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Investigation of Stability of Functionally Graded Material when Encountering Periodic Loading

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Abstract**—**In this work, functionally graded materials (FGMs), subjected to loading, which varies with time has been studied. The material properties of FGM are changing through the thickness of material as power law distribution. The conical shells have been chosen for this study so in the first step capability equations for FGM have been obtained. With Galerkin method, these equations have been replaced with time dependant differential equations with variable coefficient. These equations have solved for different initial conditions with variation methods. Important parameters in loading conditions are semi-vertex angle, external pressure and material properties. Results validation has been done by comparison between with those in previous studies of other researchers.

Keywords**—**Impulsive semi-vertex angle, loading, functionally graded materials, composite material.

NOMENCLATURE

A_{mn}	amplitude
$A_\beta, B_\beta\ (\beta = 1-6)$	defined in (15)
C_{1k}, C_{2k}, C_{3k} $(k = 0, 1, 2)$	defined in (16)
C_0	integration constant
e d	power law exponent
E, E_1, E_2	elastic moduli of the material
$e_S, e_\theta, e_{S\theta}$	strain components on the reference surface of the conical shell
F_1, F_2	Material property of the constituent's materials
h	Thickness of the conical shell
i	Power of time in the external pressure expression
I_{cr}	critical stress impulse
$j_k (k = 1,2)$	coefficient
k_d	dynamic factor
$M_s, M_\theta, M_{S\theta}$	moment resultants
m	wave number in the S direction
$N_S, N_\theta, N_{S\theta}$	forces resultants
$N_S^0, N_\theta^0, N_{S\theta}^0$	membrane forces in the fundamental configuration
n	wave number in the circumferential direction
n_{st}, n_d	wave numbers corresponding to the static and dynamic critical loads
$Q_{\alpha\beta}$	Reduced stiffness defined in (7)-(9)
q_{crs}, q_{crd}	static and dynamic critical loads, respectively
q_0, q_1	loading parameter and static external pressure, respectively
$\bar{q}_1$	defined in (36)
r_1, r_2	average radii of the small and large bases of the conical shell
$S\theta\zeta$	coordinate system on the reference surface of the conical shell
S	the axis through the vertex on the reference surface of the cone
S_1, S_2	the inclined distances of the bases of the cone from the vertex
t, t_{cr}	time and critical time, respectively
T	temperature in Kelvin
V_f	volume fractions
w	displacement of the reference surface in the inwards normal direction ζ
γ	semi-vertex angle of the cone
δ_1, δ_2	Defined in (35b) and (40), respectively
$\upsilon, \upsilon_1, \upsilon_2$	Poisson's ratios
τ	dimensionless time parameter
ρ, ρ_1, ρ_2	densities of the materials
λ	a parameter that depends on the geometry of the conical shell
θ	axis lies in the circumferential direction
$\sigma_S, \sigma_\theta, \sigma_{S\theta}$	stress components
ω	defined in (37)
$\xi_{mn}(t), \eta_{mn}(t)$	time dependent amplitudes
ζ	the axis in the inwards normal direction of the reference surface
X	defined in (35a)
$\Delta_\mu (\mu = -1, 0, 1/2)$	defined in (31)
Ψ	stress function
ϕ_k	(k = 0, 1, 2) defined in (31)
Λ	defined in (21)
Λ_1, Λ_2	Defined in (29a) and (29b), respectively
Π	potential energy defined in (31)

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I. INTRODUCTION

FGMs are now widely used in various applications to higher material strength. Initially FGM attract attentions when Japanese scientist, published his work [1]. FGMs are some kind of composite materials, in those mechanical properties varies from one surface to another. These gradual changes in properties of material microscopically occur in mechanical properties which vary slightly from one surface to another. This is gained with slowly changing the volume fraction of the component materials. FGMs were first intended to be a thermal shield material for aerospace structures and some special duty reactors. FGMs are now fabricated for high temperature environment applications as structural components [2]. Researches for FGM have been done mainly to analysis of deformation and thermal stress [3], [4]. Birman [5] represented a formulation for stability of FGM composite plates, in which a micromechanical model was developed to solve buckling problem of a rectangular plates under axial loading. Feldman and Aboudi [6] assumed that grades of material properties all through the structure are result of a spacial distribution of volume fraction for local reinforcement. Praveen and Reddy [7] studied the response of ceramic-metal FGM plates by a finite element method for transverse shear strains, rotary inertia and to a certain extend large rotations in the "Von Karman" sense. Static and dynamic responses of FGM plates were investigated by changing the volume fraction of ceramic and metallic components by a simple power law distribution. Loy [8] analyzes vibration of a thin shell cylinder with FGM supports a compound of stainless steel and nickel. For thick plates Reddy [9] developed a formulations considering shear deformation plate theory. After that Pradhan [10] continued working in this subject of FGM thin shell cylinder to various boundary conditions. Some other studies focused on parametric resonance of FGM thin shell cylinder under periodic axial loading [11]. In the earlier studies, Reddy and his colleague [9] evolved a simple theory, while material properties are graded in the direction of thickness agreeing to a volume fraction power law distribution, however their out coming results have been only validated for simple case of FGM shell in a not changing thermal environment. Woo and Mequid [12] analyzes a solution for considerable deflection in thin FGM plates and shallow shells. In their studies the thermal load considered arises from the one dimensional steady heat conduction in the direction of plate thickness, but properties of material are independent from temperature. A self-consistent constitutive framework describes the manners of a common three-layered system encompassing a FGM layer under thermal loading [13]. A post-buckling analysis for a thin shell cylinder of finite length was also carried out under external pressure in thermal environments [14]. Transient responses in a FGM cylinder to a point load [15] and transient dynamic analysis of a cracked FGM by a BIEM [16] were also performed by research groups. In addition, large deflection and post buckling responses of FGM rectangular plates subjected to transverse and in-plane loads were done by a semi-analytical method [17].

Conical FGMs have wide applications in airspace industry as structural element so several studies were conducted on vibration and stability study of conical shells. Most of these studies focused on isotropic and composite shells. [18]-[25].

For solving the stability problems of conical shells, it is impossible to obtain analytical solutions as some difficulties occur because of the real forms of the subjective loads. Practically, simple analysis expresses an acceptable approximation for load change depends on consumed time. As evidence in some cases outcomes of the wind and fluid pressure are stated as the power function of time. In some of previous studies, external pressure is considered [26]. In this research, the stability and consistency of truncated FGM conical shells under external pressure changing as a power function of time is investigated, considering different initial conditions.

II. Theoretical Studies

In the selected coordinate system, the origin O is at the vertex of the whole cone, on the reference plane of the shell, and the S axis is on the rounded reference surface of the cone, the h axis lies in the direction of circumference on the reference surface of the cone and finally the f axis, is at right angle to the plane of the first two axes, lies within normal direction of the cone. The average radiuses of the small and large bases of the conical shell are r1 and r2, and the distances between the vertex and the small and large bases are S1 and S2. It is noteworthy that the semi-vertex angle is c.

For properly model and obtain material properties of FGMs, these properties have to be dependent to temperature and position. This is accomplished with the aid of a simple mixtures rule which governing the stiffness parameters joined with the temperature dependent characteristics of the constituents. The fraction of volume is a 3 dimensional function and the properties of the elements are functions of temperature. The mixture of these functions gives increment to the effective material properties of FGMs and could be stated as:

$$F = F_1 V_{f1} + F_2 V_{f2} \tag{1}$$

F_1 and F_2 are properties of components materials. V_{f1} and V_{f2} are volume fractions of components which stated as:

$$V_{f1} + V_{f2} = 1 \tag{2}$$

Let volume fraction follows following power law:

$$V_f = \left(\bar{\zeta} + 0.5\right)^d, \bar{\zeta} = \zeta / h \tag{3}$$

index d varies through shell thickness to reach optimum value for component materials. In some previous works such this definition for V_f has also been used [11]. Regarding (1)-(3) elastic modulus of a shell made from FGM $E(\bar{\xi})$, Poisson's ratio $\nu(\bar{\xi})$ and density $\rho(\bar{\xi})$ could be written as:

$$\begin{aligned} E(\bar{\xi}) &= (E_1 - E_2)(\bar{\zeta} + 0.5)^d + E_2 \\ \nu(\bar{\xi}) &= (\nu_1 - \nu_2)(\bar{\zeta} + 0.5)^d + \nu_2 \\ \rho(\bar{\xi}) &= (\rho_1 - \rho_2)(\bar{\zeta} + 0.5)^d + \rho_2 \end{aligned} \tag{4}$$

These three equation express Poisson ration, elastic modulus and density of material 1 and 2 and as result following equations are obtained:

$$\begin{cases} E = E_1,\ \nu = \nu_1, \rho = \rho_1\ at\ \bar{\zeta} = 0.5 \\ E = E_2,\ \nu = \nu_2, \rho = \rho_2\ at\ \bar{\zeta} = -0.5 \end{cases} \tag{5}$$

The material properties are changing continuously from material 2 (inside surface) to material 1 at the (outside surface) of conic section of the shell.

Regarding the distribution in (5), the internal surface of the conic part of shell is ceramic rich and the external surface is metal rich. Let call this type A and subsequently for a metal rich conical internal part of shell and ceramic rich external surface Type B. Consequently, the properties of material and the shells thickness, like Poissons ratio $\nu(\bar{\xi})$ and elastic modulus $E(\bar{\xi})$ could be obtained by (4). By knowing properties of material, the relations between stress and strain for thin conical shells are gained as:

$$\begin{pmatrix}\sigma_S\\ \sigma_\theta\\ \sigma_{S\theta}\end{pmatrix}=\begin{bmatrix}Q_{11} & Q_{12} & 0\\ Q_{12} & Q_{22} & 0\\ 0 & 0 & Q_{66}\end{bmatrix}\begin{pmatrix}e_S-\frac{\partial^2 w}{\partial S^2}\\ e_\theta-\frac{1}{S^2}\frac{\partial^2 w}{\partial\varphi^2}-\frac{1}{s}\frac{\partial w}{\partial S}\\ e_{S\theta}-\frac{1}{S}\frac{\partial^2 w}{\partial S\partial\varphi}+\frac{1}{S^2}\frac{\partial w}{\partial\varphi}\end{pmatrix} \quad (6)$$

where, $\varphi=\theta\sin\gamma$, σ_S, σ_θ and $\sigma_{S\theta}$ are components of stress, σ_S, σ_θ and $\sigma_{S\theta}$ are components of stress on the reference plane, w is the displacement of reference plane in the normal positive direction, on the way to the axis of the cone and supposed to be much smaller than the thickness and $Q_{\alpha\beta}$ $(\alpha,\beta=1,2,6)$ are expressed as:

$$Q_{11}=Q_{22}=\frac{(E_1-E_2)(\bar{\zeta}+0.5)^d+E_2}{1-\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]^2} \quad (7)$$

$$Q_{12}=\frac{\left[(E_1-E_2)(\bar{\zeta}+0.5)^d+E_2\right]\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]}{1-\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]^2} \quad (8)$$

$$Q_{12}=\frac{\left[(E_1-E_2)(\bar{\zeta}+0.5)^d\right]+E_2}{2\left[1+(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]} \quad (9)$$

It is presumed that a unchanging outside pressure changing like a power function of time acts on shell as:

$$N_S^0=-0.5\times S(q_1+q_0t^i)tan\gamma,\ N_\theta^0=-S\,(q_1+q_0t^i)tan\gamma,\ N_{S\theta}^0=0 \quad (10)$$

N_S^0, N_θ^0 and $N_{S\theta}^0$ are forces in the basic settings, q_0 is loading parameter, q_1 is the static pressure from outside, i is a positive whole number power which is used to show time dependence of pressure from outside which meets $i\geq 1$ t here is expressed coordination of time.

By following expression force and moment could be calculated:

$$(N_s,N_\theta,N_{s\theta})=h\int_{-0.5}^{0.5}(\sigma_S,\sigma_\theta,\sigma_{S\theta})d\,\bar{\zeta},$$
$$(M_s,M_\theta,M_{s\theta})=h^2\int_{-0.5}^{0.5}(\sigma_S,\sigma_\theta,\sigma_{S\theta})d\,\bar{\zeta} \quad (11)$$

Let stress function be:

$$N_s=\frac{1}{S^2}\frac{\partial^2\Psi}{\partial\varphi^2}+\frac{1}{S}\frac{\partial\Psi}{\partial S},\ N_\theta=\frac{\partial^2\Psi}{\partial S^2}$$
$$N_{S\theta}=-\frac{1}{S}\frac{\partial^2\Psi}{\partial S\partial\varphi}+\frac{1}{S^2}\frac{\partial\Psi}{\partial\varphi} \quad (12)$$

Therefore, equations of dynamic stability and compatibility could be rewritten as:

$$\begin{aligned}L_1(\Psi,\mathrm{w})=&A_2\frac{\partial^4\Psi}{\partial\varphi^4}+\frac{2A_2}{S}\frac{\partial^3\Psi}{\partial S^3}+\frac{S\cot\gamma-A_2}{S^2}\frac{\partial^2\Psi}{\partial S^2}+\frac{A_2}{S^3}\frac{\partial\Psi}{\partial S}+\frac{A_2}{S^4}\frac{\partial^4\Psi}{\partial S^4}\\&+\frac{2(A_1-A_5)}{S^2}\frac{\partial^4\Psi}{\partial S^2\partial\varphi^2}+\frac{2(A_5-A_1)}{S^3}\frac{\partial^3\Psi}{\partial S\partial\varphi^2}\\&+\frac{2(A_1-A_5+A_2)}{S^4}\frac{\partial^2\Psi}{\partial\varphi^2}-\frac{A_3}{S^4}\frac{\partial^4\Psi}{\partial\varphi^4}\\&-\frac{2(A_4+A_6)}{S^2}\frac{\partial^4\Psi}{\partial S^2\partial\varphi^2}+\frac{2(A_4+A_6)}{S^3}\frac{\partial^3\mathrm{w}}{\partial S\partial\varphi^2}\\&-\left[\frac{q_1+q_0t^i}{Scot\gamma}+\frac{2(A_4+A_6+A_3)}{S^4}\right]\frac{\partial^2w}{\partial\varphi^2}A_3\frac{\partial^4w}{\partial S^4}\\&-\frac{2A_3}{S}\frac{\partial^3w}{\partial s}+\left(\frac{A_3}{S^2}-\frac{(q_1+q_0t^i)S}{2cot\gamma}\right)\frac{\partial^2\mathrm{w}}{\partial S^2}\\&-\left(\frac{q_1+q_0t^i}{cot\gamma}+\frac{A_3}{S^3}\right)\frac{\partial w}{\partial S}-\rho_t h\frac{\partial^2w}{\partial t^2}=0\end{aligned}$$

$$\begin{aligned}L_2(\Psi,w)=&\frac{B_1}{S^4}\frac{\partial^4\Psi}{\partial\varphi^4}+\frac{2(B_5+B_2)}{S^2}\frac{\partial^4\Psi}{\partial S^2\partial\varphi^2}-\frac{2(B_5+B_2)}{S^3}\frac{\partial^3\Psi}{\partial S\partial\varphi^2}\\&+\frac{2(B_5+B_2+B_1)}{S^4}\frac{\partial^2\Psi}{\partial\varphi^2}+\frac{B_1}{S^3}\frac{\partial\Psi}{\partial S}+\frac{B_2-B_1}{S^2}\frac{\partial^2\Psi}{\partial S^2}\\&+\frac{2B_1}{S}\frac{\partial^3\Psi}{\partial S^3}+B_1\frac{\partial^4\Psi}{\partial S^4}-\frac{B_4}{S^4}\frac{\partial^4w}{\partial S^4}\\&-\frac{2(B_6-B_3)}{S^3}\frac{\partial^4w}{\partial S^2\partial\varphi^2}+\frac{2(B_6-B_3)}{S^3}\frac{\partial^3w}{\partial S\partial\varphi^2}\\&+\frac{2(B_6-B_3-B_4)}{S^4}\frac{\partial^2w}{\partial\varphi^2}-\frac{B_4}{S^3}\frac{\partial w}{\partial S}\\&+\left(\frac{B_\$}{S^2}+\frac{cot\gamma}{S}\right)\frac{\partial^2w}{\partial S^2}-\frac{2B_4}{S}\frac{\partial^3w}{\partial S^3}-B_4\frac{\partial^4w}{\partial S^4}=0\end{aligned}$$

A_β,$B_\beta(j=1-6)$ and $\rho_t$are defined as:

$A_1=C_{11}B_1+C_{21}B_2$, $A_2=C_{11}B_2+C_{21}B_1$, $A_3=C_{11}B_3+C_{21}B_4+C_{12}$, $A_4=C_{11}B_4+C_{21}B_3+C_{22}$, $A_5=C_{61}B_5$, $A_6=C_{61}B_6+C_{62}$
$B_1=C_{10}D$, $B_2=-C_{20}D$, $B_3=(C_{20}C_{21}-C_{11}C_{10})D$, $B_4=(C_{20}C_{11}-C_{21}C_{10})D$, $B_5=\frac{1}{C_{60}}$, $B_6=\frac{C_{61}}{C_{60}}$

$$D=1/[(C_{10})^2-(C_{20})^2],\ \rho_t=\int_{-0.5}^{0.5}[(\rho_1-\rho_2)(\bar{\zeta}+0.5)^d+\rho_2]d\bar{\zeta} \quad (15)$$

C_{1k}, C_{2k} and C_{6k} (k=0, 1, 2) are expressed as:

$$C_{1k}=h^{k+1}\int_{-0.5}^{0.5}\bar{\zeta}^k\frac{(E_1-E_2)(\bar{\zeta}+0.5)^d+E_2}{1-\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]^2}d\bar{\zeta} \quad (16a)$$

$$C_{2k}=h^{k+1}\int_{-0.5}^{0.5}\bar{\zeta}^k\frac{\left[(E_1-E_2)(\bar{\zeta}+0.5)^d+E_2\right]\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]}{1-\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]^2}d\bar{\zeta} \quad (16b)$$

$$C_{6k}=h^{k+1}\int_{-0.5}^{0.5}\bar{\zeta}^k\frac{\left[(E_1-E_2)(\bar{\zeta}+0.5)^d+E_2\right]\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]}{1-\left[(\nu_1-\nu_2)(\bar{\zeta}+0.5)^d+\nu_2\right]^2}d\bar{\zeta} \quad (16c)$$

III. Solving the Problems

As conical section of shell has a simple support around the bases, the displacement and stress functions, w and Ψ, are defined as:

$$w=\sum_m\sum_n\xi_{mn}(t)e^{\lambda r}\sin m_1r\cos n_1\varphi \quad (19)$$
$$\Psi=\sum_m\sum_n\eta_{mn}(t)S_2e^{(\lambda+1)r}\sin m_1r\cos n_1\varphi$$

$m_1 = m\pi/\ln(S_2/S_1)$, $r = \ln(S/S_2)$, $n_1 = n/\sin\gamma$, $\xi_{mn}(t)$ and $\eta_{mn}(t)$ vary with amplitudes, m is the number of wave in the direction of S, n is the wave number in the peripheral direction, λ is the geometry dependent parameter on $1.2 \leq \lambda \leq 0.2$ section of the of the conical shell.

Let $r = \ln(S/S_2)$ and by regarding (13) and (14) with the aid of Galerkin method the following equations are obtained:

$$\int_0^{2\pi\sin\gamma}\int_{-\ln(S_2/S_1)}^{0} L_1(\Psi, w) w S_2^2 e^{2r} dr dc = 0$$
$$\int_0^{2\pi\sin\gamma}\int_{-\ln\left(\frac{S_2}{S_1}\right)}^{0} L_2(\Psi, w) w S_2^2 e^{2r} dr d\varphi = 0$$

The equations acquired by (17)-(19), by means of derivatives with respect to variables φ and S, each at a time, it is noteworthy that, the involved functions must increase with respect to φ and slowly varies with respect to S. For$m = 1$, regarding above properties, terms $\xi_{mn}(t)$ and$\eta_{mn}(t)$, ignoring minor terms and removing $\eta_{mn}(t)$ from equations, then the following expression is obtained:

$$\frac{d^2\xi_{mn}(\tau)}{d\tau^2} + \Lambda(\tau)\xi_{mn}(t) = 0 \quad (20)$$

Regarding the above equations $t = t_{cr}\tau$ and t_{cr} are critical and τ is the dimensionless time parameter which lies$0 \leqq \tau \leqq 1$. In (20), the below equations must be used:

$$\Lambda(\tau) = \frac{t_{cr}^2}{\rho_t h S_2^2}\left[\left(A_3 - \frac{A_2B_4}{B_1}\right)\frac{\Delta_{-1}n_1^4}{S_2^2} + \frac{m_2^2}{n_1^4}\frac{\Delta_0}{B_1}cot^2\gamma - n_1^2\Delta_{1/2}\left(q_1 + q_0 t_{cr}^i \tau^1\right)S_2\tan\gamma\right] \quad (21)$$

$$m_2^2 = (m_1^2 + \lambda^2)(m_1^2 + \lambda^2 - 1) \quad (22)$$

$$\Delta_\mu = \frac{[1-(S_1S_2^{-1})^{2(\lambda+\mu)}][m_2^2-(\lambda+1)^2](\lambda+1)}{[1-(S_1S_2^{-1})^{2(\lambda+1)}][m_2^2+(\lambda+1)^2](\lambda+\mu)} \quad \mu = -1, 0, 1/2 \quad (23)$$

The problem solution is transformed to second ODE (Order Differential Equation) with time dependent variable coefficients which meets the first condition as:

$$\varepsilon = 0, \frac{\partial\xi}{\partial\tau} = 0 \quad (24)$$

Regarding (ξ, τ) curve has a max. Value at $\tau = 1$ the initial condition is r as,

$$\xi = 0 \text{ when } \tau = 0 \text{ and } \frac{\partial\xi(1)}{\partial\tau} = 0 \text{ when } \tau = 1. \quad (24)$$

In (20), a method is employed by using Lagrange–Hamilton type principle. The approximating functions meet (24) and (25) has been selected as a 1st approximation as:

$$\xi_{mn}(\tau) = A_{mn}\xi(\tau) = A_{mn}e^{j_1\tau}\tau^2[(j_1+3)(j_1+2)^{-1} - \tau] \quad (26)$$

$$\xi_{mn}(\tau) = A_{mn}\xi(\tau) = A_{mn}e^{j_2\tau}\tau^2[(j_2+3)(j_2+2)^{-1} - \tau] \quad (27)$$

When value of critical load is minimized it will be dependent on choosing $\xi(\tau)$ then, it could depend on the values of j_k $(k = 1,2)$ too. It has been obtained that after the computations, none of the minimum values of critical load corresponds to$j_k = i + 1$ $(k = 1,2)$. Here, A_{mn} is amplitude of unknown displacement. Equation (20) is multiplied by $\xi_\tau^{'}(\tau)$ and after integration, below equation is gained:

$$\left[\frac{d\xi(\tau)}{d\tau}\right]^2 + \Lambda_1[\xi(\tau)]^2 - 2\Lambda_2\int\xi(\tau)\frac{d\xi(\tau)}{d\tau}\tau^i d\tau = C_0 \quad (28)$$

C_0 is a constant for integration and it is assumed that the initial conditions are equal to zero. In addition, in each interval points $0 < \tau < 1$, $\xi_\tau^{'}(\tau)$ is not equal (19) to zero and below definitions are applicable:

$$\Lambda_1 = \frac{t_{cr}^2}{\rho_t h S_2^2}\left[\left(A_3 - \frac{A_2B_4}{B_1}\right)\frac{\Delta_{-1}}{S_2^2}n_1^4 + \frac{m_2^2}{n_1^4}\frac{\Delta_0}{B_1}cot^2\gamma - q_1 n_1^2\Delta_{1/2}S_2\tan\gamma\right] \quad (29a)$$

$$\Lambda_2 = \frac{q_0 n_1^2 t_{cr}^{2+i}\Delta_{1/2}\tan\gamma}{\rho_t h S_2^2} \quad (29b)$$

After substituting (26) and (27) in (28) and after integration in $0 \leq \tau \leq 1$, for Lagrange–Hamilton type functional the following expression is gained:

$$\Pi = A_{mn}\left\{\frac{\Phi_1\rho_t h S_2^4}{t_{cr}^2 n_1^2\tan\gamma} - \Phi_2 q_0 t_{cr}^i S_2^3\Delta_{1/2} + \Phi_0\left[\left(A_3 - \frac{A_2B_4}{B_1}\right)\frac{\Delta_{-1}}{\tan\gamma}n_1^2 + \frac{S_2^2m_2^2\Delta_0}{B_1tan^3\gamma}\frac{1}{n_1^6} - q_1\Delta_{1/2}S_2^3\right]\right\} \quad (30)$$

where

$$\Phi_0 = \int_0^1[\xi(\tau)]^2d\tau, \Phi_1 = \int_0^1[\xi_\tau'(\tau)]^2d\tau,$$
$$\Phi_2 = 2\int_0^1\int_0^\tau \eta^i\xi_\tau'(\eta)\xi(\eta)d\eta d\tau, \quad (31)$$

TABLE I
THE VALUES OF Φ_K, K=0, 1,2 FOR DIFFERENT VALUES POWER OF TIME I

Symbol	$i = 1$	$i = 2$	$i = 3$	$i = 4$
	$\xi(\tau) = e^{j_1\tau}\tau^2[(j_1+3)(j_1+2)^{-1} - \tau]$			
Φ_0	0.8859	3.5645	15.809	75.2705
Φ_1	4.6561	24.700	142.836	870.4636
Φ_2	0.5789	1.8139	6.8670	29.3800
	$\xi(\tau) = e^{j_1\tau}\tau[(j_2+2)(j_2+1)^{-1} - \tau]$			
Φ_0	2.086	7.0527	27.7911	121.533
Φ_1	6.678	30.8768	168.032	992.333
Φ_2	1.1430	2.9326	9.9737	39.916

The values of Φ_k, $k = 0, 1, 2$ is given in Table I. Meanwhile in finite time, there is no agreement between the work done by external forces and inertia force and the minimum value of potential energy. When the minimum condition in respect of unknown amplitude A_{mn} of Π, must be supported by minimum condition in respect of n_1^2. These two conditions give the net two equations which are dependent on t_{cr} andn_1:

$$\frac{\partial\Pi}{\partial A_{mn}} = \frac{\Phi_1\rho_t h S_2^4}{t_{cr}^2\tan\gamma}\frac{1}{n_1^2} - \Phi_2 q_0 t_{cr}^i S_2^3\Delta_{\frac{1}{2}} + \Phi_0\left[\left(A_3 - \frac{A_2B_4}{B_1}\right)\frac{\Delta_{-1}}{\tan\gamma}n_1^2 + \frac{S_2^2m_2^2\Delta_0}{B_1tan^3\gamma}\frac{1}{n_1^6} - q_1\Delta_{\frac{1}{2}}S_2^3\right] = 0 \quad (32)$$

$$\frac{\partial \Pi}{\partial n_1^2} = \Phi_0 \left[\left(A_3 - \frac{A_2 B_4}{B_1} \right) \frac{\Delta_{-1}}{\tan\gamma} - \frac{3 S_2^2 m_2^2 \Delta_0}{B_1 tan^3\gamma} \frac{1}{n_1^8} \right] - \frac{\Phi_1 \rho_t h S_2^4}{t_{cr}^2 \tan\gamma} \frac{1}{n_1^4} = 0$$

After removing t_{cr} from (32) and (33), these two equation are gained:

$$(1-3x)^{\frac{i}{2}} \left(1 - X - 0.5 \overline{q_1} X^{\frac{1}{4}} \right) = q_0 w^{i/2} X^{(1+i)/4} \quad (34)$$

where these definitions are applied:

$$X = \delta_1 \frac{cot^2\gamma}{n_1^8} \quad (35a)$$

$$\delta_1 = \frac{m_2^2 S_2^2}{(A_3 B_1 - A_2 B_4)} \frac{\Delta_0}{\Delta_{-1}} \quad (35b)$$

$$\overline{q_1} = \frac{\Delta_{1/2} S_2^{5/2} B_1^{1/4} q_1}{\Delta_0^{1/4} [(A_3 B_1 - A_2 B_4)\Delta_{-1}]^{3/4}} \frac{\Delta_0}{cot^{3/2}\gamma} \quad (36)$$

$$\omega = \frac{\Phi_1 \Phi_2^{2/i} \rho_t h (\Delta_{1/2})^{2/i} S_2^{(3i+5)/i} B_1^{(2+i)/i}}{2^{2/i} \Phi_0^{(2+i)/i} [m_2^2 \Delta_0]^{(1+i)/(2i)} [(A_3 B_1 - A_2 B_4)\Delta_{-1} cot^2\gamma]^{(3+i)/(2i)}} \quad (37)$$

For $q_1 = 0$ and large values of the loading parameters, (34) for X is solved by putting this value in (35) and taking into consideration the relation$n_1 = n/\sin\gamma$. Finally, the (38) is gained as:

$$n_d^2 = \frac{1}{\sqrt{2}} \delta_1^{1/4} q_0^{1/(1+i)} \omega^{i/(2+2i)} sin^{1/2}(2\gamma) \sin\gamma \quad (38)$$

The wave number n_d, which is dependent on variation of the dynamic load. Substituting (38) in (32) demonstrates (39) and (40) as:

$$q_{crd} = q_0 t_{cr}^i = \frac{2\delta_2 \Phi_0}{\Phi_2} q_0^{1/(1+i)} w^{i/(2+2i)} \quad (39)$$

where

$$\delta_2 = \frac{\Delta_0^{1/4}}{\Delta_{1/2}} \frac{m_2^{\frac{1}{2}} \Delta_{-1}^{\frac{3}{4}} (A_3 B_1 - A_2 B_4)^{3/4}}{B_1 S_2^{5/2}} cot^{3/2}\gamma \quad (40)$$

For the static case $(t_{cr} \to \infty, q_0 \to 0)$ from (33) the next coming equation is obtained for the wave number:

$$n_{st}^2 = 0.75^{0.25} \delta_1^{\frac{1}{4}} sin^{\frac{1}{2}}(2\gamma) \sin\gamma \quad (41)$$

Replacing (41) in (32) and substituting $q_0 t_{cr}^i / \Phi_0$, the critical load is gained as:

$$q_{crs} = \frac{4\delta_2}{3^{3/4}} \quad (42)$$

From $K_d = q_{crd}/q_{crs}$ definition the dynamic factor is obtained as:

$$K_d = \frac{3^{3/4} \Phi_0}{2\Phi_2} q_0^{1/(1+i)} \omega^{i/(2+2i)} \quad (43)$$

Critical time and stress impulse can be found as:

$$t_{cr} = \left(\frac{2\delta_2 \Phi_0}{\Phi_2} \right)^{1/i} q_0^{1/(1+i)} \omega^{1/(2+2i)} \quad (44)$$

$$I_{cr} = \int_0^{t_{cr}} q_0 t^i dt = \left[\frac{2\Phi_0 \delta_2}{\Phi_2} \right]^{(1+i)/i} \frac{\omega^{1/2}}{1+i} \quad (45)$$

IV. Numerical Computations

Silicon nitride and nickel is used in this study is used is nickel. The densities and Poissons ratios of the materials are not dependent on the temperature. The density of silicon nitride and nickel is 2370 kg/m^3 and 8900 kg/m^3 respectively. The Poissons ratio is 0.24 for silicon nitride and 0.31 for nickel. The module of elasticity is obtained as: (temperature dependent):

$$E_{sn} = 348.43 \times 10^9 (1 - 3.070 \times 10^{-4} T + 2.160 \times 10^{-7} T^2 - 8.946 \times 10^{-11} T^3) \quad (46)$$

$$E_{ni} = 223.95 \times 10^9 (1 - 2.794 \times 10^{-4} T - 3.998 \times 10^{-9} T^2) \quad (47)$$

E_{sn} and E_{ni} are of silicon nitride and nickel, respectively, and $T = 300$ K is the temperature.

TABLE II
VARIATION OF THE CRITICAL PARAMETERS WITH SEMI-VERTEX ANGLE γ FOR $\left(q_0 = 225 \left(\frac{MPa}{s}\right) (\xi(0) = 0, \xi_\tau'(1) = 0, \xi(\tau) = e^{2\tau}\tau\left[\frac{3}{2} - \tau\right]\right)$

γ	$d^{SN}=0$	Type A material					$d^N=0$
		$d=0.5$	$d=1$	$d=2$	$d=3$	$d=4$	
				$q_{crd}(MPa)$			
30º	0.0791	0.0768	0.0744	0.07123	0.0694	0.0682	0.0629
45º	0.0715	0.0694	0.0672	0.06437	0.0627	0.0616	0.0569
60º	0.0601	0.0583	0.0565	0.05413	0.0527	0.0518	0.0478
				$I_{cr} \times 10^5$ $(Mpa\ s)$			
30º	13.895	13.107	12.2981	11.276	10.698	10.334	8.0803
45º	11.346	10.702	12.2981	9.2069	8.7350	8.4377	7.1875
60º	8.0224	7.5674	7.10030	6.5102	6.1766	5.9663	5.0823
				K_d			
30º	3.0458	2.5047	2.2937	2.0920	1.9871	1.9204	1.5919
45º	2.6378	2.1692	1.9864	1.8117	1.7209	1.6631	1.3786
60º	3.0458	2.5047	2.2937	2.0920	1.9871	1.9204	1.5919

TABLE III
VARIATION OF THE CRITICAL PARAMETERS WITH SEMI-VERTEX ANGLE $\boldsymbol{\gamma}$ FOR $\boldsymbol{q_0 = 225\,(\frac{MPa}{s})\,(\xi(0)=0, \xi'_\tau(1)=0, \xi(\tau)=e^{2\tau}\tau\left[\frac{3}{2}-\tau\right]}$

γ	$d^{SN}=0$			Type B material			$d^N=0$
		$d=0.5$	$d=1$	$d=2$	$d=3$	$d=4$	
				$q_{crd}(MPa)$			
30°	0.0791	0.0715	0.0744	0.0769	0.0781	0.0787	0.0629
45°	0.0715	0.0646	0.0672	0.0695	0.0705	0.0711	0.0569
60°	0.0601	0.0544	0.0565	0.0584	0.0593	0.0598	0.0478
				$I_{cr} \times 10^5\ (Mpa\ s)$			
30°	13.895	11.368	12.298	13.145	13.537	13.749	8.0803
45°	11.346	9.2817	10.141	10.733	11.053	11.226	7.1875
60°	8.0224	6.5632	7.1003	7.5894	7.8158	7.9378	5.0823
				K_d			
30°	3.0458	2.0752	2.2937	2.4973	2.5971	2.6597	1.5919
45°	2.6378	1.7972	1.9864	2.1627	2.2492	2.3034	1.3786
60°	3.0458	2.0752	2.2937	2.4973	2.5971	2.6598	1.5919

In Tables II and III, stable results for the FGM conic shell are shown. The silicon nitride nickel shell of geometrical properties $r_1 = 2.25 \times 10^{-2}$ (m), $r_2 = 8 \times 10^{-2}$ (m), $r_2 = 8 \times 10^{-2}$ (m), $h = 1.3 \times 10^{-4}$ (m), $\lambda = 1.2$ are simply supported for type A and B materials. In the case that $d = 0$ (type A), the shell is made of ceramic and in type B material, when $d \leq 0$, the shell is metallic.

Table II shows critical and dynamic load. Considering the values of dynamic factor values in Tables II and III, it would be obvious that for power law exponent $d \geq 0$, values for critical parameters in Type B material are higher than a Type A material. For power law exponent $d = 0$, critical values of parameters for Type A and B are same. When $d \leq 1$ in power law exponent the values of critical parameters for type A material are larger than a Type B material. For instance, in power law exponent when d is equal to one the dynamic critical load, dynamic factor and critical impulse for a Type B material are approximately about 13.5%, 24.84% and 27.8% (greater than a Type A material). While coefficient d become greater, there will be a little increment.

V.CONCLUSIONS

In the current research stability of conical shells of FGM which is subjected to external pressure with variation of a power function of time was studied. Considering a large values of loading parameters into account, analytic solutions are used for different primary conditions for critical parameters values. Results were changing considerably while material distribution was different by changing the values of the power law exponent. This parameter controls the material volume fraction of the different materials in the FGM_S. It has also found that reasonable control could be possible if we have a good control over critical parameters values by properly changing the power law exponent. A validation of the analysis has been done with comparison between previous results and those which has found to be accurate.

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