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Multi-Agent Coverage Control with Bounded Gain Forgetting Composite Adaptive Controller

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Abstract**—**In this paper, we present an adaptive controller for decentralized coordination problem of multiple non-holonomic agents. The performance of the presented Multi-Agent Bounded Gain Forgetting (BGF) Composite Adaptive controller is compared against the tracking error criterion with a Feedback Linearization controller. By using the method, the sensor nodes move and reconfigure themselves in a coordinated way in response to a sensed environment. The multi-agent coordination is achieved through Centroidal Voronoi Tessellations and Coverage Control. Also, a consensus protocol is used for synchronization of the parameter vectors. The two controllers are given with their Lyapunov stability analysis and their stability is verified with simulation results. The simulations are carried out in MATLAB and ROS environments. Better performance is obtained with BGF Adaptive Controller.

Keywords—Adaptive control, Centroidal Voronoi Tessellations, composite adaptation, coordination.

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I. Introduction

Multi-agent coordination is a challenging problem studied intensively in the past years. In many applications, using more than one agent is necessary to achieve better and faster results. So, a subtopic of multi-agent coordination, multi agent distributed coverage control has its importance in mobile sensor networks. It uses locational optimization and Centroidal Voronoi Tessellations to place the sensors in optimal way in order to improve coverage performance.

In literature, there are existing works related to the adaptive coverage problem. One of the works [1] proposes a method to drive the mobile robots to an optimal configuration by means of a decentralized, adaptive control law with a Lyapunov-type proof. In another work [2], an adaptive and decentralized coverage control for a team of mobile sensors is proposed with a Lyapunov stability proof. It uses non-holonomic sensors and time-varying sensory functions. In another paper [3], a control strategy for groups of vehicles for motion and reconfiguration to a sensed environment is given. Its framework uses virtual bodies and artificial potentials. The method adapts to the sensed environment to optimize the climb mission. Another work [4] discusses coverage control, spatial partitioning and dynamic vehicle routing problems in detail. Those problems include distributed optimization of the configuration of the robots in order to minimize a cost function. The distributed stochastic gradient algorithms for this purpose are described since they are related to adaptive coverage with heterogeneous agents. In another work [5], a multiscale adaptive search algorithm for seeking an unknown number of targets using multiple mobile sensors is proposed. The Sequential Ratio Probability Test and Recursive Least Squares estimation enable calculation of uncertainty of the target and its location, respectively. A similiar work [6] presents a modified model reference adaptive control (MRAC) approach which brings high performance as well as higher level of robustness. Additionally, a time delay resistant (TDR) adaptive control method is proposed. The two methods are used in simulation of the longitudinal dynamics of an aerial vehicle. Another paper [7] proposes a deployment strategy maximizing the area coverage of the mobile nodes using K neighbors constraint. It is distributed, scalable and based on potential fields.

In one of our previous works [8], a power aware adaptive control structure is proposed used with the same coverage control framework which uses the consensus protocol to synchronize the agents and simultaneously adapts to the environment.

In this paper, an adaptive controller is presented. The presented controller uses BGF Composite Adaptive Control laws which are shown to have faster convergence than MRAC and conventional adaptive control approaches as given in [9]. In none of the works mentioned above, a BGF Composite Adaptive Control law is used. The problem studied in this work is to position the non-holonomic sensor nodes optimally in response to the sensed environment in a coordinated way using BGF based adaptive control structures. This approach has better performance with respect to other adaptive control methods that are defined as a main contribution of this paper. For the given purpose, the sensors used in simulation are light sensors on the robots giving the location of the global light source. Additionally, a multi-agent adaptive control algorithm with consensus protocol is used to estimate the model parameters. The reason to choose an adaptive algorithm for the kinematic model is to improve the tracking errors which is accomplished by the composite adaptive algorithm using the sensor data in the both in the estimation loop and the control loop. To the best of the author's knowledge, this is the first work using a BGF Composite Adaptive Controller which uses both the tracking and estimation errors in adaptation law for the multi-agent coverage control problem with non-holonomic agents.

The paper is organized as follows: In Section II, mathematical background of the optimal coverage control

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problem is given. In Section III, the application of the coverage control for non-holonomic sensors is mentioned with a new adaptive controller for coverage. Then, Section IV gives the stability analysis of the controllers and in Section V, we present the simulation results. Lastly, the conclusions of the presented work are given in Section VI.

II. Problem Statement

Introductory information about the adaptive coverage control problem will be given in this section.

A. Voronoi Diagrams

In [2] and [8], the Voronoi tessellation of an open set $S \subseteq R^N$ is defined as Υ_k satisfying the condition $\Upsilon_k \cap \Upsilon_l = \varnothing$ for $k \neq l$ and $\cup_k \Upsilon_k = S$. The definition of the Voronoi region Υ_k is as:

$$\Upsilon_k = \left\{ q \in S \mid \|q - p_k\| \leq \|q - p_l\|, k \neq l \right\} \tag{1}$$

The Euclidean norm is defined in R^N by the $\| \ \|$ operator and the generator points are given with p_k. In the work, the Voronoi tessellations are calculated by using Fortune's Sweepline algorithm.

B. Optimal Coverage Formulation

Consider $S \subseteq R^N$ as a bounded environment and $\Psi : R^N \rightarrow R^+$ as a density function. Let $f : R^+ \rightarrow R$ be a non-increasing performance function. Then we define locational optimization function $\Im$ as:

$$\Im(p_1, p_2, ..., p_m) = \sum_{k=1}^{m} \int_{\Upsilon_k} f(\|q - p_k\|)\Psi(q)dq \tag{2}$$

The Υ_k is Voronoi region *k* and p_k is the generator point of the corresponding Voronoi cell and *m* is the number of the generator points.

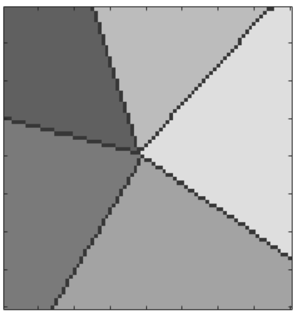

Fig. 1 Example Voronoi Tessellation

The centroid X_{Υ_k} and mass Λ_{Υ_k} of Voronoi regions defined in [10] are given by:

$$X_{\Upsilon_k} = \frac{1}{\Lambda_{\Upsilon_k}} \int_{\Upsilon_k} q\,\Psi(q)dq \tag{3}$$

$$\Lambda_{\Upsilon_k} = \int_{\Upsilon_k} \Psi(q)dq \tag{4}$$

If we define the function $f(\|q - p_k\|) = \|q - p_k\|^2$ and take the partial derivative of locational optimization function $\Im$ with respect to p_k, we get the following equations as given in [2]:

$$\Im = \sum_{k=1}^{n} \int_{\Upsilon_k} f(\|q - p_k\|)\Psi(q)dq \tag{5}$$

$$\begin{aligned} \frac{\partial \Im}{\partial p_k} &= \int_{\Upsilon_k} \frac{\partial}{\partial p_k}\left(\|q - p_k\|^2\right)\Psi(q)dq \\ &= \int_{\Upsilon_k} -2(q - p_k)\Psi(q)dq \\ &= -2\int_{\Upsilon_k} q\Psi(q)dq + 2\int_{\Upsilon_k} p_k\Psi(q)dq \\ &= 2\Lambda_{\Upsilon_k}(p_k - X_{\Upsilon_k}) \end{aligned} \tag{6}$$

The locational optimization function given in (2) is minimized by using the centroid positions given in (3). The centroid positions should be equal to the positions of the agents for the optimal solution. The definition of the Centroidal Voronoi Tessellations comes from this equality as shown in [2].

III. Coverage Control for Team of Nonholonomic Agents

An adaptive control scheme with a linear consensus protocol for a nonholonomic agent model is presented in this section. The kinematic model parameters are estimated within the adaptive control law.

A. Nonholonomic Control Law

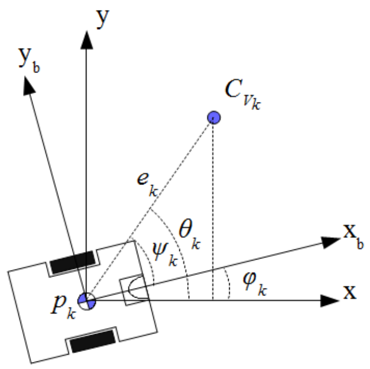

Fig. 2 Position of the agent and model parameters

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In this work, the adaptive control law given is based on the control method proposed by [2] with a unicycle model. It is used to drive the agents to the centroid locations.

In Fig. 2, p_k and C_{V_k} are the position of the agent and the centroid location for a single agent. The control inputs are defined as u_k and ω_k which represent linear and angular velocities, respectively. The heading angle is shown with φ_k and the Euclidean distance between the agent and the centroid is defined as e_k. Lastly, the angle between the agent and the centroid is given as ψ_k.

$$\begin{pmatrix} \dot{e}_k \\ \dot{\psi}_k \\ \dot{\theta}_k \end{pmatrix} = \begin{pmatrix} -u_k \cos\psi_k \\ -\omega_k + u_k \dfrac{\sin\psi_k}{e_k} \\ u_k \dfrac{\sin\psi_k}{e_k} \end{pmatrix} \tag{7}$$

The control law proposed is:

$$\begin{pmatrix} u_k \\ \omega_k \end{pmatrix} = \begin{pmatrix} \gamma \cos\psi_k e_k \\ 2\gamma \sin\psi_k \cos\psi_k + \lambda(\psi_k + \theta_k) \end{pmatrix} \tag{8}$$

$\gamma > 0$ and $\lambda > 0$ are control gains. The control law drives the agents to the centroid C_{V_k} positions.

B. Feedback Linearization Controller for Multi-Agent Coverage Problem

In order to apply Feedback Linearization control method, first, the system model given in (7) should be considered. As stated in [9], the state vector x_k can be redefined and the combined error vector s_k for k^{th} agent can be shown as:

$$x_k = \begin{pmatrix} e_k \\ \psi_k \end{pmatrix} \tag{9}$$

$$s_k = \chi_0 x_k \tag{10}$$

where χ_0 is a positive constant. The system matrix F_1 can be denoted as:

$$F_1 = \begin{pmatrix} -\dfrac{1}{\cos\psi_k} & 0 \\ -\dfrac{\sin\psi_k}{e_k \cos\psi_k} & -1 \end{pmatrix} \tag{11}$$

So, the system equations can be rewritten as:

$$\begin{pmatrix} u_k \\ \omega_k \end{pmatrix} = F_1 \dot{x}_k \tag{12}$$

The matrices K and K_u are defined as:

$$K = \begin{pmatrix} 4\cos\psi_k + \xi & 0 \\ 0 & 1 \end{pmatrix} \tag{13}$$

Then, the control law can be written as:

$$u = \begin{pmatrix} u_k \\ \omega_k \end{pmatrix} = F_1 \dot{x}_k + \dot{s}_k + K K_d s_k \tag{14}$$

It can be seen that K is positive definite for $|\psi_k| \le \frac{\pi}{2}$ and ξ is a small positive real number. Since the robot is moving both forward and backward directions, the angular error assumption holds. Also, $K_d = diag([K_{d,1} \quad K_{d,2}])$ is a positive definite gain matrix.

The nonlinear terms in the system model are canceled with the first term in the controller.

C. BGF Composite Adaptive Controller for Multi-Agent Coverage Problem

Different from the traditional adaptive controllers, the composite adaptive controllers take the tracking error as well as the prediction error into account.

Based on the system model given in (7), in order to apply BGF adaptive control method, the state vector x_k and the combined error vector s_k for k^{th} agent should be defined, as stated in [9]:

$$x_k = \begin{pmatrix} -e_k \\ -\psi_k \end{pmatrix} \tag{15}$$

$$s_k = \chi_0 x_k \tag{16}$$

The system matrix F_2 is defined in (17). The system equations can be rewritten as:

$$F_2 = \begin{pmatrix} \dfrac{1}{\cos\psi_k} & 0 \\ \dfrac{\sin\psi_k}{e_k \cos\psi_k} & 1 \end{pmatrix} \tag{17}$$

$$\begin{pmatrix} u_k \\ \omega_k \end{pmatrix} = F_2 \dot{x}_k \tag{18}$$

The known matrix $Y(\dot{x}, x)$ yields:

$$F_2 \dot{x}_k = Y a_k \tag{19}$$

where $a_k = \begin{pmatrix} a_{1,k} & a_{2,k} \end{pmatrix}^T$ is the parameter vector to be estimated defined for k^{th} agent [9]. Obviously, Y and Y' matrices can be selected as:

$$Y = \begin{pmatrix} -\dfrac{\dot{e}_k}{\cos\psi_k} & 0 \\ -\dfrac{\dot{e}_k \sin\psi_k}{e_k \cos\psi_k} & -\dot{\psi}_k \end{pmatrix} \tag{20}$$

$$Y' = \begin{pmatrix} 1 - \dfrac{\sin\psi_k}{e_k^2}\psi_k & 0 \\ 0 & 1 \end{pmatrix} \tag{21}$$

The matrix Y' is used for cancelling the term coming from K_u in Lyapunov proof which will be given in the next section. The K_u matrix is defined as:

$$K_u = \begin{pmatrix} 0 \\ 2\sin\psi_k \cos\psi_k \end{pmatrix} \tag{22}$$

A linear consensus protocol is used in the estimation of the nonlinear model parameters for the agents. In an undirected graph of the mobile agents and n vertices $V_k = \{v_1, v_2, ..., v_n\}$, the agents share their estimated parameter vector. Let the neighborhood of k^{th} agent be defined as:

$$L_k = \{ l \mid \{v_k, v_l\} \in G \} \tag{23}$$

The communication among the nodes can be represented by the m edges $G = \{g_1, g_2, ..., g_m\}$ where j^{th} element of G is $g_j = \{v_k, v_l\}$. The consensus protocol provides synchronization among the mobile agents.

The control and parameter update law as stated in [9] with linear consensus can be obtained as:

$$u = \begin{pmatrix} u_k \\ \omega_k \end{pmatrix} = F_2\left(Y\hat{a}_k - KK_d s_k + K_{d,1} K_u \right) \tag{24}$$

$$\dot{\hat{a}}_k = -P(t)\left((Y'Y)^T s_k + W^T R(t) e_{pred,k} \right) - \eta P(t) \sum_{l \in \mathcal{L}_k} (\hat{a}_k - \hat{a}_l) \tag{25}$$

where $P(t)$ is a uniformly positive definite gain matrix and $R(t)$ is a uniformly positive definite weighting matrix. The prediction error $e_{pred,k}$ for k^{th} agent is defined as:

$$e_{pred,k} = \hat{a}_k - a_k \tag{26}$$

The control input and the matrix $W(\dot{x}, x)$ can be written after passing a first order low pass filter:

$$u = Y_1(\dot{x}, x) a_k \tag{27}$$

$$W(\dot{x}, x) = \frac{\lambda_f}{p + \lambda_f} Y_1(\dot{x}, x) \tag{28}$$

where λ_f is the filter coefficient and $p = d/dt$ is the differential operator.

$$\frac{d}{dt} P^{-1}(t) = -\lambda(t) P^{-1}(t) + W^T W \tag{29}$$

$$\lambda(t) = \lambda_0 \left(1 - \|P\| / k_0 \right) \tag{30}$$

where $\lambda(t)$ is the variable forgetting factor. The λ_0 and k_0 are positive constants. The λ_0 determines the maximum value of the forgetting factor and the k_0 gives the upper bound of the gain matrix norm.

If the gain matrix $P(t)$ is calculated by using a BGF law as given in (29), then the controller is called as BGF Composite Adaptive Controller. Since $P(t)$ is positive definite, its determinant is always positive and it is non-singular.

The control and parameter update law for BGF Composite Adaptive Controller are as in (24) and (25), respectively.

By using the control and adaptation laws, the nonlinear dynamics of the system can be estimated and cancelled. The proof of convergence of the laws will be given in the stability analysis section.

IV. Stability Analysis

In this section, the Lyapunov stability of the presented controllers are given with their proofs.

A. Feedback Linearization Controller for Multi-Agent Coverage Problem

Theorem 1: Let K_d be a positive definite gain matrix and k be the agent number. For each agent k, the tracking errors s_k and the estimation errors $\tilde{a}_k$ converge to zero asymptotically and the internal dynamic φ_k is stable as $t \to \infty$.

Proof: If the nonlinearity in the model is cancelled by applying the control input in (14) to the system, the equations become:

$$\dot{s}_k = -KK_d s_k \tag{31}$$

The Lyapunov function candidate is defined as:

$$V(t) = \sum_k V_k(t) \tag{32}$$

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The matrix K is positive definite for $|\psi_k| \le \frac{\pi}{2}$ and assumed to be a constant matrix and $\xi > 0$, so the Lyapunov function candidate can be written as:

$$V_k(t) = \frac{1}{2} s_k^T K^{-1} s_k \tag{33}$$

Taking the derivative of the Lyapunov function candidate yields:

$$\dot{V}_k(t) = s_k^T K^{-1} \dot{s}_k = s_k^T(-K_d s_k) = -s_k^T K_d s_k \tag{34}$$

Since K and K_d are positive definite matrices, by Barbalat's Lemma, $\dot{V}(t) \to 0$ and the errors converge to zero asymptotically since $V(t)$ is positive and lower bounded, and $\ddot{V}(t)$ is bounded as $t \to \infty$.

The internal dynamic φ_k can be investigated by applying the control law to the system model:

$$\dot{\varphi}_k = \omega_k \tag{35}$$

$$\dot{\varphi}_k \cong -\frac{\dot{e}_k}{e_k}\frac{\sin\psi_k}{\cos\psi_k} + K_{d,2}\chi_0\psi_k \tag{36}$$

$$\psi_k = \theta_k - \varphi_k \tag{37}$$

The term $\frac{\dot{e}_k}{e_k}$ is negative and e_k is bounded since e_k is stable. From the fact that, $\sin\psi_k \approx \psi_k$ and $\cos\psi_k \approx 1$, the first term dominates the others. Thus, the following equations hold:

$$\begin{aligned} \varphi_k > 0 \Rightarrow \psi_k < 0 \Rightarrow \dot{\varphi}_k < 0 \\ \varphi_k < 0 \Rightarrow \psi_k > 0 \Rightarrow \dot{\varphi}_k > 0 \end{aligned} \tag{38}$$

So, it can be concluded that the internal dynamic φ_k is stable.

B. BGF Composite Adaptive Controller for Multi-Agent Coverage Problem

In this section, the stability analysis of the multi-agent BGF Composite Adaptive Controller is given. Using a multi-agent BGF controller with a consensus law is the novelty of this work.

Theorem 2: Let Γ and $R(t)$ be positive definite matrices and I be the identity matrix. For each agent *k,* the tracking errors s_k and the estimation errors $\tilde{a}_k$ converge to zero asymptotically and the internal dynamic φ_k is stable as $t \to \infty$.

Proof: Let us first define $\Gamma = diag([0.5 \quad 1])$. Taking $R(t) = I$ where I is the identity matrix and defining the Lyapunov function candidate yields:

$$V(t) = \sum_k V_k(t) \tag{39}$$

$$V_k(t) = \frac{1}{2}\left(\frac{1}{\chi_0} s_k^T s_k + 2\tilde{a}_k^T \Gamma P^{-1} \chi_0 \tilde{a}_k + \chi_0 \psi_k^2\right) \tag{40}$$

Since $P(t)$ is now changing with respect to time, taking the derivative the equation leads to:

$$\dot{V}_k(t) = \frac{1}{\chi_0} s_k^T \dot{s}_k + 2\tilde{a}_k^T \Gamma P^{-1} \chi_0 \dot{\hat{a}}_k + \tilde{a}_k^T \Gamma \dot{P}^{-1} \chi_0 \tilde{a}_k + \chi_0 \psi_k \dot{\psi}_k \tag{41}$$

$$\begin{aligned} \dot{V}_k(t) = &-K_{1,1}K_{d,1}\chi_0^2 e_k^2 - \chi_0^2 K_{2,2} K_{d,2}\psi_k^2 \\ &-2\chi_0 K_{d,2}\psi_k^2 - \lambda(t)\tilde{a}_k^T \Gamma P^{-1}\chi_0 \tilde{a}_k \\ &-\tilde{a}_k^T \Gamma W^T W \chi_0 \tilde{a}_k - 2\tilde{a}_k^T \Gamma \chi_0 \eta \sum_{l \in \mathcal{L}_k} (\hat{a}_k - \hat{a}_l) \end{aligned} \tag{42}$$

The first, second, third and fourth terms are negative since K, K_d and W are positive definite matrices, and $P^{-1}(t) \ge I/k_0$ and $\lambda(t)$ is positive.

$$\begin{aligned} -\mu \sum_k \sum_{l \in \mathcal{L}_k} \tilde{a}_k^T (\hat{a}_k - \hat{a}_l) &= -\frac{1}{2}\mu \sum_k \sum_l A_{kl} (\hat{a}_k - \hat{a}_l)^T (\hat{a}_k - \hat{a}_l) \\ &= -\frac{1}{2}\mu \sum_k \sum_{l \in \mathcal{L}_k} \|\hat{a}_k - \hat{a}_l\|^2 \end{aligned} \tag{43}$$

The fourth term is also negative by using (39) and (43), the detailed proof for binary protocol is given in [11]. Here, A denotes adjacency matrix.

By Barbalat's Lemma, $\dot{V}(t) \to 0$ and the errors converge to zero asymptotically since $V(t)$ is positive and lower bounded, and $\ddot{V}(t)$ is bounded as $t \to \infty$. The detailed proof is given in the Appendix A.

Since all the errors converge to zero, we can investigate the internal dynamics of φ_k. By applying the control law to the system, we obtain the following dynamics:

$$\dot{\varphi}_k = \omega_k \tag{44}$$

$$\begin{aligned} \dot{\varphi}_k \cong &-\frac{\dot{e}_k}{e_k}\left(\frac{\sin\psi_k}{\cos^2\psi_k} + \frac{\sin\psi_k}{\cos\psi_k}\right) + 4\chi_0 K_{d,1}\sin\psi_k \\ &+ K_{d,2}\chi_0\psi_k + 2K_{d,1}\sin\psi_k\cos\psi_k \end{aligned} \tag{45}$$

Hence, the following equations hold near the origin since the first term dominates the others:

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$$\begin{aligned} \psi_k > 0 &\Rightarrow \varphi_k < 0 \Rightarrow \dot{\varphi}_k > 0 \\ \psi_k < 0 &\Rightarrow \varphi_k > 0 \Rightarrow \dot{\varphi}_k < 0 \end{aligned} \tag{46}$$

Thus, it can be concluded that the internal dynamic φ_k is stable.

V. Simulation Results

In this section, the simulation results of the sensor coverage problem are studied. The simulation results carried out in MATLAB environment are given for the two controllers. The dimensions of the map are 10x10 meters and the robot count is 5.

The coefficients used in simulations are $K_d = [0.4 \quad 0.4]^T$, $P_0 = diag([0.01 \quad 0.2])$, $\chi_0 = 0.1$ and $\mu = 3$. The distributed density function $\phi(q)$ is chosen as expanding circle and it is triggered at predefined time instants in the simulation. The variance parameter of the Gaussian is taken as $\sigma^2 = 0.05$. For the adaptive controller, the parameters are $R = diag([1 \quad 1])$, $\lambda_f = 10^{-4}$, $k_0 = 0.1$ and $\lambda_0 = 10$.

The simulation starts with a trigger which corresponds a density function with an expanding circle of Gaussians. The final radius of the circle is 3.5 meters. The adaptive control algorithm enables the agents to position themselves according to the density function in approximately 20 seconds. In this time period, the angular and distance errors converge to zero as given in Fig. 3.

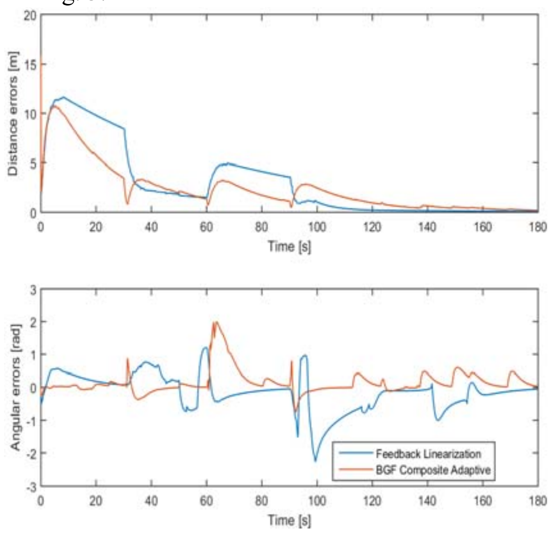

Fig. 3 Angular and Distance Errors of the Two Controllers

The second and third triggers come at 30th and 60th seconds from the start of the simulation. The final radii of the expanding circle are now 2 and 3 meters, respectively. Again, the adaptive control algorithm positions the mobile agents to their calculated positions in approximately 20 seconds. Similarly, with the fourth trigger at 90th second, the final radius of the expanding circle is 2.5 meters. The angular and distance errors converge to zero asymptotically.

The distance and angular errors of the two controllers for 5 agents are given in Fig. 3.

The distance and angular errors of BGF Adaptive Controller in Fig. 4 show that the errors of the individual agents converge to zero asymptotically. The estimation and consensus errors of BGF Adaptive Controller are given in Fig. 5.

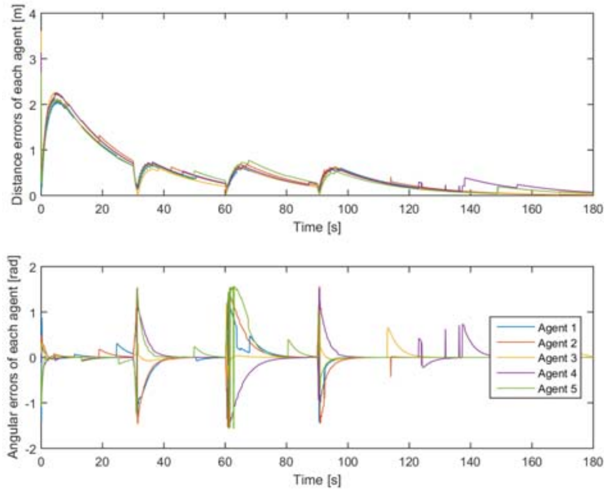

Fig. 4 Distance and Angular Errors of Each Agent for BGF Adaptive Controller

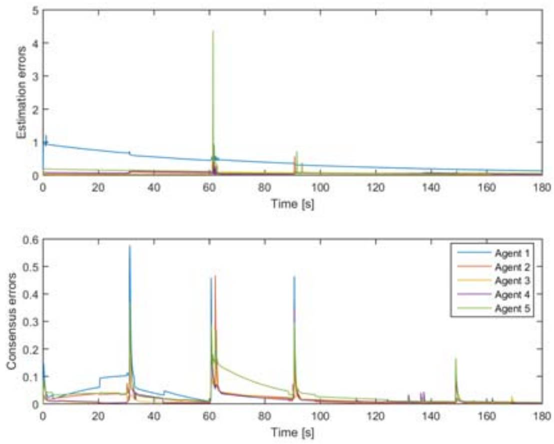

Fig. 5 Estimation and Consensus Errors of Each Agent for BGF Adaptive Controller (Filtered)

The main difference between the Feedback Linearization controller and the adaptive one is the consensus protocol. In the BGF Adaptive Controller, each agent shares its parameter vector with its neighbors. This provides synchronization among the mobile nodes since the agents communicate with each other. The ROS simulation video with three agents can be viewed at [12].

VI. Conclusions

A multi-agent adaptive controller for coverage control problem is presented. The controller estimates the model parameters of the kinematic model of the robot. The Centroidal Voronoi Tessellations provide decentralized multi-agent coordination.

Different from the traditional constant gain adaptive controllers, the composite adaptive controllers take the tracking error as well as the prediction error into account. The Feedback Linearization and BGF Composite Adaptive Controller are proven to be asymptotically stable with Lyapunov stability theory. The estimation errors of the adaptive controller converge to zero asymptotically. The theoretical results are verified with the simulation results.

With the BGF Composite Adaptive Controller, the best tracking error performance is achieved. The adaptive controller can deal with the unknown dynamics since the unknown parameters are estimated while the Feedback Linearization Controller can only deal with the nonlinearities derived in the design phase based on the system model.

Appendix A. Stability Proof of the BGF Composite Adaptive Controller

The Lyapunov function candidate is defined as:

$$V(t)=\sum_{k}V_k(t)$$

$$V_k(t)=\frac{1}{2}\left(\frac{1}{\chi_0}s_k^T s_k+2\tilde{a}_k^T\Gamma P^{-1}\chi_0\tilde{a}_k+\chi_0\psi_k^2\right)$$

For simplicity, K is taken as constant and is positive definite for $\left|\psi_k\right|\le\frac{\pi}{2}$. Since the robot is moving both forward and backward directions, this assumption holds.

Taking the derivative of the Lyapunov candidate yields:

$$\begin{aligned}\dot{V}_k(t)&=\frac{1}{\chi_0}s_k^T\dot{s}_k+2\tilde{a}_k^T\Gamma P^{-1}\chi_0\dot{\hat{a}}_k+\tilde{a}_k^T\Gamma\dot{P}^{-1}\chi_0\tilde{a}_k+\chi_0\psi_k\dot{\psi}_k\\&=\frac{1}{\chi_0}s_k^T\dot{s}_k-2\chi_0\tilde{a}_k^TP^{-1}P\Gamma W^TR(t)W\tilde{a}_k\\&\quad-2\chi_0\tilde{a}_k^T\Gamma P^{-1}P(Y'Y)^Ts_k-\chi_0\tilde{a}_k^T\Gamma\lambda P^{-1}\tilde{a}_k\\&\quad+\chi_0\tilde{a}_k^T\Gamma W^TW\tilde{a}_k+\chi_0\psi_k\dot{\psi}_k-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\end{aligned}$$

If we choose $R(t)=I$ and apply the control law to the system, we get:

$$\begin{aligned}\dot{V}_k(t)&=s_k^T\left(Y\hat{a}_k-KK_ds_k+K_{d,1}K_u\right)\\&\quad-\chi_0\tilde{a}_k^TP^{-1}P\Gamma W^TW\tilde{a}_k-2\chi_0\tilde{a}_k^T\Gamma P^{-1}P(Y'Y)^Ts_k\\&\quad-\chi_0\tilde{a}_k^T\Gamma\lambda P^{-1}\tilde{a}_k+\chi_0\psi_k\dot{\psi}_k-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\end{aligned}$$

If we use $Y'Y$ and apply the control law to the system equations we obtain the following time derivative:

$$\dot{\psi}_k=-\omega_k+u_k\frac{\sin\psi_k}{e_k}$$

$$\begin{aligned}\dot{V}_k(t)&=s_k^TY\tilde{a}_k-s_k^TKK_ds_k+s_k^TK_{d,1}K_u\\&\quad-\chi_0\tilde{a}_k^TP^{-1}P\Gamma W^TW\tilde{a}_k-2\chi_0\tilde{a}_k^T\Gamma P^{-1}P(Y'Y)^Ts_k\\&\quad-\chi_0\tilde{a}_k^T\Gamma\lambda P^{-1}\tilde{a}_k\\&\quad+\chi_0\left(\begin{aligned}&\left(-Y_{21}\tilde{a}_{k,1}-Y_{22}\tilde{a}_{k,2}\right)\psi_k-2K_{d,2}\psi_k^2\\&-2\sin\psi_k\cos\psi_k\psi_kK_{d,1}\\&+\left(Y_{11}\tilde{a}_{k,1}+Y_{12}\tilde{a}_{k,2}+4\cos\psi_kK_{d,1}e_k\right)\frac{\sin\psi_k}{e_k}\psi_k\end{aligned}\right)\\&\quad-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\end{aligned}$$

Substituting s_k with $s_k=\left(-e_k\quad-\psi_k\right)^T$ in the equations yields:

$$\begin{aligned}\dot{V}_k(t)&=-e_k\left(Y_{11}\tilde{a}_{k,1}+Y_{12}\tilde{a}_{k,2}\right)\chi_0-\psi_k\left(Y_{21}\tilde{a}_{k,1}+Y_{22}\tilde{a}_{k,2}\right)\chi_0\\&\quad-e_k^TKK_d\chi_0{}^2e_k-\psi_k^TKK_d\chi_0{}^2\psi_k-2\sin\psi_k\cos\psi_k\psi_k\chi_0K_{d,1}\\&\quad-\chi_0\tilde{a}_k^TP^{-1}P\Gamma W^TW\tilde{a}_k\\&\quad+\chi_0\left(1-\frac{\sin\psi_k}{e_k{}^2}\psi_k\right)\left(Y_{11}\tilde{a}_{k,1}+Y_{12}\tilde{a}_{k,2}\right)e_k\\&\quad+2\chi_0\left(Y_{21}\tilde{a}_{k,1}+Y_{22}\tilde{a}_{k,2}\right)\psi_k-\chi_0\tilde{a}_k^T\Gamma\lambda P^{-1}\tilde{a}_k\\&\quad+\chi_0\left(-Y_{21}\tilde{a}_{k,1}-Y_{22}\tilde{a}_{k,2}\right)\psi_k-2\chi_0K_{d,2}\psi_k{}^2\\&\quad-2\sin\psi_k\cos\psi_k\psi_k\chi_0K_{d,1}\\&\quad+\left(Y_{11}\tilde{a}_{k,1}+Y_{12}\tilde{a}_{k,2}\right)\frac{\sin\psi_k}{e_k}\chi_0\psi_k\\&\quad+4\sin\psi_k\cos\psi_k\psi_k\chi_0K_{d,1}\\&\quad-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\end{aligned}$$

The equation becomes:

$$\begin{aligned}\dot{V}_k(t)&=-e_k^TKK_d\chi_0{}^2e_k-\psi_k^TKK_d\chi_0{}^2\psi_k\\&\quad-\chi_0\tilde{a}_k^TP^{-1}P\Gamma W^TW\tilde{a}_k-\chi_0\tilde{a}_k^T\Gamma\lambda P^{-1}\tilde{a}_k\\&\quad-2\chi_0K_{d,2}\psi_k{}^2-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\\&=-K_{1,1}K_{d,1}\chi_0^2e_k^2-\chi_0^2K_{2,2}K_{d,2}\psi_k^2\\&\quad-2\chi_0K_{d,2}\psi_k^2-\lambda(t)\tilde{a}_k^T\Gamma P^{-1}\chi_0\tilde{a}_k\\&\quad-\tilde{a}_k^T\Gamma W^TW\chi_0\tilde{a}_k-2\tilde{a}_k^T\Gamma\chi_0\eta\sum_{l\in\mathcal{L}_k}(\hat{a}_k-\hat{a}_l)\end{aligned}$$

$$\dot{V}_k(t)\le 0$$

The first, second, third and fourth terms are negative since K, K_d and W are positive definite matrices, and $P^{-1}(t)\ge I/k_0$ and $\lambda(t)$ is positive.

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$$-\mu\sum_k\sum_{l\in\mathcal{L}_k}\tilde{a}_k^T(\hat{a}_k-\hat{a}_l)=-\frac{1}{2}\mu\sum_k\sum_l A_{kl}(\hat{a}_k-\hat{a}_l)^T(\hat{a}_k-\hat{a}_l)$$
$$=-\frac{1}{2}\mu\sum_k\sum_{l\in\mathcal{L}_k}\left\|\hat{a}_k-\hat{a}_l\right\|^2$$

The fourth term is also negative by using (39), the detailed proof for binary protocol is given in [11]. Here, A denotes adjacency matrix.

By Barbalat's Lemma, $\dot{V}(t)\to 0$ and the errors converge to zero asymptotically since $V(t)$ is positive and lower bounded, and $\ddot{V}(t)$ is bounded as $t\to\infty$.

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