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Nonlinear Dynamic Analysis of Base-Isolated Structures Using a Mixed Integration Method: Stability Aspects and Computational Efficiency

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***Abstract*—**In order to reduce numerical computations in the nonlinear dynamic analysis of seismically base-isolated structures, a Mixed Explicit-Implicit time integration Method (MEIM) has been proposed. Adopting the explicit conditionally stable central difference method to compute the nonlinear response of the base isolation system, and the implicit unconditionally stable Newmark's constant average acceleration method to determine the superstructure linear response, the proposed MEIM, which is conditionally stable due to the use of the central difference method, allows to avoid the iterative procedure generally required by conventional monolithic solution approaches within each time step of the analysis. The main aim of this paper is to investigate the stability and computational efficiency of the MEIM when employed to perform the nonlinear time history analysis of base-isolated structures with sliding bearings. Indeed, in this case, the critical time step could become smaller than the one used to define accurately the earthquake excitation due to the very high initial stiffness values of such devices. The numerical results obtained from nonlinear dynamic analyses of a base-isolated structure with a friction pendulum bearing system, performed by using the proposed MEIM, are compared to those obtained adopting a conventional monolithic solution approach, i.e. the implicit unconditionally stable Newmark's constant acceleration method employed in conjunction with the iterative pseudo-force procedure. According to the numerical results, in the presented numerical application, the MEIM does not have stability problems being the critical time step larger than the ground acceleration one despite of the high initial stiffness of the friction pendulum bearings. In addition, compared to the conventional monolithic solution approach, the proposed algorithm preserves its computational efficiency even when it is adopted to perform the nonlinear dynamic analysis using a smaller time step.

***Keywords*—**Base isolation, computational efficiency, mixed explicit-implicit method, partitioned solution approach, stability.

I. Introduction

THE numerical solution of nonlinear dynamic equilibrium equations of seismically base-isolated structures adopting a conventional monolithic solution approach, i.e. an implicit single-step time integration method employed in conjunction with an iterative procedure, can require considerable computational effort.

In order to achieve a substantial reduction in computation, a partitioned solution approach [1] can be used to perform the analysis. In the context of base-isolated structures, the above-mentioned approach can be easily employed being the decomposition of the discrete structural model of such structures driven by physical considerations: the base isolation system is much more flexible than the superstructure to decouple the latter from the earthquake ground motion [2].

In the last 30 years, various authors [3]-[7] developed several partitioned time integration methods allowing different time steps or time integration algorithms or both to be used in different spatial subdomains of the mesh.

Vaiana et al. [2] proposed a Mixed Explicit-Implicit time integration Method (MEIM) specifically for the nonlinear dynamic analysis of base-isolated structures under earthquake excitation: in each time step of the analysis, the nonlinear response of the base isolation system is computed first using the explicit conditionally stable central difference method, then the implicit unconditionally stable Newmark's constant average acceleration method is employed to evaluate the superstructure linear response. Thus, the proposed solution algorithm allows to evaluate the nonlinear response of base-isolated structures without adopting an iterative procedure generally required by conventional monolithic solution approaches within each time step of the nonlinear time history analysis.

Since the proposed MEIM is conditionally stable due to the use of the central difference method, in this work, a procedure to evaluate the critical time step is first developed for two-dimensional (2D) base-isolated structures and then extended to the three-dimensional (3D) case.

The main aim of this paper is to investigate the stability and the computational efficiency of the MEIM when adopted for the nonlinear dynamic analysis of base-isolated structures having seismic isolators with very high initial stiffness values, such as sliding bearings, for which the critical time step could become smaller than the one used to define accurately the earthquake excitation. To this end, the numerical results obtained from nonlinear dynamic analyses of a 3D base-isolated structure with a friction pendulum bearing (FPB) system subjected to bidirectional earthquake excitation, performed using the MEIM, are compared to those obtained using a conventional monolithic solution approach, developed by [8] specifically for the analysis of base-isolated structures.

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II. Structural Model and Equations of Motion

In this section, the discrete structural model of a typical seismically base-isolated structure is described, and the equations of motion are formulated.

The discrete structural model of such a system can be decomposed into two substructures: the n-story superstructure and the base isolation system consisting of seismic isolation bearings and a full diaphragm above the seismic devices. Fig. 1 shows the discrete structural model of a typical two-story base-isolated structure.

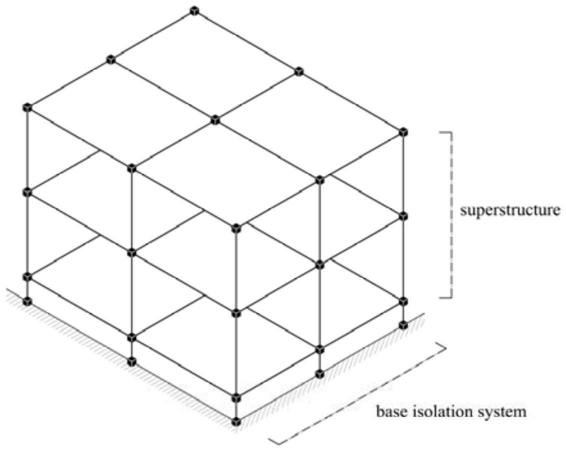


Fig. 1 Discrete structural model of a typical two-story base-isolated structure

The superstructure is considered to remain elastic during the earthquake excitation. This assumption, which is reasonable in the context of seismically base-isolated structures because the introduction of a flexible base isolation system generally reduces the earthquake response in such a way that the superstructure deforms within the elastic range, allows to decrease the computational effort of a nonlinear time history analysis. In order to adequately represent the elastic behavior of the superstructure, a 3D discrete structural model with three Degrees of Freedom (DOFs) per floor has to be adopted. In this work, the elastic superstructure is assumed to be a 3D shear building, thus floor diaphragms are considered to be rigid in its own plane, the beams are considered to be axially inextensible and flexurally rigid, and the columns are considered to be axially inextensible.

As far as the isolation system is concerned, the base isolation system diaphragm is assumed to be infinitely rigid in its own plane, the seismic isolation devices are considered rigid in the vertical direction, and torque resistance of individual bearing is neglected. The base isolation system can include linear and nonlinear isolation elements.

A global coordinate system, denoted with upper case letters X, Y, and Z, is attached to the mass center of the base isolation system. Because of this structural idealization, the total number of DOFs of the 3D structural model of a base-isolated structure is equal to $3n + 3$. The i-th floor diaphragm has three DOFs defined at the diaphragm reference point o_i, which is vertically aligned to the global coordinate system origin O. The DOFs for the i-th floor are the translation u_{ix} along the X-axis, the translation u_{iy} along the Y-axis, and the rotation $u_{i\vartheta}$ about the vertical axis Z; u_{ix} and u_{iy} are defined relative to the ground. The $3n$ superstructure DOFs are listed in the displacement vector $\boldsymbol{u}_s$, whereas the three DOFs of the base isolation system are in the displacement vector $\boldsymbol{u}_b$. The i-th diaphragm mass is lumped in its mass center (MC_i) which is also the geometric center.

The equations of motion for the elastic superstructure are expressed in the following form:

$$\boldsymbol{m}_s\,\ddot{\boldsymbol{u}}_s + \boldsymbol{c}_s\,\dot{\boldsymbol{u}}_s + \boldsymbol{k}_s\,\boldsymbol{u}_s + \boldsymbol{c}\,\dot{\boldsymbol{u}}_b + \boldsymbol{k}\,\boldsymbol{u}_b = -\boldsymbol{m}_s\,\boldsymbol{r}_s\,\ddot{\boldsymbol{u}}_g \tag{1}$$

with

$$\boldsymbol{c} = \begin{bmatrix} -\boldsymbol{c}_1 & \boldsymbol{0} \end{bmatrix}^T \tag{2}$$

$$\boldsymbol{k} = \begin{bmatrix} -\boldsymbol{k}_1 & \boldsymbol{0} \end{bmatrix}^T \tag{3}$$

where $\boldsymbol{m}_s$ is the superstructure mass matrix, $\boldsymbol{c}_s$ the superstructure damping matrix, $\boldsymbol{k}_s$ the superstructure stiffness matrix, and $\boldsymbol{r}_s$ the superstructure earthquake influence matrix. Furthermore, $\boldsymbol{u}_s$, $\dot{\boldsymbol{u}}_s$, and $\ddot{\boldsymbol{u}}_s$ represent the floor displacement, velocity, and acceleration vectors relative to the ground, respectively, $\ddot{\boldsymbol{u}}_g$ the ground acceleration vector, $\boldsymbol{c}_1$ and $\boldsymbol{k}_1$ the viscous damping and stiffness matrices of the superstructure first story.

The equations of motion for the base are:

$$\mathbf{m}_b\,\ddot{\mathbf{u}}_b + (\mathbf{c}_b + \mathbf{c}_1)\,\dot{\mathbf{u}}_b + \mathbf{c}^T\dot{\mathbf{u}}_s + (\mathbf{k}_b + \mathbf{k}_1)\,\mathbf{u}_b + \mathbf{k}^T\mathbf{u}_s + \mathbf{f}_n = -\mathbf{m}_b\,\mathbf{r}_b\,\ddot{\mathbf{u}}_g\,, \tag{4}$$

where $\boldsymbol{m}_b$ is the base isolation system mass matrix, $\boldsymbol{c}_b$ is the damping matrix of linear viscous isolation elements, $\boldsymbol{k}_b$ is the stiffness matrix of linear elastic isolation elements, and $\boldsymbol{f}_n$ is the resultant nonlinear forces vector of nonlinear elements. Furthermore, $\boldsymbol{u}_b$, $\dot{\boldsymbol{u}}_b$, and $\ddot{\boldsymbol{u}}_b$ represent the base isolation system displacement, velocity, and acceleration vectors relative to the ground, respectively, and $\boldsymbol{r}_b$ the base isolation system influence matrix.

Combining (1) and (4), the following system of $3n + 3$ ordinary differential equations of the second order in time, coupled in terms of elastic and viscous forces and decoupled in terms of inertial forces, is obtained:

$$\begin{bmatrix} \mathbf{m}_b & \mathbf{0}^T \\ \mathbf{0} & \mathbf{m}_s \end{bmatrix}\begin{Bmatrix} \ddot{\mathbf{u}}_b \\ \ddot{\mathbf{u}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{c}_b + \mathbf{c}_1 & \mathbf{c}^T \\ \mathbf{c} & \mathbf{c}_s \end{bmatrix}\begin{Bmatrix} \dot{\mathbf{u}}_b \\ \dot{\mathbf{u}}_s \end{Bmatrix} + \begin{bmatrix} \mathbf{k}_b + \mathbf{k}_1 & \mathbf{k}^T \\ \mathbf{k} & \mathbf{k}_s \end{bmatrix}\begin{Bmatrix} \mathbf{u}_b \\ \mathbf{u}_s \end{Bmatrix} + \begin{Bmatrix} \mathbf{f}_n \\ \mathbf{0} \end{Bmatrix} = -\begin{bmatrix} \mathbf{m}_b & \mathbf{0}^T \\ \mathbf{0} & \mathbf{m}_s \end{bmatrix}\begin{Bmatrix} \mathbf{r}_b \\ \mathbf{r}_s \end{Bmatrix}\ddot{\mathbf{u}}_g \tag{5}$$

This system is nonlinear because of the presence of the resultant nonlinear forces vector $\boldsymbol{f}_n$ of the base isolation system.

III. Proposed Mixed Time Integration Method

In the following, the MEIM proposed by [2] is described. The explicit time integration method adopted to predict the nonlinear response of the base isolation system is the second order central difference method, whereas the implicit time integration method employed to compute the linear response of the superstructure is the second order Newmark's constant average acceleration method, also called trapezoidal rule. Thus, in each time step of a nonlinear time history analysis, the proposed method allows first the solution of the base isolation system response, then these results are used for the evaluation of the superstructure response. As it will be shown, the proposed algorithm does not require the use of an iterative procedure within each time step of a nonlinear time history analysis.

A. Evaluation of the Base Isolation System Response

Writing the set of three equations of motion of the base isolation system at time *t* gives:

$$\mathbf{m}_b\,\ddot{\mathbf{u}}_b(t)+(\mathbf{c}_b+\mathbf{c}_1)\,\dot{\mathbf{u}}_b(t)+\mathbf{c}^T\dot{\mathbf{u}}_s(t)+(\mathbf{k}_b+\mathbf{k}_1)\,\mathbf{u}_b(t)+\mathbf{k}^T\mathbf{u}_s(t)+\mathbf{f}_n(t)=-\mathbf{m}_b\,\mathbf{r}_b\,\ddot{\mathbf{u}}_g(t) \tag{6}$$

Assuming constant time steps and substituting the central difference expressions for velocity and acceleration vectors at time *t* into (6) gives:

$$\hat{\boldsymbol{k}}_b\,\boldsymbol{u}_b(t+\Delta t)=\hat{\boldsymbol{p}}_b(t) \tag{7}$$

where

$$\hat{\boldsymbol{k}}_b=\left[\frac{1}{(\Delta t)^2}\boldsymbol{m}_b+\frac{1}{2\Delta t}(\boldsymbol{c}_b+\boldsymbol{c}_1)\right] \tag{8}$$

and

$$\begin{aligned}\hat{\mathbf{p}}_b(t)&=-\mathbf{m}_b\,\mathbf{r}_b\,\ddot{\mathbf{u}}_g(t)-\mathbf{c}^T\dot{\mathbf{u}}_s(t)-\mathbf{k}^T\mathbf{u}_s(t)-\mathbf{f}_n(t)\\&+\left[\frac{2}{(\Delta t)^2}\mathbf{m}_b-\mathbf{k}_b-\mathbf{k}_1\right]\mathbf{u}_b(t)+\left[-\frac{1}{(\Delta t)^2}\mathbf{m}_b+\frac{1}{2\Delta t}(\mathbf{c}_b+\mathbf{c}_1)\right]\mathbf{u}_b(t-\Delta t),\end{aligned} \tag{9}$$

from which $\boldsymbol{u}_b(t+\Delta t)$ can be evaluated.

In (9), $\boldsymbol{u}_b(t-\Delta t)$, $\boldsymbol{u}_b(t)$, $\boldsymbol{u}_s(t)$, and $\dot{\boldsymbol{u}}_s(t)$ are assumed known from implementation of the procedure for the preceding time steps. In order to calculate the solution at time Δt, a special starting procedure must be used. Since $\boldsymbol{u}_b(0)$, $\dot{\boldsymbol{u}}_b(0)$, and $\ddot{\boldsymbol{u}}_b(0)$ are known at time t = 0, $\boldsymbol{u}_b(-\Delta t)$ can be obtained using the following relation [9]:

$$\boldsymbol{u}_b(-\Delta t)=\boldsymbol{u}_b(0)-\Delta t\,\dot{\boldsymbol{u}}_b(0)+\frac{(\Delta t)^2}{2}\ddot{\boldsymbol{u}}_b(0)\,. \tag{10}$$

B. Evaluation of the Superstructure Response

Writing the set of $3n$ dynamic equilibrium equations of the superstructure at time $t+\Delta t$ gives:

$$\begin{aligned}&\boldsymbol{m}_s\,\ddot{\boldsymbol{u}}_s(t+\Delta t)+\boldsymbol{c}_s\,\dot{\boldsymbol{u}}_s(t+\Delta t)+\boldsymbol{k}_s\,\boldsymbol{u}_s(t+\Delta t)\\&+\boldsymbol{c}\,\dot{\boldsymbol{u}}_b(t+\Delta t)+\boldsymbol{k}\,\boldsymbol{u}_b(t+\Delta t)=-\boldsymbol{m}_s\,\boldsymbol{r}_s\,\ddot{\boldsymbol{u}}_g(t+\Delta t)\,.\end{aligned} \tag{11}$$

Substitution of the trapezoidal rule expressions for the superstructure velocity and acceleration vectors at time $t+\Delta t$ into (11) gives:

$$\hat{\boldsymbol{k}}_s\,\boldsymbol{u}_s(t+\Delta t)=\hat{\boldsymbol{p}}_s(t+\Delta t)\,, \tag{12}$$

where

$$\hat{\boldsymbol{k}}_s=\left[\frac{4}{(\Delta t)^2}\boldsymbol{m}_s+\frac{2}{\Delta t}\boldsymbol{c}_s+\boldsymbol{k}_s\right]\,, \tag{13}$$

and

$$\begin{aligned}\hat{\mathbf{p}}_s(t+\Delta t)&=-\mathbf{m}_s\,\mathbf{r}_s\,\ddot{\mathbf{u}}_g(t+\Delta t)-\mathbf{c}\,\dot{\mathbf{u}}_b(t+\Delta t)-\mathbf{k}\,\mathbf{u}_b(t+\Delta t)\\&+\left[\frac{4}{(\Delta t)^2}\mathbf{m}_s+\frac{2}{\Delta t}\mathbf{c}_s\right]\mathbf{u}_s(t)+\left[\frac{4}{\Delta t}\mathbf{m}_s+\mathbf{c}_s\right]\dot{\mathbf{u}}_s(t)+\mathbf{m}_s\,\ddot{\mathbf{u}}_s(t).\end{aligned} \tag{14}$$

In order to solve for $\boldsymbol{u}_s(t+\Delta t)$, first the base isolation system velocity vector at time $t+\Delta t$ has to be evaluated. This vector can be computed in terms of displacement vectors using the three-point backward difference approximation [10]:

$$\dot{\boldsymbol{u}}_b(t+\Delta t)=\frac{1}{2\,\Delta t}\left[-4\,\boldsymbol{u}_b(t)+3\,\boldsymbol{u}_b(t+\Delta t)+\boldsymbol{u}_b(t-\Delta t)\right]. \tag{15}$$

IV. Stability Analysis

The proposed MEIM is conditionally stable because the central difference method is adopted to compute the nonlinear response of the base isolation system. In this section, a procedure to evaluate the critical time step is first developed for 2D base-isolated structures and then extended to the 3D case.

Considering the 2D discrete structural model of a base-isolated structure having only linear isolation elements and neglecting the superstructure and base isolation system viscous damping, (6) becomes:

$$m_b\,\ddot{u}_b(t)+(k_b+k_1)\,u_b(t)-k_1\,u_1(t)=-m_b\,\ddot{u}_g(t)\,. \tag{16}$$

Since the stability of an integration method can be determined by examining the behavior of the numerical solution for arbitrary initial conditions, it is possible to consider the integration of (16) for $\ddot{u}_g(t)=0$:

$$m_b\,\ddot{u}_b(t)+(k_b+k_1)\,u_b(t)-k_1\,u_1(t)=0\,. \tag{17}$$

Dividing (17) by m_b, and expressing the displacement of the superstructure first floor in terms of u_b, i.e. $u_1 = u_b + \alpha\, u_b$, where the parameter α is generally much smaller than 1, (17) becomes:

$$\ddot{u}_b(t) + \frac{(k_b - \alpha\, k_1)}{m_b} u_b(t) = 0\,. \tag{18}$$

After substituting the central difference expression for the acceleration at time *t*, (18) can be reformulated into a recursive matrix form as follows:

$$\begin{Bmatrix} u_b(t+\Delta t) \\ u_b(t) \end{Bmatrix} = \boldsymbol{A} \begin{Bmatrix} u_b(t) \\ u_b(t-\Delta t) \end{Bmatrix}, \tag{19}$$

where $\boldsymbol{A}$ is the integration approximation operator given by:

$$\boldsymbol{A} = \begin{bmatrix} 2 - \dfrac{\Delta t^2}{m_b}(k_b - \alpha\, k_1) & -1 \\ 1 & 0 \end{bmatrix}. \tag{20}$$

In this work, the numerical stability is analyzed by using the spectral decomposition of the matrix $\boldsymbol{A}$. Since the stability of an integration method depends only on the eigenvalues of the approximation operator $\boldsymbol{A}$, the following eigenvalue problem has to be solved:

$$\boldsymbol{A}\, \boldsymbol{v} = \lambda\, \boldsymbol{v}\,. \tag{21}$$

The eigenvalues of the matrix $\boldsymbol{A}$ are the roots of the following characteristic polynomial:

$$p(\lambda) = det\,(\boldsymbol{A} - \lambda\, \boldsymbol{I})\,, \tag{22}$$

which, in this case, is defined as:

$$p\,(\lambda) = \left[2 - \frac{\Delta t^2}{m_b}(k_b - \alpha\, k_1) - \lambda \right](-\lambda) + 1\,. \tag{23}$$

Thus, the two eigenvalues of $\boldsymbol{A}$ are:

$$\lambda_1 = \left[1 - \frac{\Delta t^2}{2\, m_b}(k_b - \alpha\, k_1) \right] + \sqrt{\frac{1}{4}\left[2 - \frac{\Delta t^2}{m_b}(k_b - \alpha\, k_1) \right]^2 - 1}\,, \tag{24}$$

$$\lambda_2 = \left[1 - \frac{\Delta t^2}{2\, m_b}(k_b - \alpha\, k_1) \right] - \sqrt{\frac{1}{4}\left[2 - \frac{\Delta t^2}{m_b}(k_b - \alpha\, k_1) \right]^2 - 1}\,. \tag{25}$$

For stability, the absolute values of λ_1 and λ_2 have to be smaller than or equal to 1, that is, the spectral radius $\rho(\boldsymbol{A})$ of the approximation operator $\boldsymbol{A}$, defined as $\rho(\boldsymbol{A}) = \max_{i=1,2} |\lambda_i|$, must satisfy the condition $\rho(\boldsymbol{A}) \le 1$. It follows from this condition that the critical time step Δt_{cr} is given by:

$$\Delta t_{cr} = \frac{T}{\pi} = 2\sqrt{\frac{m_b}{k_b - \alpha\, k_1}}\,. \tag{26}$$

The same time step stability limit is also applicable when the viscous damping is not neglected [9].

It is important to observe that the highest horizontal stiffness of each seismic isolator has to be used in order to evaluate Δt_{cr} and that α can be assumed equal to zero.

Considering the 3D discrete structural model of a seismically base-isolated structure, the critical time step Δt_{cr} can be evaluated considering the lowest natural period given by the following eigenvalue problem:

$$\boldsymbol{k}_b^h\, \boldsymbol{\Phi} = \boldsymbol{m}_b\, \boldsymbol{\Phi}\, \boldsymbol{\Omega}^2\,, \tag{27}$$

where $\boldsymbol{k}_b^h$ is the stiffness matrix of the base isolation system assembled using the highest horizontal stiffness of each nonlinear element.

V. Numerical Investigation

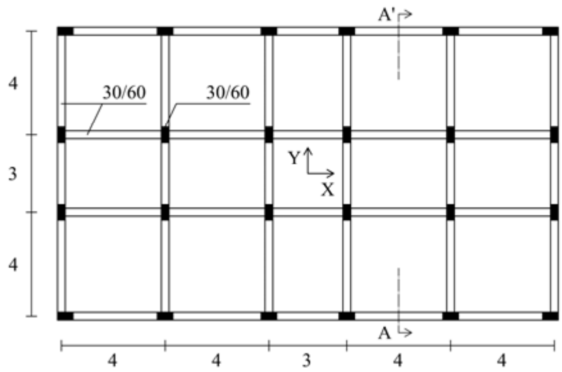
In the following, the nonlinear dynamic response of a 3D base-isolated structure with a FPB system subjected to bidirectional earthquake excitation is predicted using the proposed MEIM. The main aim of the numerical application is to investigate the stability and the computational efficiency of the MEIM when adopted for the nonlinear dynamic analysis of base-isolated structures having seismic isolators with very high initial stiffness values, such as FPBs, for which the critical time step could become smaller than the one used to define accurately the earthquake excitation. The numerical results are compared to those obtained using a conventional monolithic solution approach, i.e. the implicit unconditionally stable Newmark's constant acceleration method used in conjunction with the iterative pseudo-force procedure. For brevity, in this paper, the latter solution algorithm proposed by [8] specifically for the analysis of base-isolated structures, is referred to as the Pseudo-Force Method (PFM).

A. Analyzed 3D Base-Isolated Structure

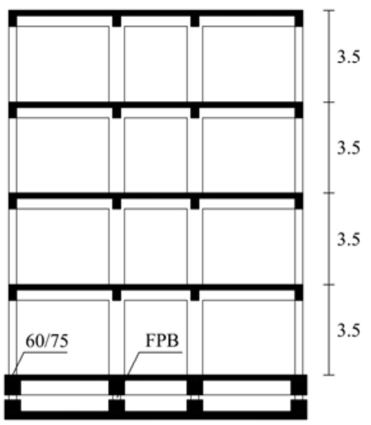
The superstructure is a four-story reinforced concrete structure with plan dimensions 19 m x 11 m, and story height *h* = 3.5 m. The weight of the superstructure is 9921.24 kN, and the first three natural periods are 0.33 s, 0.33 s, and 0.26 s, respectively. Each superstructure diaphragm mass includes the contributions of the dead load and live load on the floor diaphragm and the contributions of the structural elements and of the nonstructural elements between floors.

The base isolation system, having a total weight of 3006.44 kN, consists of an orthogonal mesh of foundation beams having rectangular cross section with dimensions 60 cm x 75 cm, and 24 identical FPBs, positioned centrically under all columns.

As a result of the kinematic constraints assumed in Section II, the total number of DOFs, defined relative to the ground, is equal to 15. The typical floor plan and a section of the analyzed 3D base-isolated structure are shown in Fig. 2.



(a)



(b)

Fig. 2 Four-story reinforced concrete base-isolated structure (a) typical floor plan (b) section A-A'

The base isolation system has been designed in order to provide an effective isolation period T_{eff} = 2.50 s and an effective viscous damping υ_{eff} = 0.10 at the design displacement d_d = 0.50 m. Each bearing has a radius of curvature of the spherical concave surface R = 1.55 m, and a sliding friction coefficient $\mu = 0.06$.

B. Model Adopted for Friction Pendulum Bearings (FPBs)

The dynamic behavior of each FPB is simulated by using a mathematical model, introduced by [8], capable of predicting the biaxial behavior of both elastomeric and sliding bearings. According to this model, the FPB nonlinear restoring forces along the orthogonal directions X and Y are described by the following equations:

$$f_{nx} = \frac{N}{R} u_x + \mu N z_x , \tag{28}$$

$$f_{ny} = \frac{N}{R} u_y + \mu N z_y , \tag{29}$$

in which N is the vertical load carried by the bearing, R is the radius of curvature of the spherical concave surface of the bearing, μ is the sliding friction coefficient, z_x and z_y are dimensionless variables governed by the following system of two coupled first order ordinary nonlinear differential equations proposed by [11]:

$$\begin{Bmatrix} \dot{z}_x \, y \\ \dot{z}_y \, y \end{Bmatrix} = \begin{Bmatrix} A \, \dot{u}_x \\ A \, \dot{u}_y \end{Bmatrix} - \begin{bmatrix} z_x^2 \left[\gamma \operatorname{sgn}(\dot{u}_x z_x) + \beta \right] & z_x z_y \left[\gamma \operatorname{sgn}(\dot{u}_y z_y) + \beta \right] \\ z_x z_y \left[\gamma \operatorname{sgn}(\dot{u}_x z_x) + \beta \right] & z_y^2 \left[\gamma \operatorname{sgn}(\dot{u}_y z_y) + \beta \right] \end{bmatrix} \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix}, \tag{30}$$

in which A, β, and γ are dimensionless quantities that control the shape of the hysteresis loop, y is the yield displacement, $\dot{u}_x$ and $\dot{u}_y$ are the velocities that occur at the isolation device in X and Y directions, respectively.

In this work, the bearing normal force N is assumed equal to the weight W acting on each isolator, the dependency of the sliding friction coefficient on bearing pressure and sliding velocity is neglected, and the yield displacement y is assumed equal to 0.0001 m. Furthermore, the unconditionally stable semi-implicit Runge-Kutta method [12] is employed to solve the differential equations governing the behavior of each nonlinear isolation element with a number of steps equal to 50.

C. Numerical Results

Bidirectional earthquake excitation is imposed with component SN and SP of the 1989 Loma Prieta motion applied along directions X and Y, respectively. The ground acceleration record time step is 0.005 s. It is important to observe that normally 200 points per second are used to define accurately an acceleration record [13].

Table I gives the Nonlinear Time History Analyses (NLTHAs) results obtained using the PFM and the proposed MEIM, both implemented on the same computer (Intel® Core™ i7-4700MQ processor, CPU at 2.40 GHz with 16 GB of RAM) by using the computer program MATLAB. In the PFM, the adopted convergence tolerance value is equal to 10^{-6}.

The comparison of the maximum and minimum values of the MC_b displacements and MC_4 accelerations in X and Y directions, obtained using the PFM and MEIM, reveals that the proposed method provides numerical results that are close enough to those obtained adopting the PFM.

As regards the stability of the MEIM, the critical time step Δt_{cr}, evaluated considering the lowest natural period given by the eigenvalue problem in (27), is equal to 0.012 s. It is evident that, in this case, being the critical time step larger than the imposed ground acceleration time step, there are no stability problems despite of the very high initial stiffness value of FPBs.

TABLE I
NLTHAS RESULTS WITH $\Delta t = 0.005$ S

			$u_x^{(MC_b)}$ [m]		$u_y^{(MC_b)}$ [m]		$\ddot{u}_x^{(MC_4)}$ [g]		$\ddot{u}_y^{(MC_4)}$ [g]	
	tct [s]	*tctp*	max	min	max	min	max	min	max	min
PFM	1425	-	0.564	-0.457	0.297	-0.183	1.178	-1.119	0.600	-0.729
MEIM	99.8	7.0%	0.564	-0.457	0.297	-0.183	1.137	-1.056	0.558	-0.743

TABLE II
NLTHAS RESULTS WITH $\Delta t = 0.001$ S

			$u_x^{(MC_b)}$ [m]		$u_y^{(MC_b)}$ [m]		$\ddot{u}_x^{(MC_4)}$ [g]		$\ddot{u}_y^{(MC_4)}$ [g]	
	tct [s]	*tctp*	max	min	max	min	max	min	max	min
MEIM	490.59	34.4%	0.559	-0.446	0.295	-0.181	0.879	-0.957	0.544	-0.654

As far as the computational efficiency is concerned, the total computational time, *tct*, required by the MEIM is significantly reduced in comparison to the PFM. It must be noted that the comparisons using the *tct* are meaningful only qualitatively because it depends on the CPU speed, memory capability, and background processes of the computer used to obtain the previous results. To this end, in order to normalize the computational time results, Table I also shows the percentage of the MEIM *tct* evaluated with respect to the PFM *tct* as:

$$\text{MEIM } tctp\,[\%] = \frac{\text{MEIM } tct}{\text{PFM } tct} \cdot 100\,. \qquad (31)$$

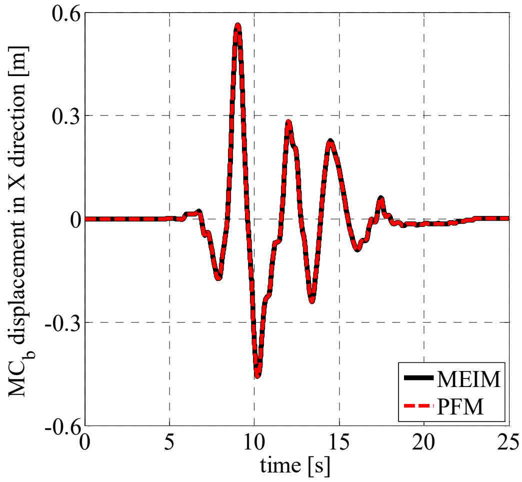
In addition, according to the numerical results listed in Table II, the proposed MEIM, performed with a smaller time step, that is, $\Delta t = 0.001$ s, requires less computational effort than the PFM even if the latter is performed using the larger time step (i.e., $\Delta t = 0.005$ s). Indeed, the MEIM *tctp*, referred to the PFM *tct* evaluated adopting $\Delta t = 0.005$ s, is equal to 34.42 %.

It can therefore be concluded that even when a time step smaller than the one used to define the ground acceleration accurately has to be adopted because of stability requirements, as in the case of base isolation systems having isolators with very high initial stiffness, such as sliding bearings, the proposed method preserves its computational efficiency with respect to the widely used PFM.

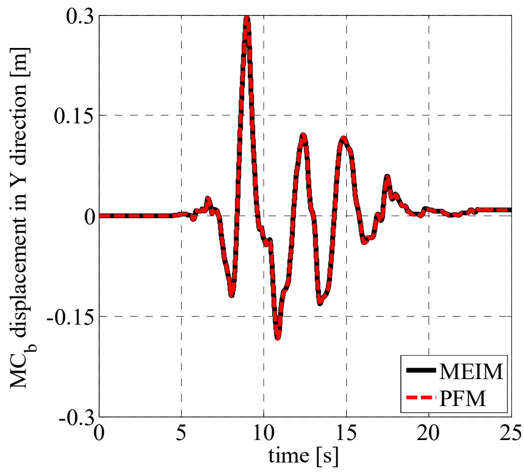
Figs. 3 and 4 illustrate, respectively, the displacement time history of the MC_b and the acceleration time history of the MC_4. It is evident the good agreement between responses computed using the PFM and the proposed MEIM.

VI. CONCLUSIONS

A MEIM, proposed by [2] to perform the nonlinear time history analysis of seismically base-isolated structures, has been presented. Adopting the explicit central difference method to compute the nonlinear response of the base isolation system and the implicit Newmark's constant average acceleration method to evaluate the superstructure linear response, the proposed solution algorithm allows to avoid the use of an iterative procedure for each time step of a nonlinear dynamic analysis.



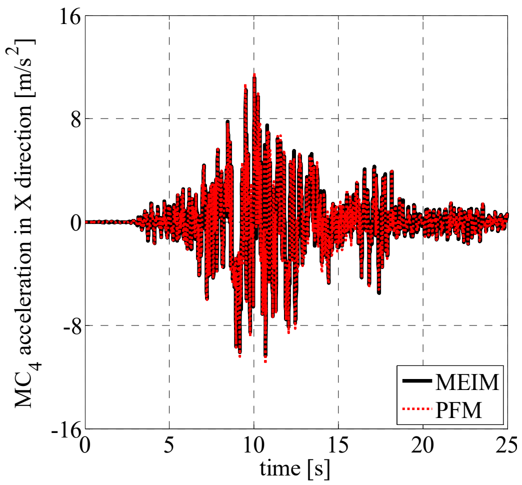
(a)



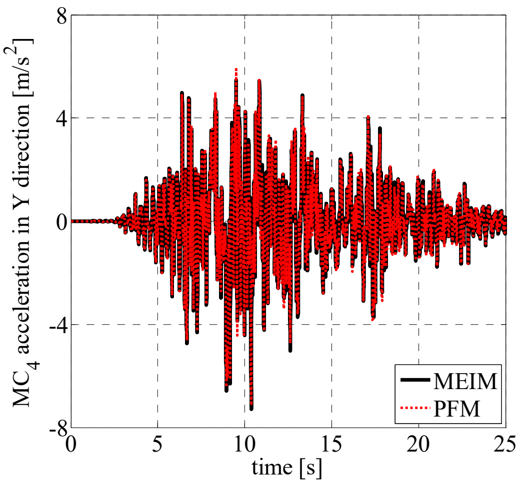
(b)

Fig. 3 Displacement time history of the base isolation system mass center in (a) *X* and (b) *Y* directions

Since the MEIM is conditionally stable because of the use of the central difference method, a procedure to evaluate the critical time step has been developed first for 2D base-isolated structures and then extended to the 3D case.



(a)



(b)

Fig. 4 Acceleration time history of the superstructure fourth story mass center in (a) *X* and (b) *Y* directions

In order to investigate the stability and the computational efficiency of the proposed MEIM when adopted to analyze base-isolated structures having seismic isolators with very high initial stiffness values, such as sliding bearings, for which the critical time step could become smaller than the one used to define accurately the earthquake excitation, the seismic response of a 3D base-isolated structure with a FPB system has been predicted using the proposed MEIM. The numerical results have been compared to those obtained adopting the widely used PFM. From the numerical results presented in the paper, the following conclusions can be drawn:

1) The proposed MEIM provides results that are close enough to those obtained adopting the PFM;
2) Despite the very high initial stiffness of FPBs, in the proposed numerical example, the MEIM does not have stability problems being the critical time step larger than the imposed ground acceleration one;
3) The *tct* required by the MEIM is significantly reduced in comparison to the PFM: the MEIM *tctp* evaluated with respect to the PFM *tct* for Δt = 0.005 s is equal to 7 %. In addition, the MEIM, performed with a smaller time step (i.e., Δt = 0.001 s), requires less computational effort than the PFM even if the latter is performed using the larger time step (i.e., Δt = 0.005 s): indeed, the MEIM *tctp*, referred to the PFM *tct* evaluated adopting Δt = 0.005 s, is equal to 34.42 %. It transpires that even when a smaller time step has to be used to avoid stability problems, the proposed solution algorithm preserves its computational efficiency.

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