# Lookup Table Reduction and Its Error Analysis of Hall Sensor-Based Rotation Angle Measurement

Young-San Shin, Seongsoo Lee

**Abstract**—Hall sensor is widely used to measure rotation angle. When the Hall voltage is measured for linear displacement, it is converted to angular displacement using arctangent function, which requires a large lookup table. In this paper, a lookup table reduction technique is presented for angle measurement. When the input of the lookup table is small within a certain threshold, the change of the outputs with respect to the change of the inputs is relatively small. Thus, several inputs can share same output, which significantly reduce the lookup table size. Its error analysis was also performed, and the threshold was determined so as to maintain the error less than 1°. When the Hall voltage has 11-bit resolution, the lookup table size is reduced from 1,024 samples to 279 samples.

*Keywords*—Hall sensor, angle measurement, lookup table, arctangent.

#### I. INTRODUCTION

HALL sensor is used to measure linear or angular displacement. It is exploited in many types of equipment such as joysticks, cameras, smartphones, motors and cars. Especially, it is widely used in automotive applications [1], as shown in Fig. 1. When a Hall sensor is displaced in the magnetic field, Hall voltage is induced in proportional to the linear displacement. To measure rotation angle, a 3-axis Hall sensor is used, as shown in Figs. 2 and 3. In the mechanical movements, either a magnet or Hall sensor rotates in two displacements ( $\theta$ ,  $\phi$  of angular coordinates), as shown in Fig. 1. In the electronic measurement, three displacements (x, y, z of rectangular coordinates) are measured by Hall voltages (Vx, Vy, Vz), and are converted into two rotation angles ( $\theta$ ,  $\phi$ ) using an arctangent function, as shown in Fig. 3.

Usually, small microcontroller unit (MCU) is embedded in the Hall sensor for sensor signal processing, as shown in Figs. 3 and 4. However, in many cases, the small embedded MCU have no floating units and it is difficult to calculate the arctangent function [2], [3]. Therefore, a lookup table (LUT) is often used to calculate the arctangent function, but still it suffers from large table size [4], [5]. Small embedded MCUs have no external memory and it has only a small embedded memory shared with a program code, used data, and parameter tables. Therefore, LUT should be optimized to reduce memory occupancy.

In this paper, a two-stage LUT reduction technique is presented as an angle measurement in a Hall sensor-based rotation angle measurement, where several addresses of the LUT shares the same sample. Also, its error analysis was performed.

### II. TWO-STAGE LUT

Fig. 5 shows the proposed Hall sensor-based rotation angle measurement system with two-stage LUT. Rotation angle is measured as two Hall voltages, i.e. Vx and Vy. Digitized Hall voltages by 11-bit analog-to-digital converter (ADC) are X and Y. Note that X and Y are 11-bit signed integers. Angle calculation is to find Angle = LUT[X], where X is the digitized Hall voltage and LUT[·] is the LUT. There are two digitized Hall voltages, X and Y, but X and Y can share the same LUT. Furthermore, positive  $X (\geq 0)$  and negative  $X (\leq 0)$  can also share the same LUT with opposite signs. Nevertheless, it requires 1,024 samples in the LUT.

Fig. 6 shows a curve between X and Angle = LUT[X]. When the magnitude of X is lower than a threshold  $X_T$ ,  $\Delta(LUT[X])/\Delta(X)$  is small. This means that several X's can share same LUT[X] in the range of  $X \le X_T$ . When  $X > X_T$ , conventional LUT is used. This two-stage LUT can significantly reduce the LUT size.

In this paper, when the Hall voltage has 11-bit resolution, threshold  $X_T$  is set to 851, where approximation error should be kept under 1°. As a result, the LUT size is reduced from 1,024 samples to 279 samples. The detailed process of the proposed two-stage LUT is described in Fig. 5.

#### **III. ERROR ANALYSIS**

In the proposed two-stage LUT, several digitized Hall voltages share a common value. So there is some error between the measured and approximated rotation angles, as shown in Fig. 7.

When X increases,  $\Delta(LUT[X])/\Delta(X)$  also increases; this is due to the fact that LUT[X] is a monotonically decreasing function of X, as shown in Fig. 6. Thus, total error increases until X reaches to X<sub>T</sub>, as shown in Fig. 8. When X exceeds X<sub>T</sub>, the number of LUT[X] samples is not reduced, and error becomes negligible.

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Fig. 3 Measurements of Hall voltages and calculation of rotation angle in Hall sensor

Embedded

MCU with

Analog

Readout

Circuit



Fig. 5 The proposed Hall sensor-based rotation angle measurement system with two-stage LUT



Fig. 6 Curve between digitized Hall voltage (X-axis) and rotation angle (Y-axis)







Fig. 8 Total error in the proposed two-stage LUT

## ACKNOWLEDGMENT

This research was supported by the Ministry of Science, ICT and Future Planning, supervised by the Institute for Information & communications Technology Promotion (R7117-16-0165). This research was also supported by Industrial Core Technology Development Program (10049095) funded by the Ministry of Trade, industry & Energy, Korea."

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