Introduction of the Fluid-Structure Coupling into the Force Analysis Technique

Océane Grosset, Charles Pézerat, Jean-Hugh Thomas, Frédéric Ablitzer

Abstract—This paper presents a method to take into account the fluid-structure coupling into an inverse method, the Force Analysis Technique (FAT). The FAT method, also called RIFF method (Filtered Windowed Inverse Resolution), allows to identify the force distribution from local vibration field. In order to only identify the external force applied on a structure, it is necessary to quantify the fluid-structure coupling, especially in naval application, where the fluid is heavy. This method can be decomposed in two parts, the first one consists in identifying the fluid-structure coupling and the second one to introduced it in the FAT method to reconstruct the external force. Results of simulations on a plate coupled with a cavity filled with water are presented.

Keywords—Fluid-structure coupling, inverse methods, naval, vibrations.

I. INTRODUCTION

THE study of aeroacoustic and hydroacoustic noises is an important industrial research field, especially in the transport industry [1]-[4]. Actually, flows induced by a fluid over a structure create a source of internal noise which cannot be neglected. To minimize its impact, it is necessary to have a good knowledge of this kind of excitation. In fact, turbulent flow is due to the presence of obstacles or boundary layers near the structure [5]. On one hand, it creates pressure fluctuations near the structure which are called the convective part of the excitation, and on the other hand, it generates acoustic waves in every direction, which correspond to the acoustic part of the excitation. Physically, the acoustic component is much smaller than the convective part, in terms of energy, which makes it really hard to measure. However, depending on the application, the acoustic component can be the main cause of acoustic radiation of the structure inside the cabin.

The aim of this study is to identify the wall pressure which excites the structure, for the aeronautic and naval domains, by using an inverse vibration method, such as FAT (Force Analysis Technique), also called RIFF (Filtered Windowed Inverse Resolution) [6], [7]. The main advantage of this technique is its ability to identify the force distribution from local vibration measurements. In order to do that, the FAT method is applied to simulations of a plate excited by a turbulent flow for both domains, aeronautic and naval [8]. In [8], the potentiality of the FAT method is pointed out.

O. Grosset is with the Institut de Recherche Technologique Jules Verne (IRT Jules Verne), Bouguenais, France, and the Laboratoire d'Acoustique de l'Université du Maine (LAUM UMR CNRS 6613), Le Mans, France (e-mail: oceane.grosset@irt-jules-verne.fr).

C. Pézerat, J.-H. Thomas, and F. Ablitzer are with the Laboratoire d'Acoustique de l'Université du Maine (LAUM UMR CNRS 6613), Le Mans, France (e-mail: charles.pezerat@univ-lemans.fr, jean-hugh.thomas@univ-lemans.fr, frederic.ablitzer@univ-lemans.fr).

However, in the case of the naval domain, with a subsonic flow in an heavy fluid, the fluid-structure coupling has been neglected. Yet, when considering a heavy fluid, the acoustic radiation of the structure can interfere with the vibration of the structure, the fluid-structure coupling need to be taken into account. This paper is the development of a previous study [8], it offers a method based on the FAT method to take into account the fluid-structure coupling, using the case of a plate coupled with a cavity.

The first part of this document details the structure studied and the calculation of the vibration for this system. The second part presents the Force Analysis Technique method (FAT). The last part introduces the method to take into account the fluid-structure coupling in FAT method and results obtained for the plate coupled with a cavity system.

II. DIRECT PROBLEM

In this study, the system studied is a plate coupled with a cavity filled with water (see Fig. 1). We consider a rigid cavity of size L_x, L_y and L_z along x, y and z. The plate is placed in $z = L_z$ and of size L_x and L_y. The parameters of the structure studied are presented in Table I.
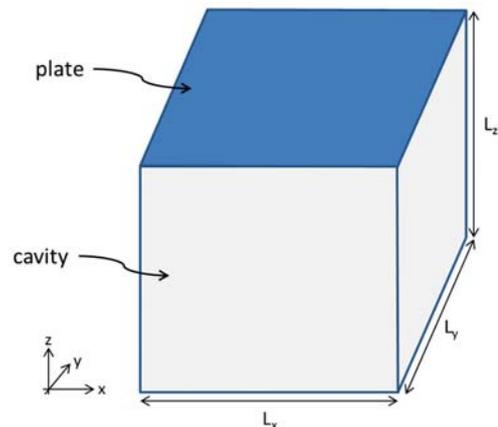
Fig. 1 Scheme of a plate coupled with a rigid cavity

The analytical problem for a plate coupled with a rigid cavity is detailed in [9]-[11] and presented here. In the cavity, the pressure is described by the inhomogeneous wave equation:

$$\nabla^2 p(\mathbf{r}, t) - \frac{\partial p(\mathbf{r}, t)}{c^2 \partial t^2} = -\rho_0 \frac{\partial q(\mathbf{r}_0, t)}{\partial t}, \qquad (1)$$

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TABLE I
PARAMETERS OF THE PLATE COUPLED WITH A CAVITY FILLED WITH
WATER

Dimension along x	$L_x = 1\ m$
Dimension along y	$L_y = 1\ m$
Dimension along z	$L_z = 1\ m$
Young modulus	$E = 210.10^9\ Pa$
Poisson coefficient	$\nu = 0.3$
Density of the structure	$\rho = 7800\ kg.m^{-3}$
Plate thickness	$h = 8.10^{-3}\ m$
Damping	$\xi = 0.005$
Density of the fluid	$\rho_0 = 1000\ kg.m^{-3}$
Speed of sound in the fluid	$c = 1500\ m.s^{-1}$

with $q(\mathbf{r}_0, t)$ the monopole source output, ρ_0 the fluid density, c the sound speed in the fluid and $p(\mathbf{r}, t)$ the pressure in the cavity. Boundary vibrations are represented as source output q_b:

$$q_b(\mathbf{r}_s, t) = -\frac{\partial^2 w(\mathbf{r}_s, t)}{\partial t^2}, \quad (2)$$

with w the plate normal displacement. The wave equation becomes:

$$\nabla^2 p(\mathbf{r}, t) - \frac{\partial^2 p(\mathbf{r}, t)}{c^2 \partial t^2} = -\rho_0 \frac{\partial q(\mathbf{r}_0, t)}{\partial t} + \rho_0 \frac{\partial^2 w(\mathbf{r}_s, t)}{\partial t^2}. \quad (3)$$

The pressure can be written as a sum over eigenmodes:

$$p(\mathbf{r}, t) = \sum_{n=0}^{\infty} p_n(t) \Psi_n(\mathbf{r}), \quad (4)$$

where $\Psi_n(\mathbf{r})$ are the cavity eigenmodes and $p_n(t)$ the pressure projected over the cavity eigenmodes:

$$p_n(t) = \int p(\mathbf{r}, t) \Psi_n(\mathbf{r})\ \mathrm{d}V. \quad (5)$$

The wave equation can be written as:

$$\sum_n -k_n^2 p_n(t) \Psi_n(\mathbf{r}) - \frac{1}{c^2} \frac{\partial^2 p_n(t)}{\partial t^2} \Psi_n(\mathbf{r}) =$$
$$-\rho_0 \frac{\partial q(\mathbf{r}_0, t)}{\partial t} + \rho_0 \frac{\partial^2 w(\mathbf{r}_s, t)}{\partial t^2}. \quad (6)$$

Multiplying by $\Psi_m(\mathbf{r})$ and using orthogonality properties, we obtain:

$$\frac{\partial^2 p_n(t)}{\partial t^2} + \omega_n^2 p_n(t) = \frac{\rho_0 c^2}{\Lambda_n} \left(Q'_n(t) - \int_S \frac{\partial^2 w(\mathbf{r}_s, t)}{\partial t^2} \Psi_n(\mathbf{r}_s)\ \mathrm{d}S \right), \quad (7)$$

with the generalised output $Q'_n(t) = \int_S \frac{\partial q(\mathbf{r}_0, t)}{\partial t} \Psi_n(\mathbf{r}_s)\ \mathrm{d}S$

and $\Lambda_n = \int_V \Psi_n^2(\mathbf{r}_s)\ \mathrm{d}V$.

The vibration field w can be decomposed over plate eigenmodes, Φ_p:

$$w(\mathbf{r}_s, t) = \sum_{p=0}^{\infty} w_p(t) \Phi_p(\mathbf{r}_s). \quad (8)$$

The wave equation is then:

$$\frac{\partial^2 p_n(t)}{\partial t^2} + \omega_n^2 p_n(t) =$$
$$\frac{\rho_0 c^2}{\Lambda_n} \left(Q'_n(t) - \int_S \sum_p \frac{\partial^2 w_p(t)}{\partial t^2} \Phi_p(\mathbf{r}_s) \Psi_n(\mathbf{r}_s)\ \mathrm{d}S \right). \quad (9)$$

In harmonic conditions and with the introduction of damping, the equation becomes:

$$-\omega^2 p_n + 2j\xi\omega\omega_n p_n + \omega_n^2 p_n = \frac{\rho_0 c^2}{\Lambda_n} \left(j\omega Q_n + \omega^2 \sum_p w_p C_{pn} \right), \quad (10)$$

with the coupling term $C_{pn} = \int_S \Phi_p(\mathbf{r}_s) \Psi_n(\mathbf{r}_s)\ \mathrm{d}S$.

The vibration of the plate is described by the motion equation:

$$D\nabla^4 w(\mathbf{r}_s, t) + \rho h \frac{\partial^2 w(\mathbf{r}_s, t)}{\partial t^2} = f(\mathbf{r}_{s0}) + p(\mathbf{r}_s, t), \quad (11)$$

where D is the flexural rigidity, ρ the plate density, h the plate thickness, $f(\mathbf{r}_{s0})$ the external pressure and $p(\mathbf{r}_s, t)$ the pressure in the cavity. With a projection over plate eigenmodes and orthogonality properties, the motion equation becomes:

$$\omega_p^2 w_p + \frac{\partial^2 w_p}{\partial t^2} = \frac{1}{\rho h \Lambda_p} \left(F_p + \int_S p(\mathbf{r}_s, t) \Phi_p(\mathbf{r}_s)\ \mathrm{d}S \right), \quad (12)$$

with $\Lambda_p = \int_S \Phi_p^2(\mathbf{r}_s)\ \mathrm{d}S$ and $F_p = \int_S f(\mathbf{r}_{s0}) \Phi_p(\mathbf{r}_s)\ \mathrm{d}S$.

Using (4), harmonic conditions and introduction of damping, it gives:

$$\left(\omega_p^2 + 2j\xi\omega\omega_p - \omega^2 \right) w_p = \frac{1}{\rho h \Lambda_p} \left(F_p + \sum_n p_n C_{pn} \right). \quad (13)$$

We obtain two coupled equation, (10) and (13), which under matrix form gives:

$$\begin{bmatrix} \mathbf{Z_{11}} & -\mathbf{C} \\ -\omega^2 \mathbf{C^T} & \mathbf{Z_{22}} \end{bmatrix} \begin{bmatrix} \mathbf{W} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{Q'} \end{bmatrix}, \quad (14)$$

with

$$\mathbf{Z_{11}} = diag((\omega_p^2 + 2j\xi\omega\omega_p - \omega^2)\rho h \Lambda_p),$$
$$\mathbf{Z_{22}} = diag((\omega_n^2 + 2j\xi\omega\omega_n - \omega^2)\frac{\Lambda_n}{\rho_0 c^2}),$$
$$\mathbf{C} = [C_{pn}],$$
$$\mathbf{W} = [w_1 w_2 ... w_p],$$
$$\mathbf{P} = [p_1 p_2 ... p_n],$$
$$\mathbf{F} = [F_1 F_2 ... F_n],$$
$$\mathbf{Q'} = j\omega[Q_1 Q_2 ... Q_n].$$

The pressure p and displacement w are obtained by inverting system (14):

$$\begin{bmatrix} \mathbf{W} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{Z_{11}} & -\mathbf{C} \\ -\omega^2 \mathbf{C^T} & \mathbf{Z_{22}} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{F} \\ \mathbf{Q'} \end{bmatrix}. \quad (15)$$

III. INVERSE PROBLEM

The FAT method developed by Pézerat and Guyader [6] allows to identify the force distribution from local vibration field. It is based on the inverse resolution of the motion equation. For a plate, the motion equation is presented in (11).

Partial derivatives are estimated by a finite difference scheme developed at the first order with a regular spatial mesh [7]. In order to estimate the force distribution on the central

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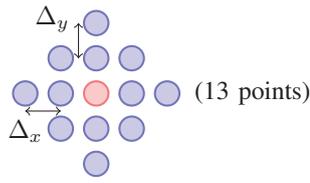
Fig. 2 Finite difference scheme which allows the estimation of the force on the central point of the scheme

point of the scheme, 13 points are needed (see Fig. 2), where Δ_x and Δ_y are spatial steps along x and y:

$$F^{FAT}(x,y,\omega) = D\left(\delta_{\Delta_x}^{4x} + \delta_{\Delta_y}^{4y} + 2\delta_{\Delta_{x,y}}^{2x2y}\right) - \rho h\omega^2 w(x,y,\omega). \tag{16}$$

The advantage of this method is that it is not necessary to measure the vibration field on the entire structure or to know the boundary conditions, because the motion equation describes a local dynamic equilibrium which is sufficient to calculate F^{FAT}. However, when the vibration field is noisy, the inverse problem becomes unstable, and it gives results far from the force really applied, with higher levels. Indeed, derivatives increase fast variations (small wavelengths linked to measurement noise). For the inverse problem studied, spatial derivatives are of the fourth order so this considerably increases this phenomenon. In order to avoid this, it is necessary to remove the high wavenumber component polluted by noise in the force distribution obtained. The regularization notion is then introduced, it consists in filtering the high wavenumbers using a low pass filter with a finite impulse response. The filter response generally used is a sinc function weighted by a Hanning window [2].

The filtering operation consists in a discrete convolution between F^{FAT} and the filter impulse response. In order to avoid to inject the filter response in the motion equation, the filter is applied to the force distribution, not to the vibration field. Filtering allows to remove the high wavenumbers of the force distribution. However, beyond this domain, the force distribution is unknown so uncertainties on the edges of the domain studied may appear. In order to avoid edge effects, the force distribution F^{FAT} is windowed before being filtered, to annul the force on the edges of the domain.

A Tukey window is used, made of half Hanning windows at extremities and equal to 1 on the remaining domain [2] (see Fig. 3). The width of the half Hanning windows is the cut-off wavelength. The choice of the cut-off wavelength is essential in the regularisation, it has to be high enough not to degrade results and small enough to eliminate aberrant forces due to measurement errors. The cut-off wavelength is chosen proportional to the structural wavenumber [6]:

$$k_c = ak_f, \tag{17}$$

where a is the regularisation parameter.

IV. INTRODUCTION OF THE FLUID-STRUCTURE COUPLING INTO THE FAT

In order to take into account the fluid-structure coupling in the FAT method, a method based on the identification of an
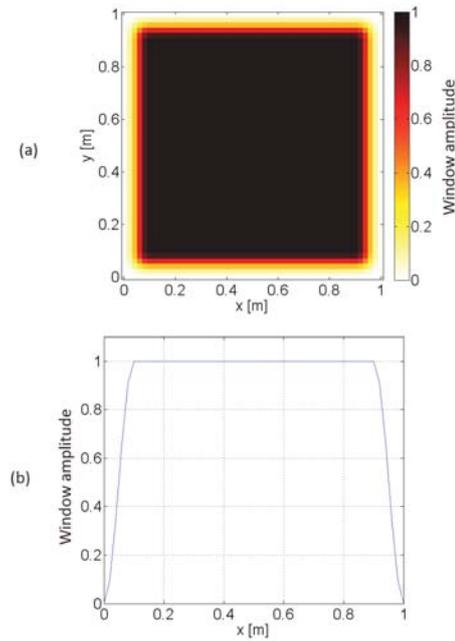
Fig. 3 (a) Spatial windowing used in FAT at 500 Hz with a=4 and in one dimension (b) at y = 0.5 m

effective wavenumber which takes into account the coupling is used. If we consider a finite plate with a fluid on one side, the plate motion equation considering fluid-structure coupling can be written as:

$$\nabla^4 w(x,y,\omega) - k_f^4 w(x,y,\omega) = \frac{p(x,y,\omega)}{D} + \frac{p_{coupling}(x,y,\omega)}{D}. \tag{18}$$

The motion equation can also be written as:

$$\nabla^4 w(x,y,\omega) - \gamma^4 w(x,y,\omega) = \frac{p(x,y,\omega)}{D}, \tag{19}$$

with γ the effective wavenumber which takes into account the fluid-structure coupling:

$$\gamma^4 = k_f^4 + \frac{p_{coupling}(x,y,\omega)}{Dw(x,y,\omega)}. \tag{20}$$

The aim of this method is to calculate the effective wavenumber γ^4. In order to calculate the effective wavenumber γ^4, a punctual force is applied to the plate (here in $x = 0.2\ m$ and $y = 0.5\ m$) and the FAT method is applied to a zone without stress (see Fig. 4). The second member of (19) is then equal to zero:

$$\Delta^2 w - \gamma^4 w = 0. \tag{21}$$

It is then possible to determine γ^4:

$$\gamma^4 = \frac{\Delta^2 w}{w}. \tag{22}$$

Results obtained for the identification of γ are presented in Fig. 5. The analytical effective wavenumber, γ_{an}, is calculated using the pressure and displacement obtained with the analytical problem, see (23). The effective wavenumber

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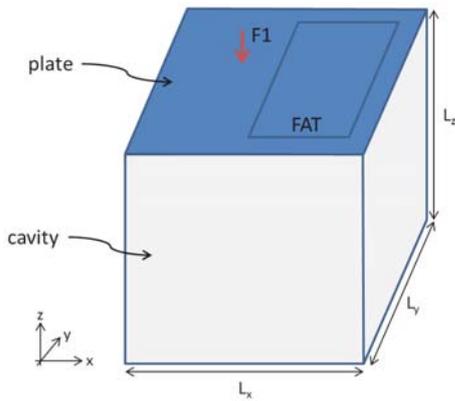
Fig. 4 Scheme of a plate coupled with a rigid cavity, with a punctual force F_1 outside the zone where FAT is applied

identified by FAT, γ_{FAT} coincides well with the one calculated analytically.

$$\gamma_{an} = \sqrt[4]{k_f^4 + \frac{p}{Dw}}, \qquad (23)$$

where p and w are respectively the pressure and displacement obtained with the analytical problem.
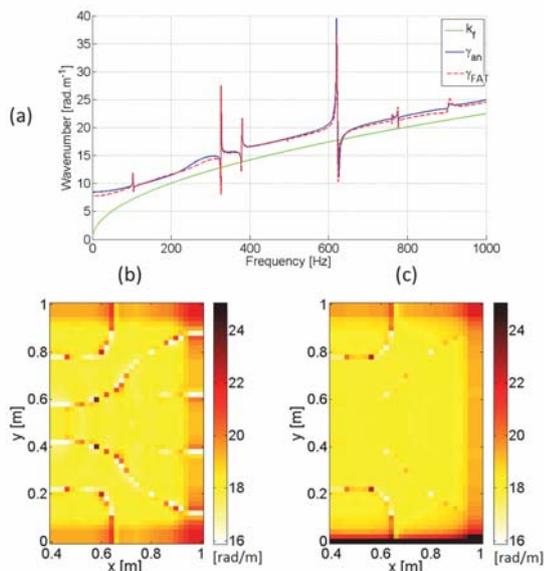
Fig. 5 (a) Evolution of γ (analytical and identified by FAT) and k_f with frequency in one point. Spatial evolution at 500 Hz of (b) γ_{an} obtained analytically, (c) γ_{FAT} identified by FAT

Then, once γ is known, a punctual force is applied to the zone where γ is identified, here in $x = 0.6\ m$ and $y = 0.5\ m$ (see Fig. 6), and FAT method is used with the effective wavenumber previously determined. The force distribution reconstructed by FAT, using either the wavenumber γ_{an}, calculated analytically, or the wavenumber γ_{FAT} identified by FAT, or k_f the flexural wavenumber, are presented in Fig. 7. On the one hand, results show that the flexural wavenumber is not sufficient to correctly identify the force distribution. On the other hand, the use of the effective wavenumber in FAT allows to take into account the fluid structure coupling and to

accurately identify the force distribution.

Fig. 6 Scheme of a plate coupled with a rigid cavity, with a punctual force F_2 inside the zone where FAT is applied
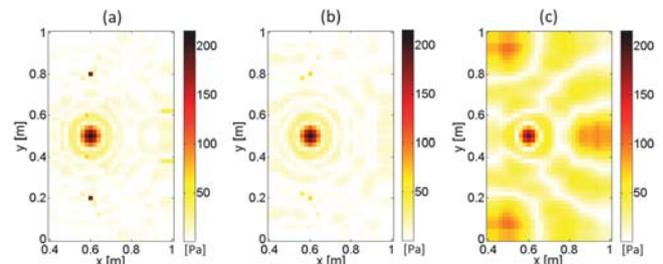
Fig. 7 Reconstructed force distribution at 500 Hz using (a) γ_{FAT} obtained with FAT, (b) γ_{an} obtained analytically, (c) k_f

V. CONCLUSION

This paper presents a method to take into account the fluid-structure coupling in the FAT method. This method is based on the one hand, on the identification of an effective wavenumber, γ, which takes into account the fluid-structure coupling, by using the FAT method. On the other hand, the method uses the re-introduction of this effective wavenumber γ into the FAT method to only identify the external forces applied to the structure. This method is applied to a plate coupled with a rigid cavity full of water. Results involving a punctual force excitation show that the effective wavenumber γ is correctly identified. The use of this effective wavenumber γ into FAT allows us to only identify the force distribution due to the punctual force, contrary to results obtained using the flexural wavenumber, k_f. Further work will focus on the one hand, on an experimental validation of this method to take into account the fluid-structure coupling, on the other hand, on its optimisation and its application in the case of a structure excited by turbulent flow.

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