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# Signal Processing Approach to Study Multifractality and Singularity of Solar Wind Speed Time Series

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***Abstract*—**This paper investigates the nature of the fluctuation of the daily average Solar wind speed time series collected over a period of 2492 days, from 1st January, 1997 to 28th October, 2003. The degree of self-similarity and scalability of the Solar Wind Speed signal has been explored to characterise the signal fluctuation. Multi-fractal Detrended Fluctuation Analysis (MFDFA) method has been implemented on the signal which is under investigation to perform this task. Furthermore, the singularity spectra of the signals have been also obtained to gauge the extent of the multifractality of the time series signal.

***Keywords*—**Detrended fluctuation analysis, generalized Hurst exponent, holder exponents, multifractal exponent, multifractal spectrum, singularity spectrum, time series analysis.



## I. Introduction

SOLAR wind [1] consists of ionized plasma along with some residual solar magnetic field. It radially outflows from the hot corona of the Sun into interplanetary space. The massive differential pressure of the gas between the solar corona and interstellar space is the primary cause of this solar wind flow. This colossal differential pressure difference forces the charged solar plasma to overcome the intense gravitational force of the Sun and causes it to stream out through the solar surface in the form of solar wind. The nature of the fluctuation of this solar wind is a matter of great curiosity to the scientists and astrophysicists. In this paper, the authors have tried to implement the statistical signal processing methodology to shed some light on this aspect. The solar wind speed time series from 1st January, 1997 to 28th October, 2003 (Space Weather Prediction Center, National Oceanic and Atmospheric Administration) has been taken as the signal under investigation. The complex behaviour of the signal has been characterized by probing the multifractality and inspecting the singularities in it. A system is said to be *fractal* when it has a non-integer dimension or *fractal dimension*. A fractal system which has more than one fractal dimensions are known as a *multifractal system*. So, for a multi-fractal system instead of a single fractal dimension, a continuous spectrum of the fractal dimension is required to explain its dynamics. This continuous spectrum is referred to as a *singularity spectrum.*

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MFDFA, which is immune to the effect of the non-stationarity of the signal, has been used to investigate the presence of multifractality, to learn about the degree of multifractality and nature of the singularity spectrum of the solar wind speed time series. MFDFA removes the underlying polynomial trends and focuses on the study of the fluctuation [2]-[5].

Here the detailed analysis of fractal properties of the solar wind speed time series has been unearthed. The signal parameters like scaling exponents $\tau(q)$, multiracial scaling exponents $h(q)$ and generalised multifractal dimensions $D(q)$, which quantifies the multifractality of the signal have been computed. For tracking the singularities in the time series signal, the singularity strength or Holder exponent ($\alpha$) is computed and the Hausdorff dimension or singularity spectrum $f(\alpha)$ is obtained. The results from all these methods indicate that the time series exhibits strong multifractality.

## II. Methodology

The method of MFDFA [6], [7] is the modified edition of generalized detrended fluctuation analysis (DFA) [8]. The intrinsic fluctuation of the time series signal is taken out by discriminating the polynomial trends from the signal. For a time series *X(i)*, MFDFA basically consists of five steps [9], [10]. The first three steps are essentially identical to the conventional DFA procedure.

**Step 1:** The signal profile *Y(j)* is determined as:

$$Y(j) = \sum_{i=1}^{j} (X(i) - \langle X \rangle) \quad j = 1, 2, ......N \tag{1}$$

Here $\langle X \rangle$ is the mean of the time series.

**Step 2:** The profile *Y(j)* is segmented into $N_s$ non-overlapping segments of equal length *s*. Since the length *N* of the time series may not be a multiple of the considered time scale *s*, a short tail at the end of the profile may remain. In order to take this tail of the series into account, the same procedure is repeated starting from the opposite end. Thereby, 2 $N_s$ segments are obtained altogether. In each segment, we fit the integrated time series by using a polynomial function $\rho_v(j)$ which is regarded as the local trend. The local trend in each segment is subtracted and the detrended fluctuation function $Y_s(j)$ is obtained as

$$Y_s(j) = Y(j) - \rho_v(j) \tag{2}$$

**Step 3:** In each segment of size $s$, from the detrended fluctuation function, we determine the variance $F^2(s, v)$:

$$F^2(s,\nu) = \frac{1}{s}\sum_{j=1}^{s}\{Y[(\nu-1)s+j]-\rho_\nu(j)\}^2 \tag{3}$$

for each segment v, v= 1, 2,….$N_s$ and

$$F^2(s,\nu) = \frac{1}{s}\sum_{j=1}^{s}\{Y[N-(\nu-N_s)s+j]-\rho_\nu(j)\}^2 \tag{4}$$

for v= $N_s$+ 1,……….., $2N_s$.

The order of the polynomial $\rho_\nu(j)$ determines the order of trend in the time series which will be removed. For the linear, quadratic, cubic or higher order polynomials, MFDFA are denoted accordingly like MFDFA1, MFDFA2, MFDFA3 and so on [11], [12]. MFDFA2 which will remove trend up to quadratic one is chosen.

**Step 4:** The averages of $F^2(s, v)$ over all segments $N_s$ are taken to find the q[th]-order fluctuation function $F_q(s)$ as

$$F_q(s) \equiv \{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}[F^2(s,\nu)]^{\frac{q}{2}}\}^{\frac{1}{q}} \tag{5}$$

Here the moment $q$ can be any real value except zero. Because when $q \rightarrow 0$, $F_q(s)$ will diverge. So a logarithmic average has been taken to find $F_q(s)$ at $q \rightarrow 0$ as

$$F_0(s) \equiv \exp\{\frac{1}{2N_s}\sum_{\nu=1}^{2N_s}\ln\left|F^2(s,\nu)\right|\} \tag{6}$$

This procedure is repeated for different scale length $s$ and $F_q(s)$ for different values of $q$ is computed.

**Step 5:** The final step is to estimate the slope of the log-log plot of $F_q(s)$ versus $s$ for each value of $q$. From this plot, the scaling behaviour of the fluctuation functions can be determined. If the series $X(i)$ are long-range power-law correlated, $F_q(s)$ increases with increase of $s$, as a power-law

$$F_q(s) \sim s^{h(q)} \tag{7}$$

Here, the moment $q$ from -10 to +10 with the increment of 0.25 and the scale $s$ from 10 to 300 ($\leq N/5$) [13] with the increment of 5 has been taken. Form the log–log plot of $F_q(s)$ against $s$, $h(q)$ is calculated for various values of $q$. If $h(q)$ varies with $q$, one can conclude that the scaling is multifractal. For mono fractal time series, $h(q)$ is a constant $H$. If $H = 0.5$, there is no correlation and the data is an uncorrelated signal (white noise). For $H < 0.5$, the data is anti-correlated. The data is long-range correlated for $H < 0.5$. The exponent $h(2)$ is identical to the well-known Hurst exponent $H$. Therefore, the exponent $h(q)$ is called generalized Hurst exponent.

For positive values of $q$, the segments $v$ with large variance $F^2(s, v)$ will dominate the average $F_q(s)$. Thus, for positive values of $q$, $h(q)$ describes the scaling behaviour of the segments with large fluctuations. For negative values of $q$, the segments $\nu$ with small variance $F^2(s, v)$ will dominate the average $F_q(s)$. Hence, for negative values of $q$, $h(q)$ describes the scaling behavior of the segments with small fluctuations.

The generalized Hurst exponent $h(q)$ is related to multifractal exponent $\tau(q)$ which is regarded as a characteristic function of the fractal behavior. For a time series, $\tau(q)$ can be obtained through the generalized Hurst exponent $h(q)$ as:

$$\tau(q) = qh(q) - 1 \tag{8}$$

If $\tau(q)$ versus $q$ is linear, the time series is monofractal. On the other hand, non-linear $\tau(q)$ vs. $q$ curve is the signature of non-homogeneous functions that exhibit multifractal properties under the time series signal. On the basis of the obtained $\tau(q)$ characteristics, we can estimate the well-known curves of $f(\alpha)$- $\alpha$ (where $\alpha$ is the Lipschitz–Holder exponent) and $D(q)$ curve. The curve of $f(\alpha)$ versus $\alpha$ is called the multifractal spectrum for the time series data $X(i)$. This spectrum is an important tool in fractal investigation for the time series. The Holder exponents $\alpha$ and the singularity spectrum $f(\alpha)$ are calculated by Legendre transform as:

$$\alpha(q) = \frac{d\tau(q)}{dq} \tag{9}$$

$$f(\alpha) = q\alpha - \tau(q) \tag{10}$$

A single-humped spectrum curve is an indication towards multifractality. In the case of a monofractal time series, the curve of the spectrum converges to a point. The range of the fractal exponents present within the multifractal signal is reflected from the width of the single-humped spectrum curve, whereas the maximum of the spectrum provides the information about the dominant fractal exponent within the range. The generalized multifractal dimension $D(q)$ is related to $\tau(q)$ as:

$$D(q) = \frac{\tau(q)}{q-1}, \qquad q \neq 1 \tag{11}$$

For multifractal time series the $D(q)$ values have a strong non-linear dependence on $q$ values which is a typical characteristic of multifractals.

## III. Results

Fig. 1 represents the exponentially smoothed and denoised time series [14] of the daily Solar Wind Speed from 1[st]January, 1997 to 28[th] October, 2003.

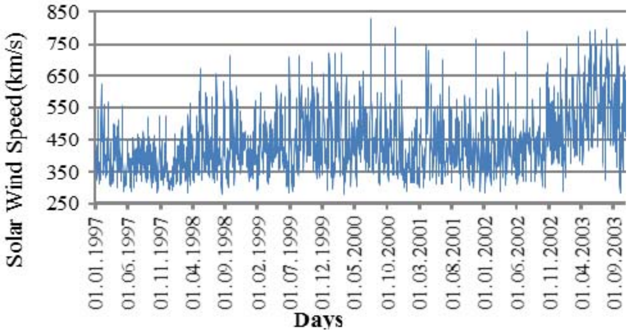


Fig. 1 Solar Wind Speed Time series

Fig. 2 gives the comparison of log-log plot of $F_q(s)$ against $s$ for various values of $q$ using MFDFA1, MFDFA2, MFDFA3 and MFDFA4 to the solar wind speed time series data. From Fig. 2 it is found that the difference between the slopes of the log-log plot of $F_q(s)$ against $s$ for MFDFA1 and MFDFA2 remains very small for every values of $q$, whereas those for MFDFA3 and MFDFA4 varies for every $q$. Thus, it can be said that the time series for the solar wind speed may have trend up to second-order and hence MFDFA2 is suitable for further analysis of the signal.

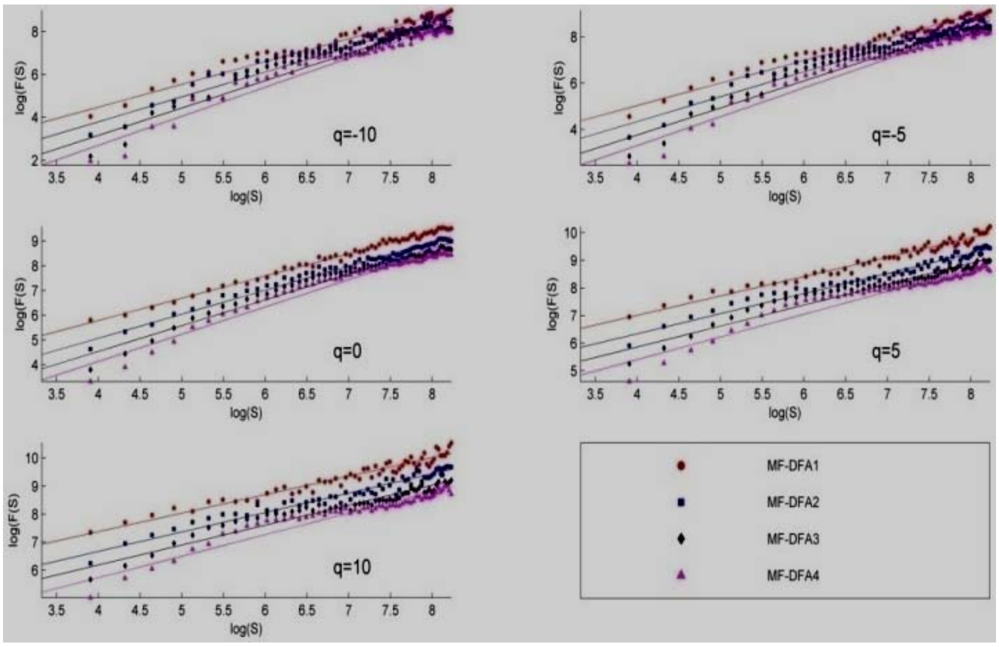


Fig. 2 log-log plot of $F_q(s)$ against $s$ of the solar wind speed time series data for various values of $q$ =-10, -5, 0, 5 and 10 using MFDFA1, MFDFA2, MFDFA3 and MFDFA4

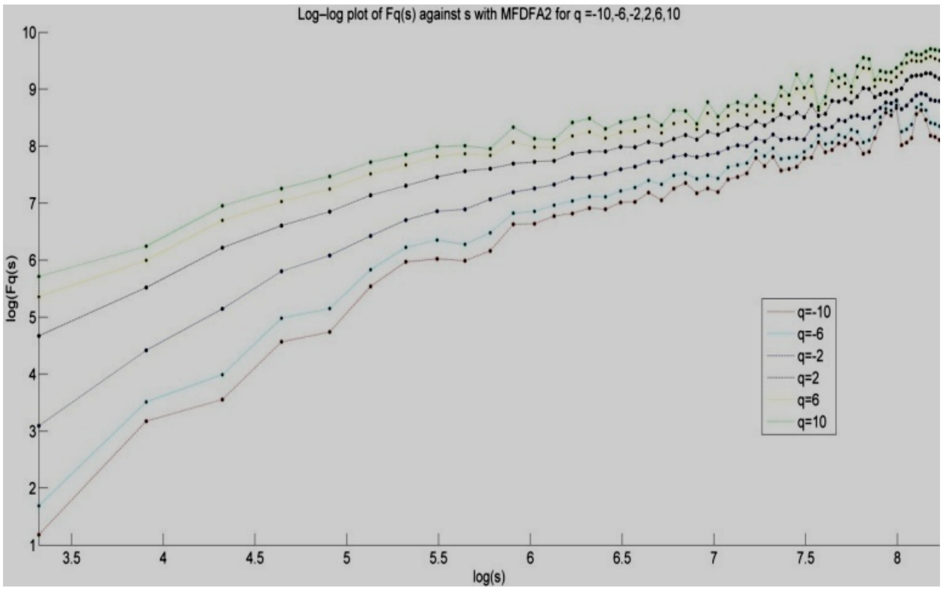


Fig. 3 *log*-log plot of $F_q(s)$ against $s$ with MFDFA2 for $q$= -10, -6, -2, 2, 6, 10

Fig. 3 shows the variation in the $\log-\log$ plot of $F_q(s)$ against $s$ with MFDFA2 for $q$ = -10, -6, -2, 2, 6, 10for the solar wind speed time series signal. The mean $F_q(s)$ taken over all the scales $s$ is denoted by $F(q)$. Fig. 4 gives the comparative view of the plots of $F(q)$ against $q$, $h(q)$ against $q$, $\tau(q)$ against $q$, $D(q)$ against $q$, $\alpha(q)$ against $q$, $f(\alpha)$ against $\alpha(q)$ and $f(\alpha)$against $q$ for the solar wind speed time series data.

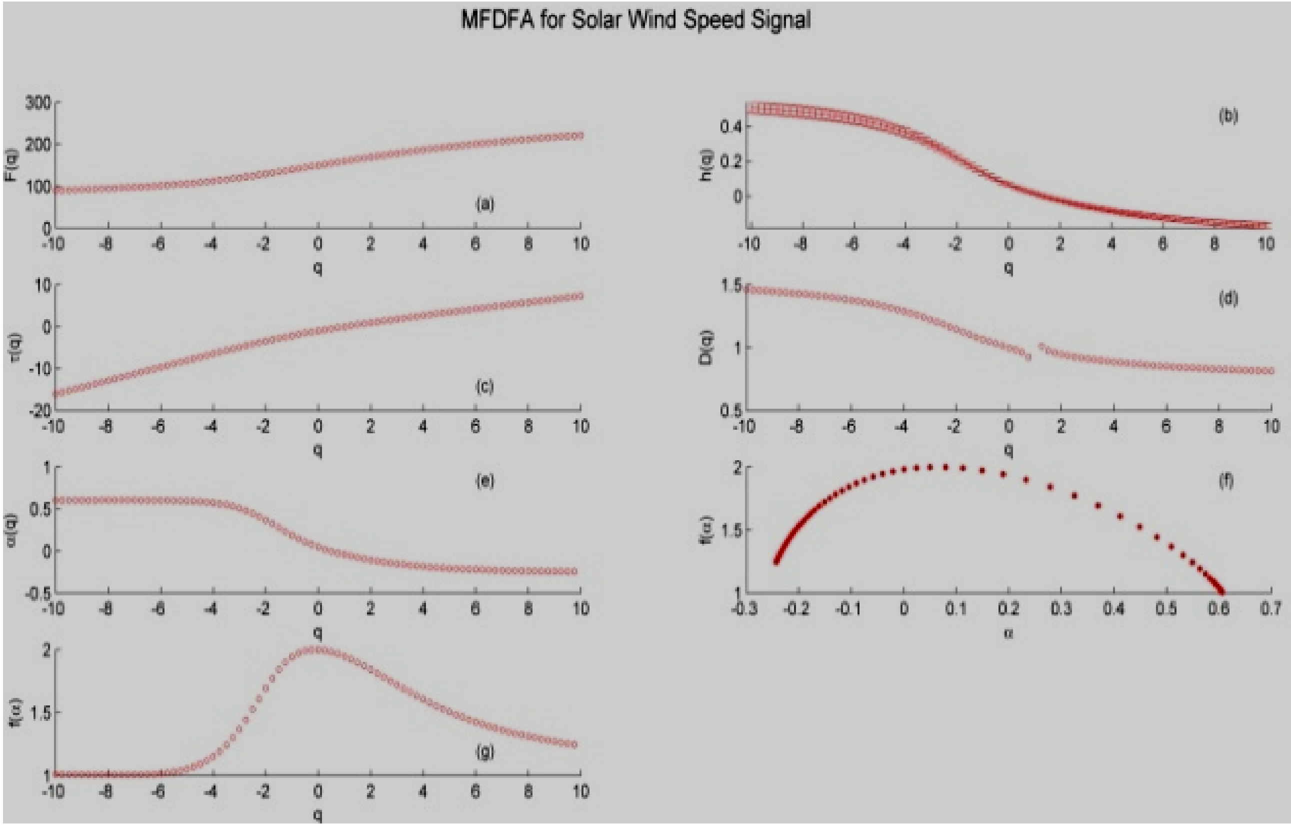


Fig. 4 Comparative view of the plots for the solar wind speed time series data (a) $F(q)$ against $q$, (b) $h(q)$ against $q$, (c) $\tau(q)$ against $q$, (d) $D(q)$ against $q$, (e) $\alpha(q)$ against $q$, (f) $f(\alpha)$ against $\alpha$, (g) $f(\alpha)$against $q$

Extrapolating the curve in Fig. 4 (f), we obtain the results like $\alpha_{\min}$ ($f$ = 0), $\alpha_{\max}$ ($f$ = 0) in the Table I.

There are two strong causes in a signal to exhibit multifractal behaviour: (i)due to the broad probability density function (PDF) values of the time series and (ii) due to the different long-range correlations for small and large fluctuations [15]. If the cause of multifractality is the first one, then multifractality in the signal cannot be removed by randomising the series, whereas if the cause of the multifractality is the second one, then the series may have PDF with finite moments that are a Gaussian distribution. If the cause of multifractality is both the randomised series will show weaker multi fractility than the original signal. The easiest way to detect the cause of the multifractality is to analyse the corresponding randomised (shuffled and surrogate) [16] time series. As the values are being put in randomly due to shuffling or surrogating, all the correlations are being destroyed but the PDF stay unaffected. So, the time series whose randomised version gives $h(q)$ = 0.5 for all values of $q$ can be said to have multifractality due to type (ii) reason. On the contrary if the randomized version of the signal reveals that the $h(q) \neq 0.5$ and is dependent on $q$, the multifractality within the signal is due to type (i) reason. The multifractality nature due to the fatness of the PDF signals is not affected by the shuffling procedure. On the other hand, the correlations in the surrogate series do not change, but the probability function changes to the Gaussian distribution. If multifractality in the time series is due to a broad PDF, $h(q)$ obtained by the surrogate method (amplitude-adjusted Fourier transform) will be independent of $q$. Fig. 5 shows the comparative view of the plots of $F(q)$ against $q$, $h(q)$ against $q$, $\tau(q)$ against $q$, $D(q)$ against $q$, $\alpha(q)$ against $q$, $f(\alpha)$ against $\alpha$ and $f(\alpha)$ against $q$ for the original, surrogated and shuffled solar wind speed signals.

TABLE I
NUMERICAL RESULTS FOR THE SINGULARITY SPECTRUM FROM FIG. 4 (F)

| Spectrum Parameter | Obtained Value |
|---|---|
| $\alpha_{\min}$ ($f$ = 0) | -0.3110 |
| $\alpha_{\max}$ ($f$ = 0) | 0.6850 |
| $W = \alpha_{\max} - \alpha_{\min}$ | 0.9960 |
| $f_{\max}(\alpha)$ | 2 |
| $\alpha$ ($f = f_{max}(\alpha)$) | 0.0792 |

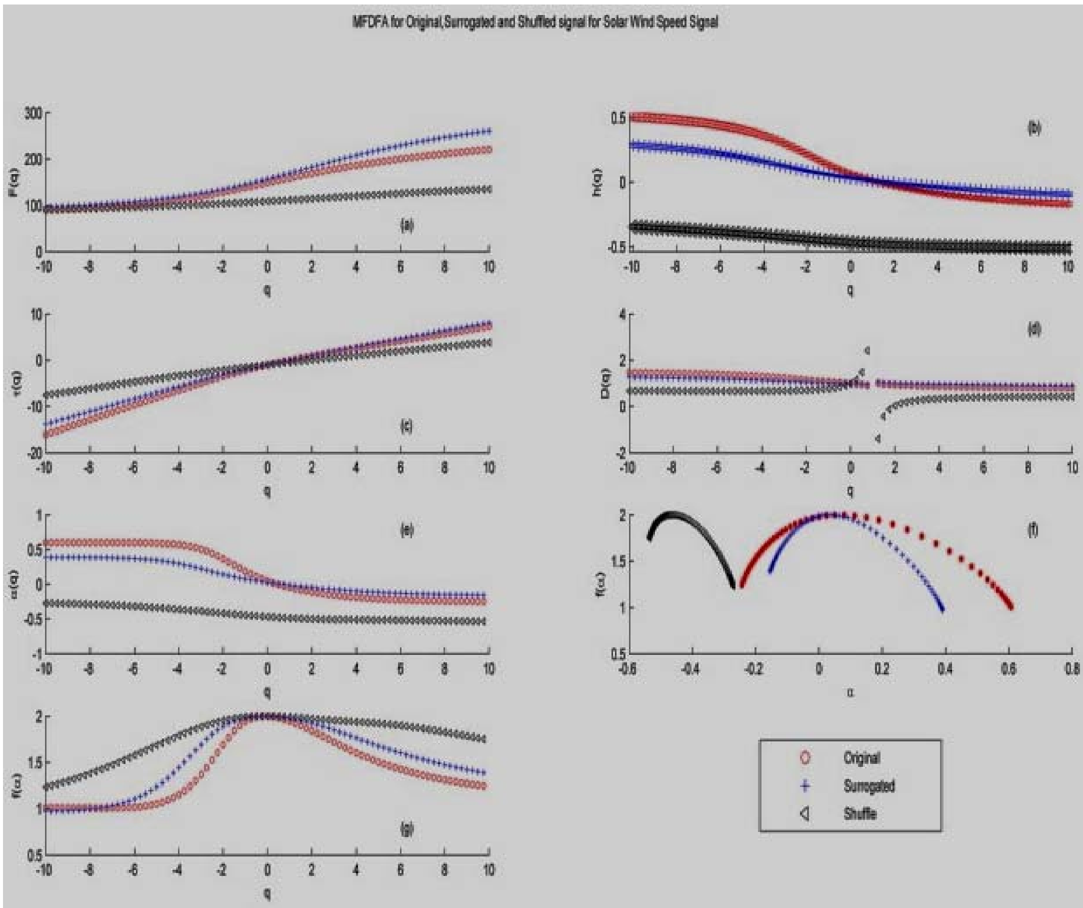


Fig. 5 Comparative view of the plots for original, surrogated and shuffled time series data: (a) $F(q)$ against $q$, (b) $h(q)$ against $q$, (c) $\tau(q)$ against $q$, (d) $D(q)$ against $q$, (e) $\alpha(q)$ against $q$, (f) $f(\alpha)$ against $\alpha$, (g) $f(\alpha)$ against $q$

## IV. Discussion and Conclusion

As shown in Fig. 4 (b), the generalized Hurst exponent $h(q)$ obviously depends on $q$. The result that the dependence of exponent $h(q)$ on $q$ indicates the solar wind speed time series possesses multifractality. The relationship between $\tau(q)$ and $q$, as shown in Fig. 4 (c), is non-linear which is another indication of multifractal behaviour under the fluctuations of the solar wind speed time series signal. From the singularity spectrum as in Fig. 4 (f) the curve $f(\alpha)$ is single-humped which also shows the multifractality of the solar wind speed time series signal. The width of the singularity spectrum indicates the range of fractal exponents present, and thus, it measures the degree of the multifractality of the series. The width of the singularity spectrum is measured by extrapolating the fitted curve to zero. We calculate the width of spectrum function is $W$ = 0.9960. The maximum of the spectrum function is $f(\alpha)_{max}$= 2, which occurs at $\alpha$ = 0.0792. These results indicate the time series have intense multifractality. From Figs. 5 (b) and (c) it is observed that variation in $h(q)$ and $\tau(q)$ for the randomised data series are much flatter than those of the original signal. Fig. 5 (b) shows that $h(q)$ for the randomised data series are not equal to 0.5. This signifies that the multifractality in the signal is mainly due to the fat–tailed PDF of the time series.

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