

# Optimal Tuning of Linear Quadratic Regulator Controller Using a Particle Swarm Optimization for Two-Rotor Aerodynamical System

Ayad Al-Mahturi, Herman Wahid

**Abstract**—This paper presents an optimal state feedback controller based on Linear Quadratic Regulator (LQR) for a two-rotor aero-dynamical system (TRAS). TRAS is a highly nonlinear multi-input multi-output (MIMO) system with two degrees of freedom and cross coupling. There are two parameters that define the behavior of LQR controller: state weighting matrix and control weighting matrix. The two parameters influence the performance of LQR. Particle Swarm Optimization (PSO) is proposed to optimally tune weighting matrices of LQR. The major concern of using LQR controller is to stabilize the TRAS by making the beam move quickly and accurately for tracking a trajectory or to reach a desired altitude. The simulation results were carried out in MATLAB/Simulink. The system is decoupled into two single-input single-output (SISO) systems. Comparing the performance of the optimized proportional, integral and derivative (PID) controller provided by INTECO, results depict that LQR controller gives a better performance in terms of both transient and steady state responses when PSO is performed.

**Keywords**—Linear quadratic regulator, LQR controller, optimal control, particle swarm optimization, PSO, two-rotor aero-dynamical system, TRAS.

## I. INTRODUCTION

VARIOUS methods have been developed to control aerodynamic systems [1]. The control of aerodynamic systems such as helicopters has become one of the most challenging engineering problems due to its nonlinearities and strong cross coupling between its parameters [2]. TRAS is a highly nonlinear with magnificent cross couplings, highly interactive and complex system [1]-[8]. Conventional controllers such as PID are being used widely in industries due to its simplicity. However, tuning PID using classical approaches such trial and error is a tedious practice and does not guarantee the desired performance [4], [8]. In [5], it was found that tuning PID parameters can be done using a machine learning but the robust performance is not guaranteed. Several control methods have been used to improve the robustness such as robust deadbeat control [8], sliding mode control technique [3], and intelligent control [7], [9]-[12]. Moreover, genetic algorithm (GA) is used in various applications as an optimization technique, but it is time consuming as it searches

for the optimal value from a population of points rather than one point [2]. In addition, the conventional optimization approaches aim only to minimize the cost function and do not consider other control objectives such reducing overshoot, settling time, rise time and steady state error. On the other hand, PSO technique is simple, gives satisfactory results and takes into account the control objectives [13].

Feedback control systems are extensively used in hardware applications to increase the efficiency and reliability. In practical, one of the most efficient techniques that is used in order to improve the performance of complex processes is to increase the number of sensors and actuators which lead to a MIMO system. Hence, it is required for any feedback control method to have the ability to handle multiplicity of sensors and actuators. LQR optimal control is a great achievement in modern feedback control that handles MIMO systems with respect to a quadratic cost function [14]. Two parameters that determine the behavior of LQR controller are: state and control weighting matrices. These two parameters influence the performance of LQR significantly and must be optimally adjusted in order to get the desired performance. Moreover, tuning LQR parameters using classic approaches such as trial and error, pole placement, and Bryson's method is a tedious work and time consuming. Thus, PSO is proposed to optimally tune the weighting matrices of LQR controller.

In [5], an optimal control with integral action is investigated. It was noticed that LQI controller gives better results compared to sliding mode control. However, the weight matrices of LQR were tuned manually which do not give the optimal value. Similarly, [15] has chosen a random value of Q and R and varied them till the desired performance is met.

In this work, a linear model of TRAS has been developed. The designed LQR controller using PSO is also investigated. In order to provide a point of comparison, an optimized PID controller provided by the manufacturer is used. Step and square waves input were used to analyze the system in terms of transient and steady state responses. TRAS with 2-DOF is decoupled into two independent SISO subsystems; 1-DOF for vertical plane and 1-DOF for horizontal plane. The cross coupling between the parameters is considered as a disturbance.

This study is trying to address the need of an optimization technique to tune controller parameters. The main objective of this research work is to stabilize the TRAS by making the

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beam move quickly for tracking a trajectory or to reach a desired altitude.

The paper is structured as follows: In first section, a summary of pervious works was addressed. Section II describes the modeling of TRAS, followed by the principle of PSO algorithm in section III. In section IV, an optimal control method is presented. In the last two sections, simulation results are described in graphs and tables using MATLAB/Simulink followed by the conclusion section.

## II. TRAS SYSTEM DESCRIPTION AND MODELLING

A laboratory set-up called TRAS is designed by INTECO company in Poland, that resembles helicopters in certain aspects shown in Fig. 1 [11]. The difference between the helicopter and TRAS is that the control in helicopter is done by changing the angle of attack, while in TRAS controlling is obtained by changing the speed of the rotors and the angle of attack is designed to be fixed. Hence, the supply voltage of the DC motor is the control inputs to TRAS.

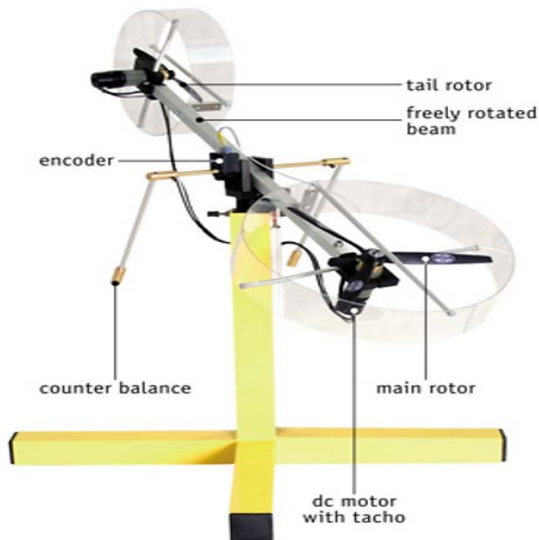


Fig. 1 TRAS set-up [16]

TRAS has two rotors known as main and tail rotors which are driven by two DC motors with two propellers that are identical to each other joined by beam pivoted on its base that can rotate freely in both horizontal and vertical planes, known as yaw and pitch, respectively [1]. If necessary, either one or both axes of rotation can be locked by means of two locking screws provided for physically restricting the horizontal or vertical plane rotation. Hence, the system allows both one and two DOF experiments [8]. The joined beam can be moved by changing the input voltage and controlling the rotation speed of these two propellers. Two counterbalance arms with a weight at their ends are fixed to the beam at the pivot, that determines the steady-state pitch angle without propeller actuation [3]. The four state variables can be described as: yaw and pitch angles that measured by position sensors, and yaw and pitch angular velocities of the beam. Two tachometers are coupled to the driving DC motors to measure the other

additional velocities of the rotors. The detailed mathematical model of TRAS is given in [16]. Table I defines TRAS parameters.

TABLE I  
PARAMETERS DEFINITIONS OF TRAS

| Symbol                         | Parameter definition   |
|--------------------------------|--|
| $\alpha_h, \alpha_v$           | Horizontal and vertical angular position of TRAS beam (rad)                        |
| $\Omega_h, \Omega_v$           | Horizontal and vertical angular velocity of TRAS beam (rad/s)                      |
| $U_h, U_v$                     | horizontal and vertical DC-motor PWM control input                                 |
| $\omega_h, \omega_v$           | rotational speed of tail and main rotor (rad/s) - nonlinear function               |
| $F_h(\omega_h), F_v(\omega_v)$ | nonlinear aerodynamic force from tail and main rotor (N)                           |
| $l_h(\alpha_v)$                | effective arm of aerodynamic force from tail rotor (m)                             |
| $J_h(\alpha_v)$                | nonlinear function of moment of inertia wrt to vertical axis, (kg m <sup>2</sup> ) |
| $M_h, M_v$                     | Horizontal and vertical turning torque (Nm)  |
| $K_h, K_v$                     | Horizontal and vertical angular momentum (N m s)                                   |
| $f_h, f_v$                     | moment of friction force in vertical and horizontal axis (N m)                     |
| $l_v$                          | arm of aerodynamic force from main rotor (m)                                       |
| $J_v$                          | moment of inertia with respect to horizontal axis (kg m <sup>2</sup> )             |
| $R_v$                          | vertical returning moment (Nm)   |
| $J_{hv}$                       | vertical angular momentum from tail rotor (Nms)                                    |
| $J_{vh}$                       | horizontal angular momentum from main rotor (Nms)                                  |

### A. Nonlinear Model

The mathematical model of the main rotor is presented in (1)-(3):

$$\frac{d\Omega_v}{dt} = \frac{1}{J_v} [l_m F_v(\omega_m) - \Omega_v k_v + g((A - B)\cos\alpha_v - C\sin\alpha_v)] + \frac{1}{J_v} \left[ -\frac{1}{2} \Omega_h^2 (A + B + C) \sin 2\alpha_v U_h k_{hv} + U_h k_{hv} - a_1 \Omega_v \text{abs}(\omega_v) \right] \quad (1)$$

$$\Omega_v = \frac{d\alpha_v}{dt} \quad (2)$$

$$I_v \frac{d\omega_v}{dt} = U_v - H_v^{-1}(\omega_v) \quad (3)$$

$$\omega_v \cong -5.2 \times 10^3 U_v^7 - 1.1 \times 10^2 U_v^6 + 1.1 \times 10^4 U_v^5 + 1.3 \times 10^2 U_v^4 - 9.2 \times 10^3 U_v^3 - 31 U_v^2 + 6.1 \times 10^3 U_v - 4.5 \quad (4)$$

$$F_v \cong -1.8 \times 10^{-18} \omega_v^5 - 7.8 \times 10^{-16} \omega_v^4 + 4.1 \times 10^{-11} \omega_v^3 + 2.7 \times 10^{-8} \omega_v^2 + 3.5 \times 10^{-5} \omega_v - 0.014 \quad (5)$$

$$A = (m_{ts} + m_{tr} + \frac{m_t}{2}) l_t ; B = (m_{mr} + m_{ms} + \frac{m_m}{2}) l_m ; C = \left( \frac{m_b}{2} l_b + m_{cb} l_{cb} \right)$$

The mathematical model of tail rotor is addressed in (6)-(8):

$$\frac{dK_h}{dt} = \frac{l_t F_h(\omega_h) \cos\alpha_v - \Omega_h k_h + U_v k_{vh} - a_2 \Omega_h \text{abs}(\omega_h)}{D \sin^2 \alpha_v + E \cos^2 \alpha_v + F} \quad (6)$$

$$\frac{d\alpha_h}{dt} = \Omega_h, \quad \Omega_h = \frac{K_h}{J_h(\alpha_v)} \quad (7)$$

$$I_h \frac{d\omega_h}{dt} = U_h - H_h^{-1}(\omega_h) \quad (8)$$

$$\omega_h \cong -2.2x10^3U_h^5 - 1.7x10^2U_h^4 - 4.5x10^3U_h^3 + 3x10^2U_h^2 + 9.8x10^3U_h - 9.2 \quad (9)$$

$$F_h \cong -2.6x10^{-20}\omega_h^5 + 4.1x10^{-17}\omega_h^4 + 3.2x10^{-12}\omega_h^3 - 7.3x10^{-9}\omega_h^2 + 2.1x10^{-5}\omega_h + 0.0091 \quad (10)$$

where,

$$D = \left(\frac{m_b}{3}l_b^2 + m_{cb}l_{cb}^2\right);$$

$$E = \left(\frac{m_m}{3} + m_{mr} + m_{ms}\right)l_m^2 + \left(\frac{m_t}{3} + m_{tr} + m_{ts}\right)l_t^2;$$

$$F = \left(m_{ms}r_{ms}^2 + \frac{m_{ts}}{2}r_{ts}^2\right).$$

### B. Linearized Model

Implementing LQR controller in TRAS requires a linear model. In this paper, the model is linearized using 'linmod' MATLAB function with some assumptions. The linearization is done with respect to an operating point (0,0) using Taylor series expansion. Assuming  $\Delta\theta = \theta - \bar{\theta}$  is small, the linear approximation of  $f(\theta)$  is achieved by retaining the first two terms, while neglecting the remaining terms with higher powers of  $\theta - \bar{\theta}$  as shown in (11):

$$f(\theta) = f(\bar{\theta}) + \frac{df}{d\theta} \Big|_{\bar{\theta}} \Delta\theta \quad (11)$$

In trigonometric functions linearization such as, in our case, the mathematical equations for rotational motion, the angle  $\theta$  is assumed to be small. Therefore,  $\sin \theta \cong \theta$ ,  $\cos \theta \cong 1$  and  $\theta\dot{\theta}^2 \cong 0$  [17].

From (1)-(3), the linearized model of the main rotor was obtained in transfer function form as:

$$\frac{\alpha_v}{U_v} = \frac{0.3916}{s^3 + 1.7437s^2 + 3.6725s + 4.1131} \quad (12)$$

Similarly, from (6)-(8), the linearized model of the tail rotor in horizontal plane is given in transfer function form as:

$$\frac{\alpha_h}{U_h} = \frac{7.2058}{s^3 + 5.8984s^2 + 1.2467s} \quad (13)$$

### III. PRINCIPLE OF PSO ALGORITHM

The principle of PSO optimization technique is to search for the best solution in n-dimension in the search space. That can be done by a collaborative share between each individual particle in a swarm. PSO was introduced by James Kennedy and Russell Eberhart and it is used for optimizing nonlinear control systems for its satisfactory results. For each step in PSO, particles have two main operators; position  $x_i$  and velocity  $v_i$ . In each iteration, the position and velocity are updated according to (14) and (15).

$$v_{ij}(t+1) = \gamma v_{ij}(t) + c_1 r_1 (p_{ij} - x_{ij}(t)) + c_2 r_2 (g_j - x_{ij}(t)) \quad (14)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \quad (15)$$

$\gamma$  is the inertia weight factor,  $c_1$  and  $c_2$  are the acceleration coefficients. For each iteration, a new velocity value for each particle is updated according to its current velocity, distance from its previous best position, and distance from the global best position. The next position of a particle is calculated from the new velocity value in the search space. The process is repeated until a satisfactory result is obtained. In this paper, PSO is used to search for the best weighting matrices R and Q of LQR controller [13], [18], [19].

### IV. OPTIMAL CONTROL

#### A. LQR Controller

In modern control theory, LQR is used to analyze such a system in state-space representation. Using state-space approach is relatively simple especially when it comes to MIMO systems. LQR controller is used to obtain the best control sequence that minimizes quadratic cost function which is given by:

$$J = \int_0^{\infty} (x(t)^T Q x(t) + u(t)^T R u(t)) dt \quad (16)$$

The control energy is represented by  $u(t)^T R u(t)$ , while the transient energy is expressed as  $x(t)^T Q x(t)$ . R is a positive definite matrix, and Q is a weighting matrix that can be positive semi-definite. The control signal  $u(t)$  represents the optimal control that controls the signal, which is

$$u(t) = -Kx(t) = -R^{-1}B^T P x(t) \quad (17)$$

K is the linear optimal feedback matrix. P is the solution of Riccati equation which can be found from (18):

$$PA + A^T - PBR^{-1}P + Q = 0 \quad (18)$$

By considering the closed loop system as asymptotically stable, P is an optimal matrix. PSO is used to obtain the best Q and R values in order to obtain the feedback controller matrix, K.

#### B. LQR with Integral Action

To reduce steady state error, an integral action is added to the system. Integral action is also used to cancel the disturbance input at steady state. Full state feedback LQR with reference input and integral action is shown in Fig 2. The constant gain Nbar is used as pre-compensator and it is a user-defined function written in m-file code.

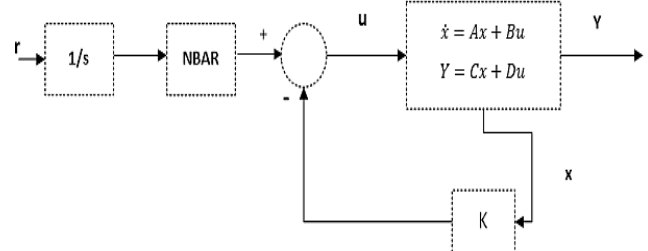


Fig. 2 Full state feedback with reference input and Nbar

### V. SIMULATION RESULTS

In PSO, the swarm size is chosen to be 50 for both pitch and yaw position control. The range of the search must be initialized when using PSO technique. The maximum iteration is the stopping condition that determines the convergence of the values. Choosing large iteration number requires fast processor to finish the task.

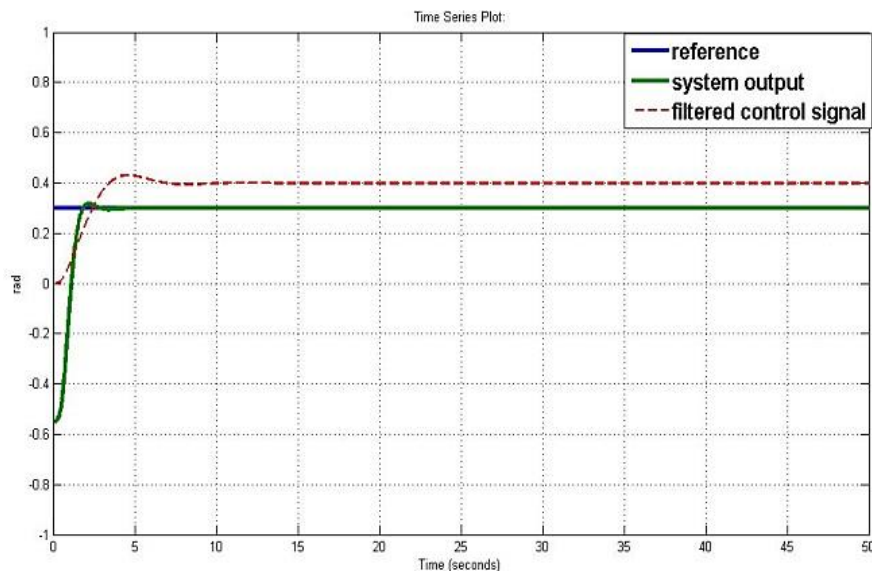
After finding the best  $Q$  and  $R$  values offline, the controller gain matrix can be found using LQR MATLAB function;  $K = lqr[A, B, Q, R]$ .

Fig. 3 illustrates the responses of TRAS in vertical plane. Step reference with 0.3 rad is presented. The initial condition is -0.5 rad for the compensation of gravity as in [4]. For evaluating the tracking performance, a square wave with a frequency of 0.025 Hz is also presented in Fig. 3 (b).

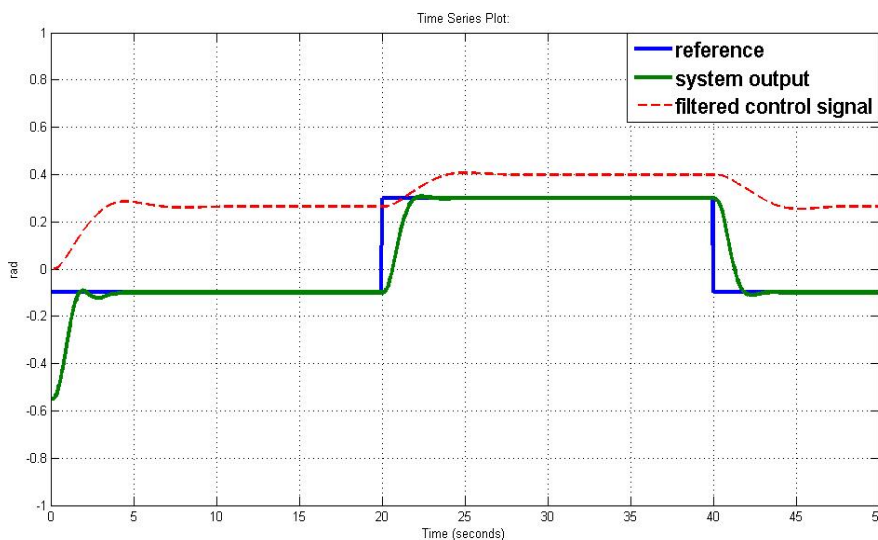
Simulation results in [16] show that the settling time is 7.58 s with an overshoot of 27.73%, whereas using LQR, the settling time is reduced to 1.7257 s and the overshoot is minimized to 1.9140% as summarized in Table II. The filtered control signal is also plotted in both figures. Compared with a step input in [16], as shown in Fig. 3 (c), the settling time, rise time, and overshoot are reduced significantly.

TABLE II  
 STEP REFERENCE PERFORMANCE CHARACTERISTICS

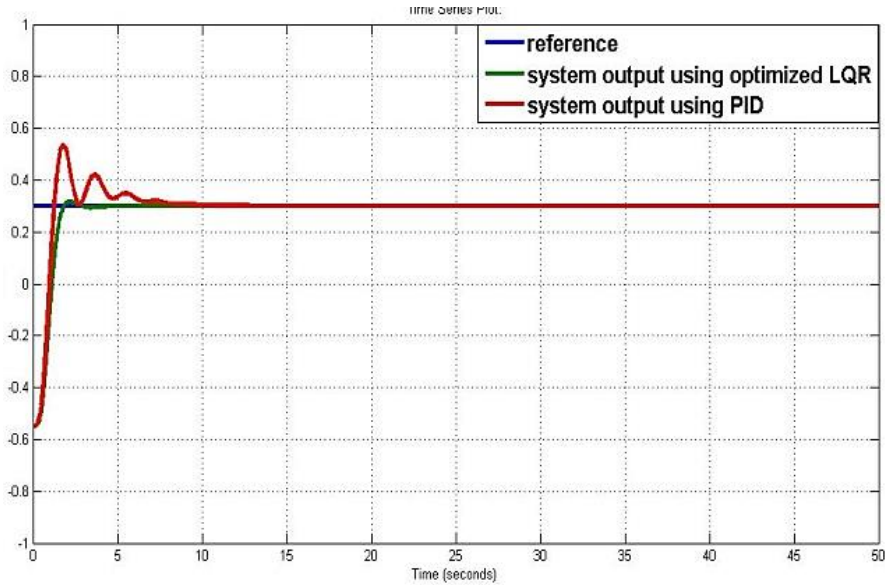
| Controller Type | Plane      | Step reference value | Rise time (s) | Settling time (s) | Overshoot (%) | Steady State Error |
|-----------------|------------|----------------------|---------------|-------------------|---------------|--------------------|
| PID [16]        | Horizontal | 1.0                  | 1.20          | 3.4722            | 0.03          | 0.0                |
|                 | Vertical   | 0.3                  | 1.24          | 7.5817            | 27.7323       | 0.0                |
| LQR             | Horizontal | 1.0                  | 1.18          | 2.2326            | 0.0           | 0.0                |
|                 | Vertical   | 0.3                  | 0.85          | 1.7257            | 1.9140        | 0.0                |



(a)



(b)



(c)

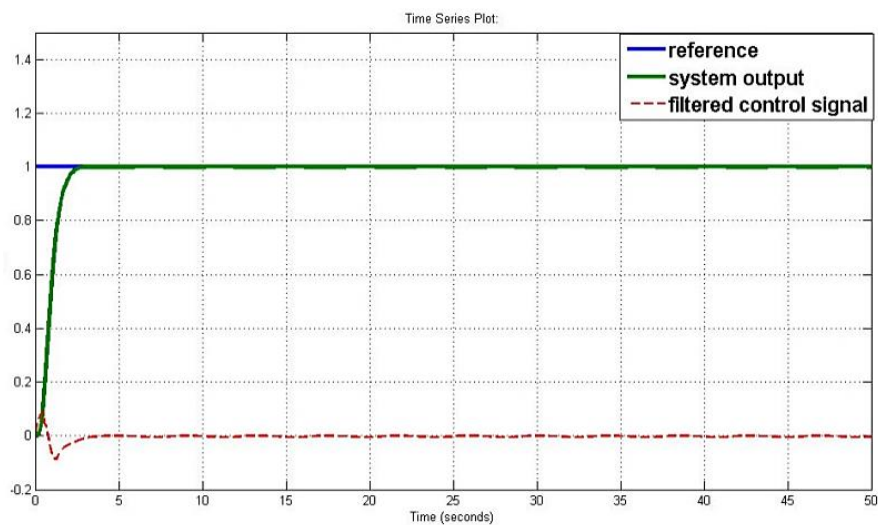
Fig. 3 System output in vertical plane. (a) Step response using LQR (b) Square wave response using LQR (c) Comparison of responses using PID and optimized LQR

For yaw position control, a step reference with 1.0 rad and initial condition of 0.0 rad are investigated shown in Fig. 4. A square wave with a frequency of 0.025 Hz is given for evaluating the tracking of the desired trajectory. It can be seen in Fig. 4 that the settling time is reduced from 3.4722 s as in [16] to 2.2326 s with 0.0% overshoot which summarized in Table II. Fig. 4 (c) illustrates the comparison of the response using PID and LQR controllers.

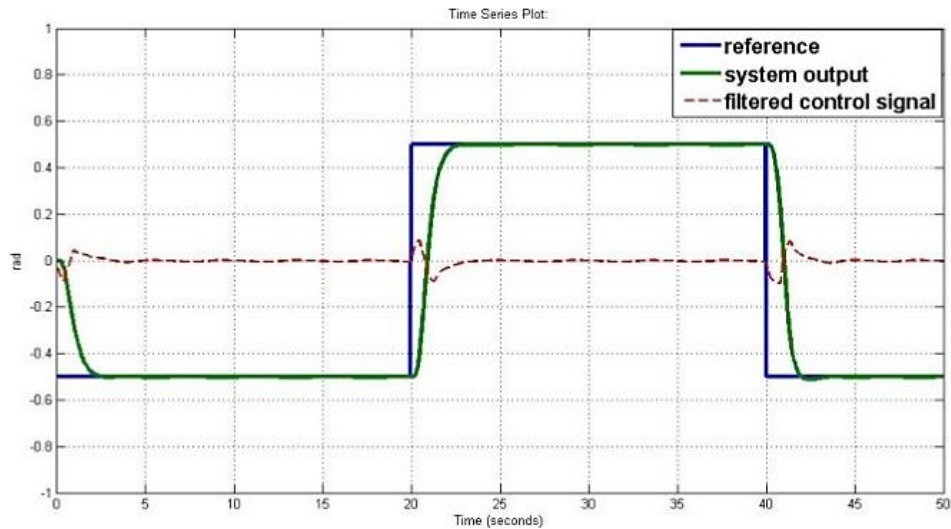
### VI. CONCLUSION

In this paper, the mathematical model of TRAS has been developed. The model is then decoupled into two SISO

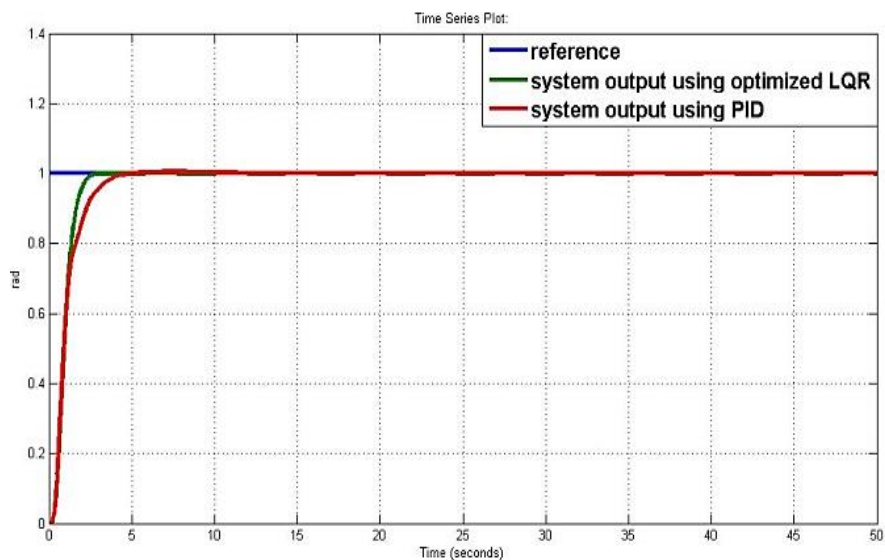
systems and cross coupling is considered as a disturbance. To control TRAS system, PSO is employed for tuning the weighting matrices of LQR controller. The performance of LQR controller has been evaluated using step and square wave inputs. The response of the LQR controller with integral action has also been compared with the existing PID controller tuned by manufacturer. Results in simulation illustrate that there is a magnificent reduction in overshoot percentage for vertical plane, and the settling time has also been minimized remarkably for both vertical and horizontal planes using the proposed LQR controller.



(a)



(b)



(c)

Fig. 4 System output in horizontal plane (a) Step response using LQR (b) Square wave response using LQR (c) Comparison of responses using PID and optimized LQR

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#### REFERENCES

- [1] J.G. Juang, R.W. Lin, and W.K. Liu, "Comparison of classical control and intelligent control for a MIMO system," *Applied Mathematics and Computation*, vol. 205, no. 2, pp. 778-791, 2008.
- [2] G.D. Prasad, P.S. Manoharan, and A.P.S. Ramalakshmi, "PID control scheme for twin rotor MIMO system using a real valued genetic algorithm with a predetermined search range," in *Power, Energy and Control (ICPEC)*, 2013.
- [3] S.S. Butt, and H. Aschemann, "Multi-Variable Integral Sliding Mode Control of a Two Degrees of Freedom Helicopter," *IFAC-PapersOnLine*, vol. 48, no. 1, pp. 802-807, 2015.
- [4] J.G. Juang, M.T. Huang, and W.K. Liu, "PID Control Using Presearched Genetic Algorithms for a MIMO System," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 38, no. 5, pp. 716-727, 2008.
- [5] A. Phillips, and F. Sahin, "Optimal control of a twin rotor MIMO system using LQR with integral action," *World Automation Congress (WAC)*, 2014.
- [6] Rahideh and M.H. Shaheed, "Real time hybrid fuzzy-PID control of a twin rotor system," *IEEE International Conference*, 2009.
- [7] C.W. Tao, J.S. Taur, Y.H. Chang and C-W. Chang, "A Novel Fuzzy-Sliding and Fuzzy-Integral-Sliding Controller for the Twin-Rotor Multi-Input Multi-Output System," *IEEE Transactions on Fuzzy Systems*, vol. 18, no. 5, pp. 893-905, 2010.
- [8] P. Wen and T.W. Lu, "Decoupling control of a twin rotor MIMO system using robust deadbeat control technique," *IET Control Theory & Applications*, vol. 2, no. 11, pp. 999-1007, 2008.
- [9] A. Boulkroune, M. M'Saad, and H. Chekireb, "Design of a fuzzy adaptive controller for MIMO nonlinear time-delay systems with unknown actuator nonlinearities and unknown control direction," *Information Sciences*, vol. 180, no. 24, pp. 5041-5059, 2010.

- [10] J. Juang, W. Liu, and R. Lin, "A hybrid intelligent controller for a twin rotor MIMO system and its hardware implementation," *ISA Transactions*, vol. 50, no. 4, pp. 609-619, 2011.
- [11] A. Rahideh and M.H. Shaheed, "Hybrid Fuzzy-PID-based Control of a Twin Rotor MIMO System," *In IEEE Industrial Electronics, IECON, 32nd Annual Conference 2006*.
- [12] C.W. Tao, J.S. Taur, and Y.C. Chen, "Design of a parallel distributed fuzzy LQR controller for the twin rotor multi-input multi-output system," *Fuzzy Sets and Systems*, vol. 161, no. 15, pp. 2081-2103, 2010.
- [13] J. Thomas, "Particle swarm optimization based model predictive control for constrained nonlinear systems," *in Informatics in Control, Automation and Robotics (ICINCO), 11th International Conference*. 2014.
- [14] L. Chrif and Z.M. Kadda, "Aircraft Control System Using LQG and LQR Controller with Optimal Estimation-Kalman Filter Design," *Procedia Engineering*, vol. 80, pp. 245-257, 2014.
- [15] S.K. Pandey and V. Laxmi, "Optimal Control of Twin Rotor MIMO System Using LQR Technique," *in Computational Intelligence in Data Mining - Volume 1: Proceedings of the International Conference on CIDM*, pp. 11-21, Dec. 2014.
- [16] "INTECO", *Two Rotor Aero-Dynamical System User's Manual, 2013*; Available: <http://www.inteco.com.pl/products/two-rotor-aerodynamical-system/>. Accessed: Oct, 15, 2016.
- [17] R.S. Esfandiari and B. Lu, *Modeling and Analysis of Dynamic Systems*. Second Edition. CRC Press, 2014.
- [18] Y. Han, Q. Li, H. Yang and W. Chen, *Design optimal temperature control system based on effective informed adaptive particle swarm optimization for proton exchange membrane fuel cell*, 35th Chinese Control Conference (CCC), 2016.
- [19] S.K. Meena and S. Channa, *Load Frequency Control of multi area system using Hybrid Particle Swarm Optimization*. 2nd International Conference on Recent Advances in Engineering & Computational Sciences (RAECS), 2015.