows the mean of the momentum transverse of the leading lepton, the pseudo-rapidity of the next to leading lepton, two electrons invariant mass, two muons invariant mass and the cross section for each model. The distribution of the previous variables is described in Figs. 2-5. The results indicate that the most significant difference is the distribution of two muons invariant mass.

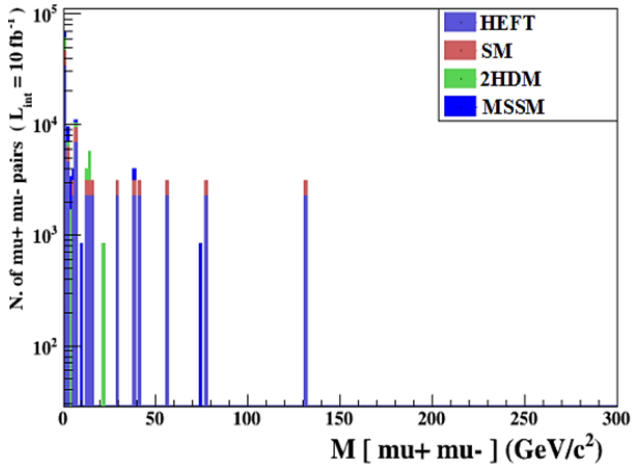


Fig. 3 The distribution of two muons invariant mass

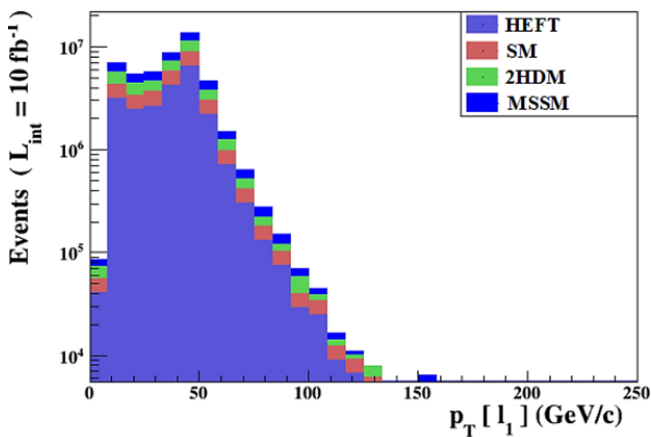


Fig. 4 The distribution of the momentum transverse of the leading lepton

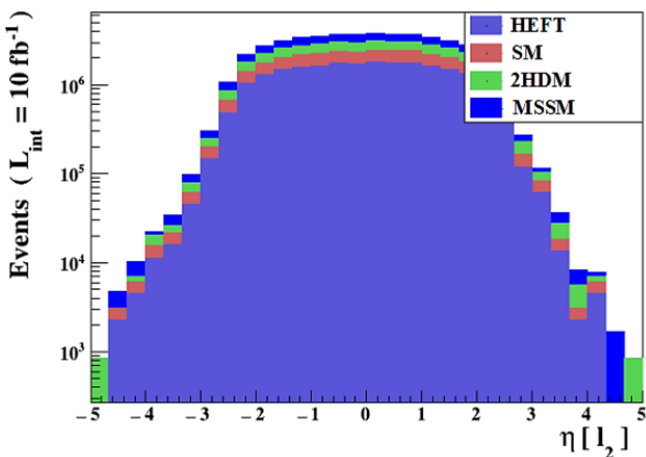


Fig. 5 The distribution of the pseudo-rapidity of the next to leading lepton

V. CONCLUSION

With Monte Carlo tools, it became possible to carry out simulations of models which were intractable using theoretical techniques. As a result, computer simulations have become

another way to understand the experimental results, and in other instances, simulations provide data with which theoretical models may be compared. In this study, three tools are used to compare the elementary particles models. The FeynRules package is used to generate the Feynman Rules, MadGraph 5 is used to mimic what happens in the detectors, and MadAnalysis 5 is used to draw the curves and interpret the results.

ACKNOWLEDGMENT

This research is funded by the Deanship of Scientific Research in Zarqa University /Jordan.

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