Using Thinking Blocks to Encourage the Use of Higher Order Thinking Skills among Students When Solving Problems on Fractions

Abdul Halim Abdullah, Nur Liyana Zainal Abidin, Mahani Mokhtar

***Abstract*—**Problem-solving is an activity which can encourage students to use Higher Order Thinking Skills (HOTS). Learning fractions can be challenging for students since empirical evidence shows that students experience difficulties in solving the fraction problems. However, visual methods can help students to overcome the difficulties since the methods help students to make meaningful visual representations and link abstract concepts in Mathematics. Therefore, the purpose of this study was to investigate whether there were any changes in students' HOTS at the four highest levels when learning the fractions by using Thinking Blocks. 54 students participated in a quasi-experiment using pre-tests and post-tests. Students were divided into two groups. The experimental group (n=32) received a treatment to improve the students' HOTS and the other group acted as the control group (n=22) which used a traditional method. Data were analysed by using Mann-Whitney test. The results indicated that during post-test, students who used Thinking Blocks showed significant improvement in their HOTS level (p=0.000). In addition, the results of post-test also showed that the students' performance improved significantly at the four highest levels of HOTS; namely, application (p=0.001), analyse (p=0.000), evaluate (p=0.000), and create (p=0.000). Therefore, it can be concluded that Thinking Blocks can effectively encourage students to use the four highest levels of HOTS which consequently enable them to solve fractions problems successfully.

***Keywords*—**Thinking blocks, higher order thinking skills, fractions, problem solving.

I. Introduction

FRACTIONS are considered as one of the important topics in mathematics. One of its importance is to establish and develop mathematical ideas to be applied in everyday situations [1]-[4]. In addition, fractions are the basics of mathematics and act as continuity towards other mathematical topics [5], for example, percentages and algebra. Reference [1] stated that primary school students should master the fractions topic prior to learning algebra. Furthermore, fractions are vital in science. Reference [6] stated that division and multiplication of fractions is important in Physics. Fractions can assist students to correlate variables with certain formulas. Reference [6] also found that students who understand the concept of multiplication and division of fractions had successfully determined the physics formula used to solve problems. Hence, the fraction topic must be mastered by students since knowledge on the topic can provide mental structure towards the continuous intellectual development [1]. Fractions are also perceived as numbers that have a unique characteristic, which can cause difficulties for the students to understand and learn its contents, and unable to master the real meaning of fractions [7], [8]. Therefore, the students think that fractions are a difficult topic in mathematics. In addition, the students are having difficulties in learning fractions is because of the teaching method which is abstract in nature and too complex. This situation led to the students' inability to grasp the concept of fractions, have a vague understanding on the concept of fractions, low proficiency level [1], and failed to apply the concept of fractions in solving fraction-based problems. According to [9], there are certain students who think that fractions are meaningless and irrelevant in life because their proficiency level is low and obscure. The students were unable to understand the reason for the need of learning fractions and its usage in the real world. Therefore, the existing knowledge of the students is disconnected from the symbolic fraction [10].

TIMSS mathematics assessment consists of two types of domains tested on the students; namely, the content domain and the cognitive domain. The cognitive domain focuses on the students' thought process while engaging the students with the mathematical content. The cognitive domain assessment involves four components of higher-order thinking in the revision of the Revised Bloom's Taxonomy; namely, to apply, to analyse, to evaluate, and to create [11]. TIMSS 2011 apparently affirms that the students in Malaysia are very weak in the cognitive domain application and interpretation [12] that requires the students to use HOTS. The weak achievement of the Malaysian students in the cognitive domain illustrates that they do not have the HOTS. The content domain which refers to the content of the subjects that will be assessed include four areas of mathematics; namely, number, geometry, algebra, and data and probability. The number and algebra domains are the domains that are most often assessed at 30%, followed by geometry (20%), and data and probability (20%). The average score earned by the Malaysian students for the number content domain in TIMSS 2007 is 494 and is 451 in the TIMSS 2011, which indicated a drastic decline compared to the other countries involved [12]. In this study, the fraction is one of the selected topics of mathematics as a field of study and Fraction is part of the number domain.

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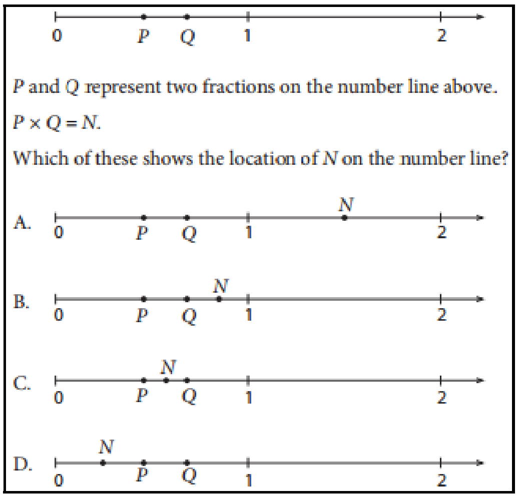

Fig. 1 TIMSS 2011 Sample Question [11]

Based on Fig. 1, the question posed to students is a form of reasoning cognitive domain and number content domain involving fractions. Students were asked to identify the result of the multiplication of fractions. The findings show that only 18% of students in Malaysia answered correctly where the correct percentages are below the international standard of 23%. The situation illustrates the inability of Malaysian students who are unable to grasp the facts, correlate and use it to solve a given problem, and faced difficulties in applying HOTS in mathematics. The findings are reinforced with the findings by [13] who found that the factors that cause difficulties for students in mastering the basic concepts of HOTS are low proficiency of basic concepts, difficulties in depicting the mental picture, and inability to connect the information given in solving the problem. This shows that the students' level of HOTS proficiency in solving mathematical problems is still at a low level [14].

Fractions are one of the mathematics topics that had been introduced to the students since primary education level. According to [15], fractions are the most complex concepts among children in primary schools. In fact, some previous studies such as by [16], [17] found that the problems faced by the students in primary schools in learning fractions can continue till secondary schools and higher education. Meanwhile, a study by [18] demonstrated that the students' misconceptions regarding numbers (whole numbers, sequences and number patterns, fractions, decimals, percentages, and integer) can be found throughout the mathematics curriculum. Reference [18] also found that students are unable to visualise or understand the solutions related to fractions and equivalent fractions. This will cause the failure of the students in solving mathematical problems in fractions.

The ability to illustrate a problem by using diagram is the most critical concept in solving the problem. However, there are studies which showed that one of the strategies used in problem-solving is using the visual method [19]-[22]. The visual method defined by Presmeg (1986a) in [19] is a visual image of either in a mind or in a diagram. A study by [23] found that low achievers showed a significant improvement when solving fraction problems using virtual manipulatives and pictorial models. Through the given stimulus, e.g. diagrams, students can build a thorough understanding and increase high visual thinking of the problem to be solved. Reference [25] has identified that the role of visual in mathematics is to understand the problem, to simplify the problem, to connect the dots, to meet individual learning styles, as a substitute to computation, as a tool to check the solution and to convert the problem into a mathematical form. Through visual too, the problem can be represented in a situation where it assists students to improve their understanding, thinking, and knowledge while providing them experience [26]. In addition, visuals can assist students to understand concepts in a short period of time although the concept seems difficult and complex [27]. This is because the visual information can be received by the brain as much as 80%. Therefore, visual in teaching and learning process are very important in providing a clear picture for the students to understand a given problem and build concepts in mathematics.

There is a visual tool used during mathematical problem-solving, namely, the Representative Model method. Studies by [19]-[22], [24] on the Representative Model found that most students in Singapore are using this method to solve mathematical problems. The Representative Model is a heuristic method for solving mathematical problems using an image as representative [19], [28], [29], [21]. In formulating a model, the students translate information from words into a diagram. Based on the diagram, the students can describe the situation, understand the operations involved, and the relationship between the variables involved [28]. As stated by [28], the use of diagrams can assist in problem-solving. Therefore, the visual method in the representative model that uses the draw-the-diagram approach is a result of visual thinking that can motivate students to solve mathematical problems effectively. In short, the objective of this study is to examine the changes in the top four HOTS students' level in fractions by using Thinking Blocks. The specific objectives of this study are:

- To identify the changes in the students in terms of application level in fractions learning by using Thinking Blocks.
- To identify the changes in the students in terms of analysis level in fractions learning by using Thinking Blocks.
- To identify the changes in the students in terms of evaluation level in fractions learning by using Thinking Blocks.
- To identify the changes in the students in terms of creation level in fractions learning by using Thinking Blocks.

II. Thinking Blocks

The learning and teaching in this study used Thinking Blocks which was adapted from the Representative Model (Bar Modelling) that has been used in all primary schools in

Singapore since the 90s. Representative model is a heuristic method of problem-solving using a pictorial representation of rectangular bars in solving mathematical word problems [28], [29]. Besides, it is a method of visual learning. Presmeg (1986a) (in [19]) defined visual method as a visual image either in the mind or a diagram. Reference [30] stated that the Representative Model is a structured and systematic mathematics teaching method. The method of teaching mathematics using the Representative Model is able to a) assist students to describe the situations involved, b) assist students to understand in detail the operations involved, and c) assist students to see the relationship between the variables involved [28]. According to [20], there are two elements that form the Representative Model; namely, the Schema Theory and the Problem Solving Theory. The Schema Theory is part of a two-phase Mayer model related to problem-solving. This theory was formulated in the technique of using a schematic diagram to access the schema knowledge. A schematic diagram is central to the Representative Model. The Representative Model has the potential to provide an effective effect by acting as a bridge between the representation phase (visual) and Mayer model solution phase. In addition, the Representative Model improves problem-solving by physically enabling problem-solving in realising the first phase of Mayer through a schematic diagram. The drawing also simplifies the problem-solving design in the Mayer second phase. In other words, when a child is asked to draw a model to solve a problem, the child can create a schematic physical representation that enhances the understanding of the situation, and then facilitate the solution to select the correct mathematical operations. Fig. 2 describes how the Representative Model react to the mathematical problem-solving theory as stated by Mayer (1985) in Reference [20]. The Mayer model helps to understand the potential of model drawings as one effective teaching strategy. The first step requires the students to categorise problems based on the schema type and then ask the students to draw a schematic diagram which includes Mayer representation phase in solving the problem. The generated diagrams within the students' minds can be used as a solution, hence improve the Mayer problem-solving phase. Reference [28] stated that the use of images as a precursor to the Representative Model is consistent with the theory of Bruner (1961), which uses enactive representation, iconic representation, and symbolic representation.

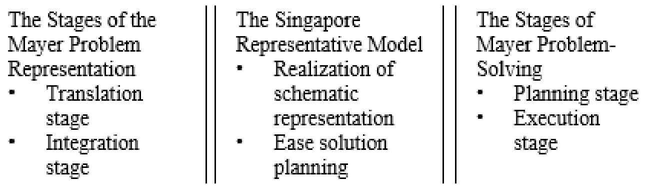

Fig. 2 Two-phase Mayer problem-solving model [20]

III. Research Methodology

The method used in the design of the experimental study is the quasi-experimental method since the samples cannot be selected randomly because of the rules and no permission given from the school to use other classes than the given classes [31]. In this study, the researchers used the design of pre- and post-test. The design segregated the sample into two groups; namely, the experimental group and the control group. In conducting this study, the experimental group was treated with the process of learning and teaching of fractions based on the Thinking Blocks, which the teaching activities had been planned by the researchers. In contrast, the control group was treated with the traditional teaching and learning process. Pre- and post-test was given to both groups to compare the Thinking Blocks application towards top four HOTS level namely to apply, to analyse, to evaluate, and to create [11]. As stated by [31], the pre- and post-test for the quasi-experimental study is suitable to be used to determine the effectiveness of a treatment or intervention. At the beginning of the process of learning and teaching for the fractions, the samples from both groups received a pre-test (Y1 and Y2). Then, the samples from the experimental group were given treatment on the teaching and learning process based on Thinking Blocks, X, while the control group was given the process of learning and teaching using traditional methods. After a month, the students also received a post-test (Y3 and Y4). The benefits of the conducted pre-test are to obtain information on the state of the samples prior to the treatment. Then, it can be compared with the post-test results after the samples had been treated. The sample consisted of 54 students who have similar academic backgrounds. The number of students per class is 22 people representing the control group and 32 people representing the experimental group as shown in Table I. In the first phase, data collection was performed quantitatively where the samples were given a pre- and post-test.

TABLE I
Study Sample Profile

Group	Gender		Total
	Male	Female	
Control Group	9	13	22
Experimental Group	14	18	32
	23	31	54

The items in the pre- and post-test were developed by the researcher based on the Integrated Curriculum for Secondary Schools (KBSM) Mathematics Form 1 as outlined by the Ministry of Education (MOE). The pre- and post-test set was divided into two parts. Part A contains demographic information of the study sample. Part B consists of the 12 questions related to fractions which was designed based on top four HOTS levels: to apply, to analyse, to evaluate, and to create, with reference to the revised Bloom's Taxonomy by [36]. The items in Part B were developed to examine the students' changes in terms of their level of application, analysis, evaluation, and creation in fractions learning using Thinking Blocks. Table II shows the distribution of items in a pre- and post-test done.

TABLE II
ITEMS IN THE PRE AND POST-TEST

Part	Variable	Item No.
A	Personal information	1, 2, 3 (Gender, Race, Class)
B	Application	1, 2, 3
	Analysis	4, 5, 6
	Evaluation	7, 8, 9
	Creation	10, 11, 12

TABLE III
MANN-WHITNEY TEST FOR HOTS LEVEL

		Pre-test	Post-test
To Apply	*Z*	-1.440	-3.188
	Asymp. Sig. (2-tailed)	.150	.001
To Analyse	*Z*	-1.655	-3.947
	Asymp. Sig. (2-tailed)	.098	.000
To Evaluate	*Z*	-1.138	-5.023
	Asymp. Sig. (2-tailed)	.255	.000
To Create	*Z*	-1.058	-5.262
	Asymp. Sig. (2-tailed)	.290	.000
Overall	*Z*	-2.368	-4.575
	Asymp. Sig. (2-tailed)	.018	.000

Items that were built for the pre- and post-test were in accordance with Test Specification Table (TST) to determine its validity. Validity refers to a measurement conducted to assess the suitability of the test scores in the real values of concept in the hypothesis [31]. Three mathematicians were selected based on specific criteria; namely, three teachers who are experienced in fractions and HOTS and they are mathematicians who have more than 10-year experience to evaluate the built items. The feedbacks received from these experts were taken into account to improve the items that do not meet the criteria. Reliability is closely related to the internal consistency of the built items in a test [32]. Thus, the *Cronbach Alfa* is referred to measure the internal consistency of the developed items. The data collection during the pilot study was analysed to determine the reliability of the instrument. According to [31], [33], built items with an alpha value of less than 0.6 will be removed from the research instrument due to the low level of reliability. In this pilot study, the *Cronbach Alpha* was ranged from 0.659 to 0.737, and the total value was 0.722, indicated that the internal consistency of the built items is high.

IV. DATA ANALYSIS

For normality, in the Kolmogorov-Smirnovaa test, the obtained values (Sig.) were 0.027 (post-control), 0.000 (post-experiment), 0.071 (pre-control), and 0.042 (pre-experiment). These indicate that the data collected in this study were found to be non-normally distributed. According to the Based on Mean, the values of Sig. (p-value) were 0.516 and 0.739, which exceed the value of 0.05; hence, the null hypothesis was accepted (not significant), which means that the variance of the two groups is homogeneous or uniform. Therefore, the assumption that the two groups are homogeneous was met.

To answer the research question, the changes in the students in terms of the level to apply, to analyse, to evaluate, and to create in the fractions learning by using Thinking Blocks, the researcher used the Mann-Whitney test to show the significant difference between the treated group and the control group. Referring to Table III, the Z-value = -1.440 for the pre-application test with significant level at 0.150, where the p-value (0.150) > α (0.05). These results indicate that there is no significant difference between the two variables; hence, the null hypothesis is accepted. For the post-application test, Z = -3.188 with significant level at 0.001 where p (0.001) < α (0.05). This indicates that there is a significant difference between the two variables; thus, the alternative hypothesis is accepted. For the pre-analysis test, Table III shows that the Z-value = -1.655 with significant level at 0.098 where p (0.098) > α (0.05). This result indicates that there is no significant difference between the two variables; so, the null hypothesis is accepted. For the post-analysis test, Z = -3.947 with significant level at 0.000 where p (0.000) < α (0.05). This indicates that there is a significant difference between the two variables; therefore, the alternative hypothesis is accepted. The pre-analyse test also indicates the significant level at 0.255, where the p-value (0.255) > α (0.05) and the Z-value = -1.138. This indicates that the alternative hypothesis is rejected because there is no significant difference between the two variables. For the post-evaluation test, the significant level is 0.000 where the p-value (0.000) < α (0.05) with the Z-value = -1.138. This situation shows that the null hypothesis is rejected because there is a significant difference between the two variables. The significant level of the pre-creation test value is 0.290 with Z = -1.058. These results demonstrate that the p-value (0.290) > α (0.05) where there is no significant difference between the two variables; hence, the alternative hypothesis is rejected. For the post-creation test, the significant level is at 0.000 with the Z-value = -5.262. These results demonstrate that the p-value (0.000) > α (0.05), where there is a significant difference between the two variables; then, the alternative hypothesis is accepted. Overall, Table III displays that the p-value for the pre-HOTS test is 0.018 < α (0.05), and post-HOTS test is 0.000 < α (0.05). The results showed that there are significant differences between the two variables; hence, the alternative hypothesis is accepted, and the null hypothesis is rejected. The Z-value for the pre-test and post-test is at -2.368 and -4.575, respectively. When analysing the mean for the post-application, post-analysis, post-assessment, and post-creation tests, the mean of the experimental group post-test surpassed the mean of the control group post-test for all four HOTS level (post-application: treated: M = 5.27, control M = 4:41; post-analysis: treated: M = 5.37, control: M = 3.64; post-evaluate: treated: M = 5.63, control: M = 3.14; post-creation: treated: M = 5.00, control: M = 2.14).

V. DISCUSSION

The intensive treatment used in this study is the Thinking Blocks, which were found to be able to assist the students in improving their HOTS. The thinking blocks used in this study was adapted from the Representation Model (Bar Modelling). According to [28], [29], the Representation Model is the heuristic method in solving problems by using images of rectangular bars. Analysis performed on the students' results found that more than 70% of the students from the experimental group completed the post-test problems by using images. The students drew rectangles to represent the problem. This matter is reinforced by the statement of Reference [28] that solving problems using the Representative Model help the students to describe situations involved, assist the students to understand in detail the operations involved, and assist the students to connect the dots between the variables involved. Thus, through this study, it cannot be denied that the usage of Thinking Blocks can improve HOTS. The result of this study is consistent with the findings of studies that have been conducted by Singaporean researchers; namely, [34], [19]. According to [34], the Representation Model (also known as visualisation) assists the students in checking the accuracy of their comprehension in solving mathematical problems. Reference [19] found that the Representation Model assists the students in solving mathematical problems, either routine or non-routine problems. In the other studies, [33], [35] show a diagram drawn by the students which can assist them to view the relationship between these elements. Therefore, the mathematical problems can be solved by using the Representation Model which indirectly help the students to develop their mathematical thinking.

A. The Change in the Students' Application Level in Fractions Learning using Thinking Blocks

The first HOTS level is to apply what is defined as cognitive abilities to use information in new situation [11], [36]. The study found that the students apply the Thinking Blocks in solving a given problem. Prior to this, the students seem to have difficulties in problem-solving due to lack of understanding on the question's requirement. However, with the help from Thinking Blocks, the students were able to transfer all the information and can easily translate the question's requirement. Reference [37] explained that a good problem solving built a representation of the problem to ease the understanding in general. For example, the students in this study built a rectangular as a representation to translate required problem in solving questions during post-test. The study is supported by the researchers such as [19]-[22]. The researchers found that most of the students in Singapore solved the mathematical problems using heuristic methods which are pictorial representations. Representative images were drawn by the students to translate information from the mathematical questions. According to [28], the representative image assists the students to illustrate the problem, and to identify mathematical operations and the variables' relationship. Therefore, the intensive treatment given to the students in the experimental group helps them to answer the questions in which there is a significant increase compared to pre-test.

B. The Change in the Students' Analysis Level in Fractions Learning using Thinking Blocks

The next HOTS stage is to analyse what is defined as analysing data on the components to understand the organisation, the structure, and the relationship between components [38], [11]. In this study, the students need to identify the suitable components that correspond to the requirements of the question. Subsequently, the Thinking Block is perceived as ablility to assist students to obtain an initial overview for the arrangement of fractions. The situation also causes the students to get visual overview for each required component. According to [39], the visualisation process is known as a good representation of the problem in solving mathematical problems. The use of visualisation in teaching and learning process provides an opportunity for the students to solve mathematical problems and to help in improving the students' thinking [40], as well as assisting students in a clearer understanding of a concept, and also to improve their performance in Mathematics [41].

Quoting [42], mathematical concepts can be well understood by the students using the visual method. The visual method assists the students in acquiring mathematical ideas through the drawn diagrams. References [43], [28] further explain that the visual method can be manipulated to obtain enactive knowledge. Therefore, the visual method assisted the students from the experimental group to analyse information in solving the Question no 4, 5, and 6 during the post-test. This study is consistent with a study by [44] that used visual method as a tool to assist the students in developing understanding, and can reduce the mistakes in solving mathematical problems. Thus, the visual method in teaching and learning can assist the students in achieving the maximum HOTS level.

C. The Change in the Students' Evaluation Level in Fractions Learning using Thinking Blocks

The definition of evaluation according to [38], [11] is to make an assessment based on specific criteria. The students in this study were required to evaluate a given problem with the help of drawn thinking blocks. This study is in line with the curriculum introduced by Davydoy, a Russian researcher [21]. Davydov curriculum developed the skills to solve complex word problems among children with drawing. Visual models were used by the children to analyse, to state the quantitative relationship, and to manipulate the symbolic relationship. The curriculum portrayed the fact that drawings assist students to analyse information more easily, and subsequently, allows them to solve a problem easily. The questions require the students to provide justifications for each given reason; however, prior to that process, they need to do the calculations. The analysis of the students' work found that the knowledge of the students from the experimental group is lower than the students from the control group because the students did not attempt to answer the questions. This situation illustrates the difficulty of the students to complete the stated questions because they were unable to state the relationship and manipulate the symbolic relationship.

D. The Change in the Students' Creation Level in Fractions Learning Using Thinking Blocks

Creation can be defined as a combination of elements to form a new idea or structure [38]. In this study, one of the questions require the students to make connections on the equal value fractions with square plot shading (rectangle), despite having different shapes. Through the process of learning through visual representation i.e. Thinking Blocks, the students can perceive the relationship of the involved variable [28]. Thus, this assists the students in determining the fractions with equal value. The students' understanding of the schematic representation (diagram on squares) can also be enhanced and then, enable them to make a proper selection of mathematical operations and procedures involved [20].

VI. Conclusion

Overall, this study examines the changes in the level of students in HOTS in fractions learning. Based on the information which had been collected and analysed, the research question is answered. The changes in the students from the experimental group to apply, analyse, evaluate, and create in fractions learning showed a very encouraging improvement in answering the questions in the form of HOTS, compared to the students of the control group. The performance of the students from the experimental group is better, compared with the students from the control group. This is because the students of the experimental group received the visual learning treatment (Thinking Blocks). This indicates that the HOTS domain in application, analysis, evaluation, and creation can be achieved by each individual through a variety of methods. One of the methods that can be used to achieve HOTS is by using Thinking Blocks. Teachers who act as facilitators can use this method to help the students in achieving excellent level by solving the problem; thus assist the students in achieving the HOTS highest level. This is due to the visual learning method which can stimulate the students to think and provide an initial overview of a problem. This also indirectly attract the students to learn and solve the problems. The results of the previous studies and the facts related to this study supported the discussions that have taken place. Finally, the application of HOTS through Thinking Blocks should be emphasised in order to create a brilliant generation in solving problems, especially in fractions.

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