

# Orbit Determination Modeling with Graphical Demonstration

Assem M. F. Sallam, Ah. El-S. Makled

**Abstract**—In this paper, there is an implementation, verification, and graphical demonstration of a software application, which can be used swiftly over different preliminary orbit determination methods. A passive orbit determination method is used in this study to determine the location of a satellite or a flying body. It is named a passive orbit determination because it depends on observation without the use of any aids (radio and laser) installed on satellite. In order to understand how these methods work and how their output is accurate when compared with available verification data, the built models help in knowing the different inputs used with each method. Output from the different orbit determination methods (Gibbs, Lambert, and Gauss) will be compared with each other and verified by the data obtained from Satellite Tool Kit (STK) application. A modified model including all of the orbit determination methods using the same input will be introduced to investigate different models output (orbital parameters) for the same input (azimuth, elevation, and time). Simulation software is implemented using MATLAB. A Graphical User Interface (GUI) application named OrDet is produced using the GUI of MATLAB. It includes all the available used inputs and it outputs the current Classical Orbital Elements (COE) of satellite under observation. Produced COE are then used to propagate for a complete revolution and plotted on a 3-D view. Modified model which uses an adapter to allow same input parameters, passes these parameters to the preliminary orbit determination methods under study. Result from all orbit determination methods yield exactly the same COE output, which shows the equality of concept in determination of satellite's location, but with different numerical methods.

**Keywords**—Orbit determination, STK, MATLAB-GUI, satellite tracking.

## I. INTRODUCTION

IN the early decades of the space era, the Soviet Union launched Sputnik-1 satellite at October 1957 [1]. More than 5,000 satellites are launched into Low Earth Orbit (LEO) and more than 5% are still functional today and the rest is considered as space debris. The LEO is a wide range of orbital inclinations (up to 90 deg.) and altitudes up to 1500 km, a particular altitude range from LEO (300-800 km) is commonly used. Below that LEO level, a satellite's orbit will rapidly decay due to the resistance of the earth's atmosphere, thus restricting extremely low altitude orbits to short term missions. Remote sensing (RS) satellites benefit from a higher spatial resolution at lower altitudes. Some countries are interested in injection of their satellites into highly inclined polar orbits, to obtain a maximum coverage of the earth surface.

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RS satellites afford surveillance services, Earth mapping and mining exploration, which are critical information for economic vitality and national security. The importance of RS satellites increases the attention of owners and organizations concerned with space to detect and monitor this kind of satellites, and also to protect their satellites from dangerous space debris.

As a consequence of the fast-growing number of satellites orbiting the earth, observation and determination of their COE has gained a vital importance.

Earth satellite monitoring site can observe any object in space. These sites use radar, laser, or optical sensors. The Doppler radar or laser can measure the satellite's position and velocity vectors, the radar or laser sensors can measure the satellite's position vector only and the optical sensors can measure the satellite's range direction (right ascension and declination angle) [9]. Optical sensors are investigated in this study because most observation facilities in our country (Egypt) use this type of sensors in satellites orbit determination.

## II. MODEL DESCRIPTION

There are three main methods for passive preliminary orbit determination that are used in this paper; the Gibbs method using three state position vectors ( $r_1$ ,  $r_2$ , and  $r_3$ ), the Lambert's method using two position state vectors ( $r_1$  and  $r_2$ ) with duration between the vectors ( $\Delta t$ ) together with direction of rotation, and finally the Gauss method using the angles only method (Right ascension (RA), declination (decl) and local sidereal angle ( $\theta$ )) followed by the Gaussian iterative improvement to enhance the obtained initial COE observed [2]. The aforementioned methods have their own algorithm in obtaining the initial COE of satellite under observation; a brief description of the used algorithm will be discussed.

### A. Gibbs Algorithm: Fig. 1

1. A Test:  $r_1, r_2, r_3 = 0$  for coplanar vectors, calculate magnitude of each vector ( $r_1$ ,  $r_2$  and  $r_3$ ).
2. Calculate cross product to get parameter C, where

$$C_{12} = r_1 \times r_2, C_{23} = r_2 \times r_3 \text{ and } C_{31} = r_3 \times r_1 \quad (1)$$

3. Verify that three vectors used in calculation are coplanar

$$\hat{u} \cdot r_1 \cdot C_{23} = 0 \quad (2)$$

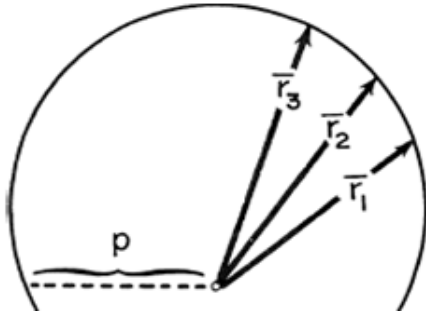


Fig. 1 Gibbs observation vectors

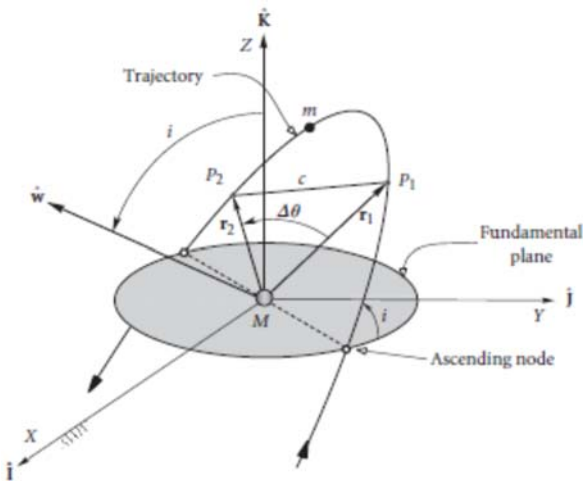


Fig. 2 Lambert's observations points p1 and p2

4. Calculate vectors (**N**, **D** and **S**) using (3), (4) and (5), respectively.

$$\mathbf{N} = \mathbf{r}_1(\mathbf{r}_2 \times \mathbf{r}_3) + \mathbf{r}_2(\mathbf{r}_3 \times \mathbf{r}_1) + \mathbf{r}_3(\mathbf{r}_1 \times \mathbf{r}_2) \quad (3)$$

$$\mathbf{D} = (\mathbf{r}_1 \times \mathbf{r}_2) + (\mathbf{r}_2 \times \mathbf{r}_3) + (\mathbf{r}_3 \times \mathbf{r}_1) \quad (4)$$

$$\mathbf{S} = \mathbf{r}_1(r_2 - r_3) + \mathbf{r}_2(r_3 - r_1) + \mathbf{r}_3(r_1 - r_2) \quad (5)$$

5. Calculate  $\mathbf{v}_2$  using

$$\mathbf{v}_2 = \sqrt{\frac{\mu}{ND}} \left( \frac{\mathbf{D} \times \mathbf{r}_2}{r_2} + \mathbf{S} \right) \quad (6)$$

6. Use  $\mathbf{r}_2$  and  $\mathbf{v}_2$  to compute the orbital elements by classical transformation method [2].

**B. Lambert Algorithm [2]-[4]: Fig. 2**

1. Calculate the magnitude of  $\mathbf{r}_1$  and  $\mathbf{r}_2$ .
2. Choose either a prograde or retrograde trajectory and calculate  $\Delta\theta$  using

$$\Delta\theta = \begin{cases} \cos^{-1}\left(\frac{r_1 r_2}{r_1 r_2}\right) & \text{if } (r_1 \times r_2)_Z \geq 0 \\ 360^\circ - \cos^{-1}\left(\frac{r_1 r_2}{r_1 r_2}\right) & \text{if } (r_1 \times r_2)_Z < 0 \\ \cos^{-1}\left(\frac{r_1 r_2}{r_1 r_2}\right) & \text{if } (r_1 \times r_2)_Z < 0 \\ 360^\circ - \cos^{-1}\left(\frac{r_1 r_2}{r_1 r_2}\right) & \text{if } (r_1 \times r_2)_Z \geq 0 \end{cases} \quad \begin{matrix} \text{prograde} \\ \text{trajectory} \\ \text{retrograde} \\ \text{trajectory} \end{matrix} \quad (7)$$

3. Calculate parameter A in (8).

$$A = \sin \Delta\theta \sqrt{\frac{r_1 r_2}{1 - \cos \Delta\theta}} \quad (8)$$

4. By iteration, using (10)-(12), solve (9) for z, which is for simplicity, finding the value of z that makes (10) equal to zero. The sign of z tells us whether the orbit is a hyperbola ( $z < 0$ ), parabola ( $z = 0$ ) or ellipse ( $z > 0$ ).

$$\sqrt{\mu} \Delta t = \left[ \frac{y(z)}{c(z)} \right]^{\frac{3}{2}} S(z) + A \sqrt{y(z)} \quad (9)$$

$$F(z) = \left( \frac{y(z)}{c(z)} \right)^{\frac{3}{2}} S(z) + A \sqrt{y(z)} - \sqrt{\mu} \Delta t \quad (10)$$

$$F'(z) = \begin{cases} \left[ \frac{y(z)}{c(z)} \right]^{\frac{3}{2}} \left\{ \frac{1}{2z} \left[ C(z) - \frac{3S(z)}{2C(z)} \right] + \frac{3S(z)^2}{4C(z)} \right\} \\ + \frac{A}{8} \left[ 3 \frac{S(z)}{C(z)} \sqrt{y(z)} + A \sqrt{\frac{C(z)}{y(z)}} \right] & (z \neq 0) \\ \frac{\sqrt{2}}{40} y(0)^{\frac{3}{2}} + \frac{A}{8} \left[ \sqrt{y(0)} + A \sqrt{\frac{1}{2y(0)}} \right] & (z = 0) \end{cases} \quad (11)$$

$$z_{i+1} = z_i - \frac{F(z_i)}{F'(z_i)} \quad (12)$$

5. Calculate y using (13):

$$y(z) = r_1 + r_2 + A \frac{zS(z)-1}{\sqrt{C(z)}} \quad (13)$$

6. Calculate the Lagrange f, g, and  $\dot{g} = 1 - \frac{y(z)}{r_2}$  functions using (14):

$$f = 1 - \frac{y(z)}{r_1} \quad (14a)$$

$$g = A \sqrt{\frac{y(z)}{\mu}} \quad (14b)$$

$$\dot{g} = 1 - \frac{y(z)}{r_2} \quad (14c)$$

7. Calculate  $\mathbf{v}_1$  and  $\mathbf{v}_2$  from (15) and (16).

$$\mathbf{v}_1 = \frac{1}{g} (\mathbf{r}_2 - f \mathbf{r}_1) \quad (15)$$

$$\mathbf{v}_2 = \frac{1}{g} (\dot{g} \mathbf{r}_2 - \mathbf{r}_1) \quad (16)$$

Use  $\mathbf{r}_1$  and  $\mathbf{v}_1$  (or  $\mathbf{r}_2$  and  $\mathbf{v}_2$ ) to compute the orbital elements by the classical transformation method [2], [3].

**C. Gauss Algorithm [2]-[4]: Fig. 3**

Given the location of observer (longitude  $\lambda$  and geocentric latitude  $\phi$ ) and the topocentric equatorial angles (right ascension, declination and the local sidereal time) of three successive observations, Gauss method can be used for obtaining the initial COE of the flying object (satellite).

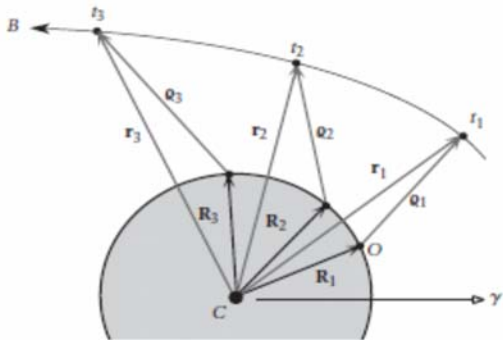


Fig. 3 Gauss observation vectors

1. Calculate the observers position vectors ( $R_1$ ,  $R_2$ , and  $R_3$ ) and the direction cosine unit vectors ( $\hat{q}_1$ ,  $\hat{q}_2$  and  $\hat{q}_3$ ) using the given observer location, time, and the topocentric equatorial angles.
2. From the local sidereal angle, calculate the time intervals  $\tau_1$ ,  $\tau_2$ , and  $\tau_3$  between the given observations.
3. Calculate the cross products

$$P_1 = \hat{q}_2 \times \hat{q}_3, P_2 = \hat{q}_1 \times \hat{q}_3 \text{ and } P_3 = \hat{q}_1 \times \hat{q}_2.$$

4. Calculate  $D_0 = \hat{q}_1 \cdot P_1$ .
5. Calculate the following cross products

$$D_{ij} = R_i \cdot P_j \quad (17)$$

$$A = \frac{1}{D_0} \left( -D_{12} \frac{\tau_3}{\tau} + D_{22} + D_{32} \frac{\tau_1}{\tau} \right) \quad (18)$$

$$B = \frac{1}{6D_0} \left[ D_{12}(\tau_3^2 - \tau^2) \frac{\tau_3}{\tau} + D_{32}(\tau^2 - \tau_1^2) \frac{\tau_1}{\tau} \right] \quad (19)$$

$$E = R_2 \cdot \hat{q}_2 \quad (20)$$

$$\begin{aligned} a &= -(A2 + 2AE + R_2^2) \\ b &= -2\mu B(A + E) \\ c &= -\mu 2B2 \end{aligned} \quad (21)$$

6. Find the roots of  $F(x) = x^8 + ax^6 + bx^3 + c$  using the Newton's iterative method.
7. Calculate  $q_1$ ,  $q_2$  and  $q_3$  using (22), (23) and (24), respectively.

$$q_1 = \frac{1}{D_0} \left[ \frac{6(D_{31} \frac{\tau_1}{\tau_3} + D_{21} \frac{\tau}{\tau_3}) r_2^3 + \mu D_{31} (\tau^2 - \tau_1^2) \frac{\tau_1}{\tau_3} - D_{11}}{6r_2^3 + \mu(\tau^2 - \tau_3^2)} - D_{11} \right] \quad (22)$$

$$q_2 = A + \frac{\mu B}{r_2^2} \quad (23)$$

$$q_3 = \frac{1}{D_0} \left[ \frac{6(D_{13} \frac{\tau_3}{\tau_1} + D_{23} \frac{\tau}{\tau_1}) r_2^3 + \mu D_{13} (\tau^2 - \tau_3^2) \frac{\tau_3}{\tau_1} - D_{33}}{6r_2^3 + \mu(\tau^2 - \tau_3^2)} - D_{33} \right] \quad (24)$$

8. Now the object (satellite) position vectors can be calculated by

$$r_i = R_i + q_i \hat{q}_i \quad (25)$$

9. Calculate Lagrange coefficients  $f_1$ ,  $g_1$ ,  $f_3$ , and  $g_3$

$$f_i \approx 1 - \frac{1}{2} \frac{\mu}{r_2^3} \tau_i^2 \quad (26)$$

$$g_i \approx \tau_i - \frac{1}{6} \frac{\mu}{r_2^3} \tau_i^3 \quad (27)$$

10. Velocity related to the middle observation ( $V_2$ ) is calculated by (28)

$$v_2 = \frac{1}{f_1 g_3 - f_3 g_1} (-f_3 r_1 + f_1 r_3) \quad (28)$$

11. Use  $r_2$  and  $v_2$  to get the observed satellite COE by the classical transformation method from state vector to COE [3].

The obtained COE from the Gauss method are then refined by the Gaussian iterative improvement method where the exact Lagrange coefficients  $f$  and  $g$  from their universal formulation values are calculated.

### III. ORDET IMPLEMENTATION (GUI APPLICATION)

The three preliminary orbit determination methods stated in the previous section are implemented on MATLAB [5]. A GUI is built on the implemented MATLAB program for each method, which are then used to make the modified GUI that combines all methods in one interface.

Modified method comprises an adapter that is tailored to produce the required input for each method. The solution strategy that has been adapted to determine the orbital parameters can be described by the following scheme as in Fig. 4.

As in most of satellite observation centers that uses an optical sensor to capture flying objects, outputs from optical sensor (telescope) are the general topocentric horizon angular coordinate data, which are azimuth, elevation, and time. So, these data will be the default input to the modified method, the data then will be transferred to the topocentric equatorial coordinate which is right ascension (RA) and declination (decl). To provide the position vectors in the inertial coordinate system, the data then flow to the Gauss method, where the input angular data will be computed through the improved Gaussian iterative method, to obtain the position vectors relative to the input observation times. Position vectors can then be used as an input to the Gibb's and Lambert's modules. The three-orbit determination methods output, which is the state vector of flying object, can enter the module of transformation from state vector to COE.

Fig. 5 shows the overall outline structure of the OrDet main window. A graphical determination, input, and output from the three orbit determination methods can be compared with each other and verified with the data obtained from STK [7].

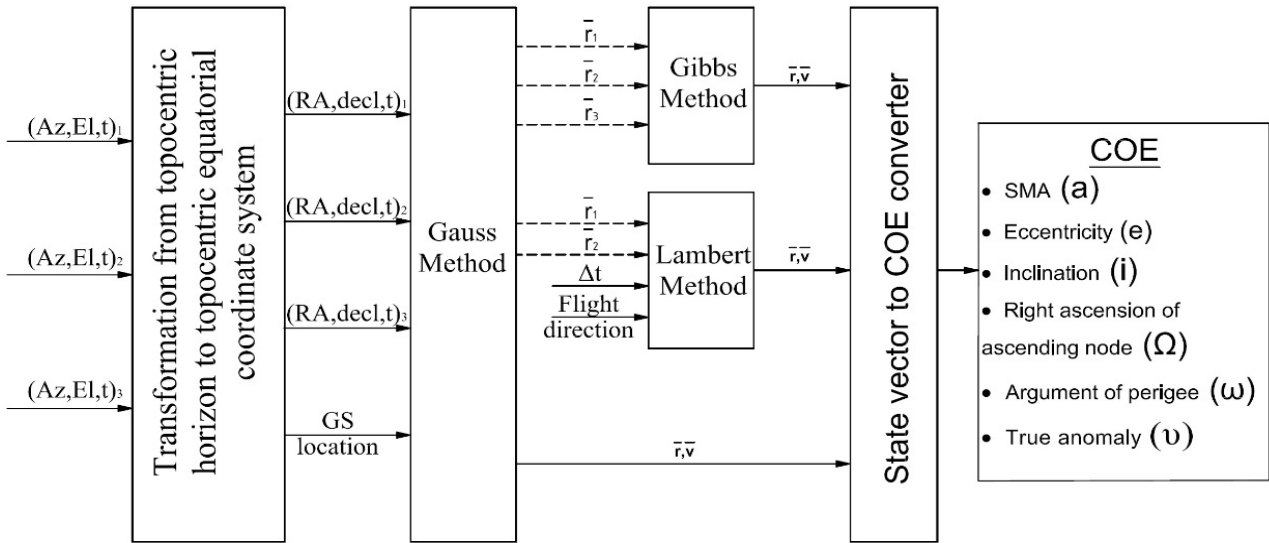


Fig. 4 Modified OrDet scheme

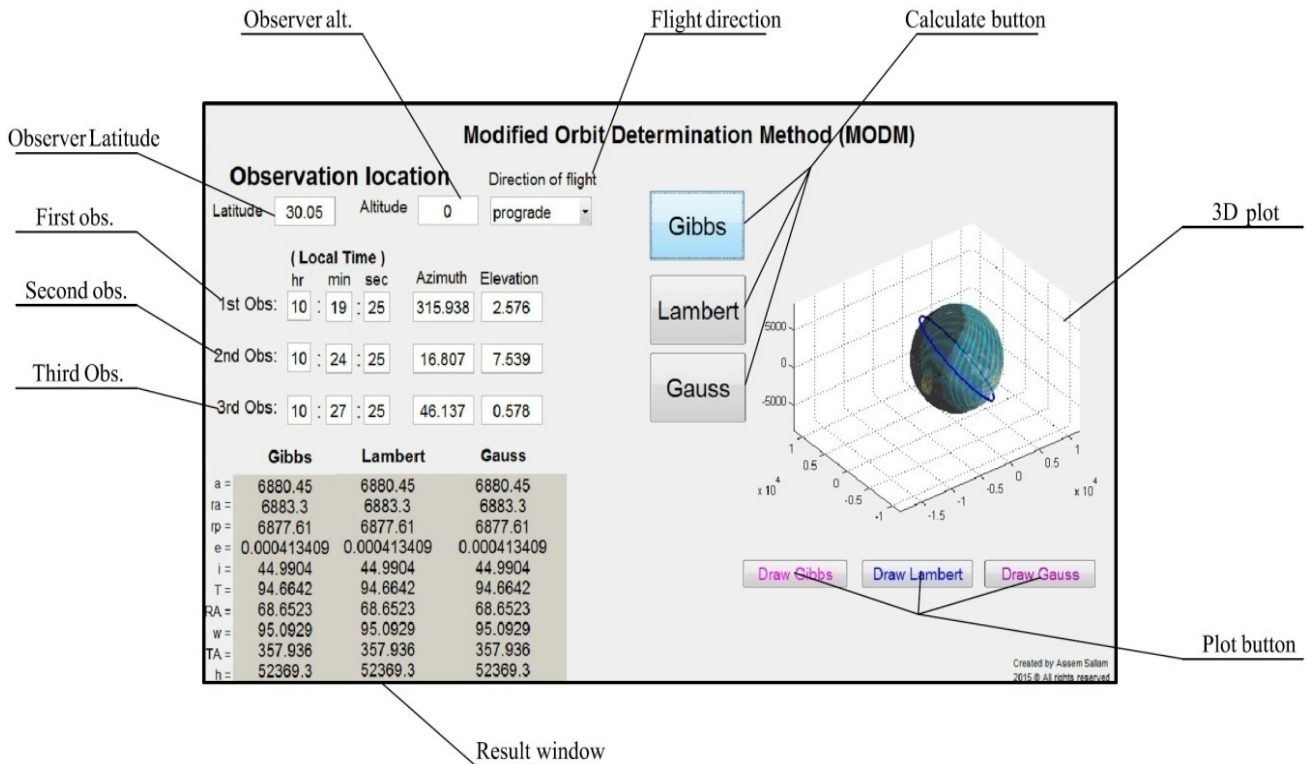


Fig. 5 OrDet modified interface description

#### IV. GRAPHICAL VISUALIZATION

The obtained COE from the three described methods are further more represented in the OrDet GUI application by a 3D visualization graph. Adding such 3D representation of satellite location has two advantages; first is to visually verify that COE are logic, which means that obtained orbit is complete and does not intersect Earth, and second advantage is the use of this visual representation for educational purposes, that helps users in imagination of orbital elements.

Simply, as the preliminary orbit determination method used

indicates the initial location of satellite (initial COE), this initial data together will be propagated [6] over a whole revolution (over the orbital period) to get the change of the COE during one complete cycle, the obtained data are then plotted on a 3D graph as shown in Fig. 6.

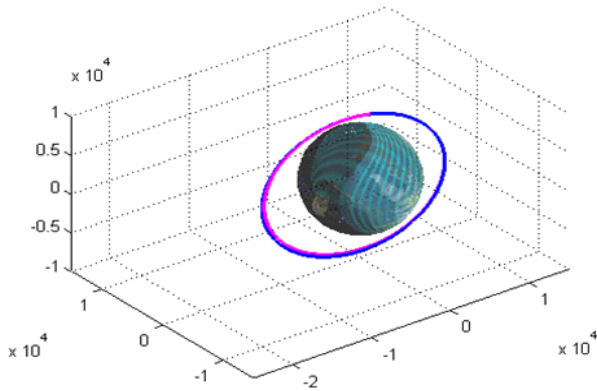


Fig. 6 Graphical representation of COE

V.DATA VERIFICATION

In order to check the validity of the developed model, data verification is carried out on two stages; first is verifying whether the modified orbit determination method with the three methods integrated together works properly, and second is to check the model output with a verified data of a guaranteed source of propagated satellite in orbit using STK [7].

The first requirement of verification showed output from the three methods with identical results, which means the accurate built of the model. In the second stage of verification, output is carried by using the commercial software STK [7], where a scenario is built with a ground station placed at Cairo. An Azimuth-Elevation-Range (AER) report is generated for the access between the ground station and the satellite. The report data are entered in the produced MATLAB software and its output is compared with the STK initial data of satellite.

Data are verified with STK using different orbit configurations, and satellite under investigation gives a constant eccentricity equal to zero (circular orbit) with inclination 60°. Altitude is changed from 400 km to 800 km as shown in Figs. 7 and 8, and other data entry for the satellite uses different orbit eccentricity (0 ÷ 1) for a constant orbit perigee radius 700 km and inclination 60° as shown in Figs. 9-11.

Orbital parameters under study are the semi-major axis (a), radius of apogee (ra), and radius of perigee (rp). The comparison shows a good agreement for different altitudes and eccentricity for the orbital parameters taken in study (a, ra and rp). The general error is less than 1% for the altitude variation, but for the eccentricity variation, error is less than 1% for the range of eccentricity less than 0.4 and increases for high eccentricity values to an average 5%, except for the radius of perigee, which shows an error about 7%. A fact that makes the model output for high eccentricity values is not confident for rp.

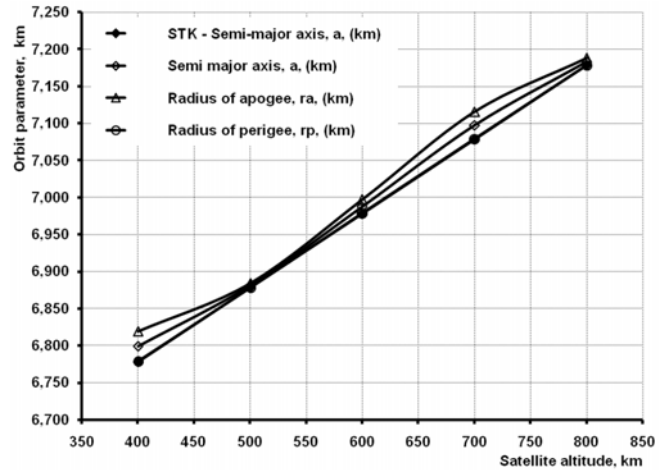


Fig. 7 Semi major axis, radius of apogee and radius of perigee for different altitudes for inclination 60°

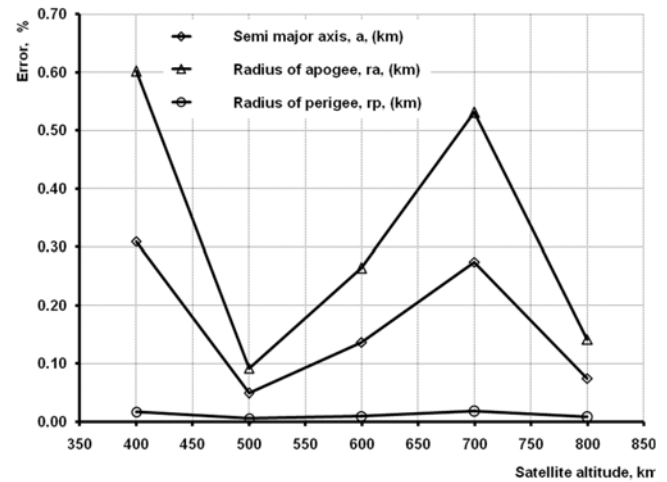


Fig. 8 Verification error for different LEO altitudes

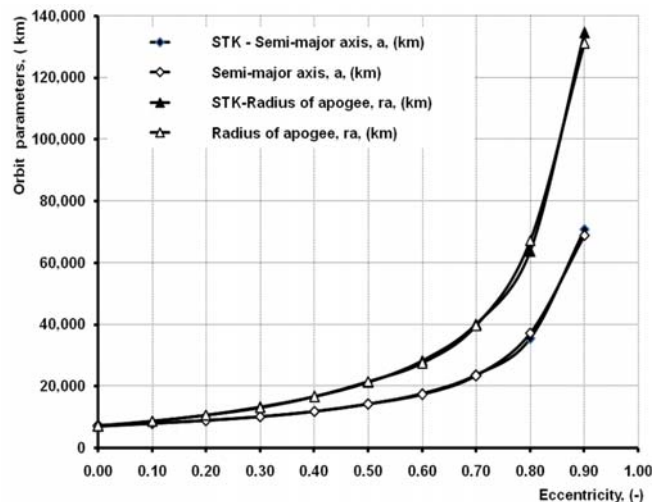


Fig. 9 Semi major axis and radius of apogee for different eccentricity for inclination 60° and rp=700 km

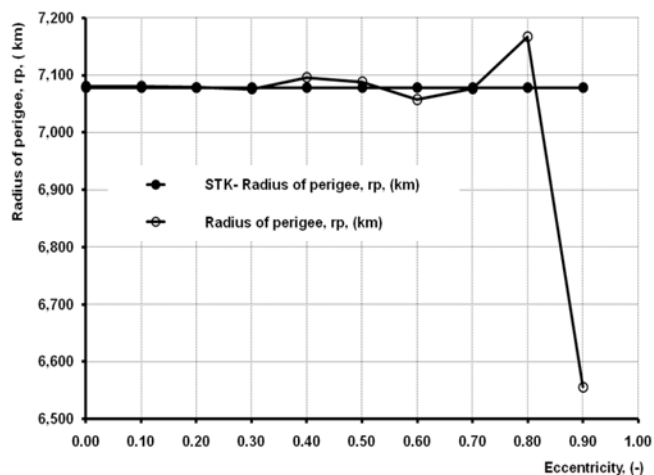


Fig. 10 Radius of perigee for different eccentricity

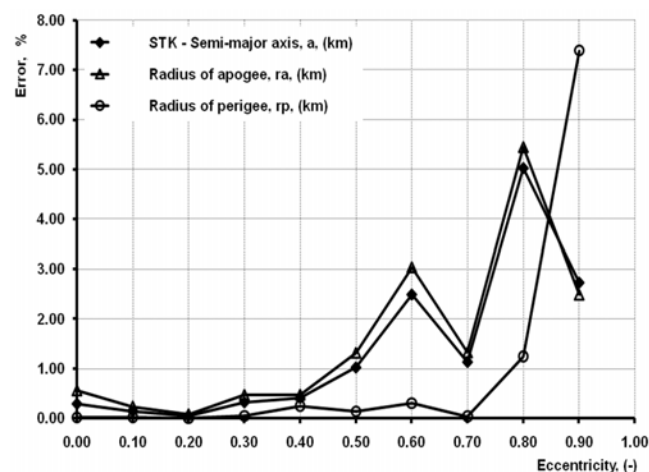


Fig. 11 Verification error for different eccentricity

## VI. CONCLUSION

A computer code OrDet based on a simplified theoretical model is developed to predict the COE parameters. The flexibility of the program, which can be run on a personal computer, makes it a valuable preliminary design tool for prediction, design, and analysis of COE parameters of LEO.

Main aim during the production of this paper is to investigate the difference between different preliminary orbit determination methods. Early at the beginning of 1970, Huseonicu [8] studied the difference between 12 preliminary orbit determination methods to find which method is the fastest from computational point of view and which method is with the least numerical error in propagation within an order of  $10^{-10}$ . However, nowadays, computational speed is very fast to even discover difference between different methods, and the accuracy taken ( $10^{-10}$ ) is for comparison order that is not used in real satellite orbit determination stations. The adapter module, which unifies the input observation data to the methods under study as described in the Modified orbit determination method (MODM), shows that all the main methods yield the exact solution with no difference in any parameter. Also, the

graphical representation of the obtained COE helps in visual verification of model output.

Changing the altitude of satellite in the LEO altitudes has no effect on the accuracy of the obtained COE, where the maximum error less than 1%. Different shapes of orbit (different eccentricity) show an error in its output less than 1% for eccentricity ranged from 0 to 0.5, and error starts to increase for eccentricity value greater than 0.5 specially for the radius of perigee, where error reaches its maximum value at  $e = 0.9$  with an error value 7.5%. For the eccentricity greater than 0.9 and less than 0.99, the model output shows singularities with no results. Close observations with difference in azimuth or elevation angle less than  $0.1^\circ$  show inaccurate results, and using the Herrick-Gibbs [9] method can be introduced to the model to give more accurate results for the close observations data.

The comparison between design software OrDet results and computed predictions from STK shows good agreement. The presented code OrDet can be considered as a powerful tool for the design, analysis, and prediction of the LEO satellites observation.

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