

# Dynamic Synthesis of a Flexible Multibody System

Mohamed Amine Ben Abdallah, Imed Khemili, Nizar Aifaoui

**Abstract**—This work denotes an insight into dynamic synthesis of multibody systems. A set of mechanism parameters design variable are synthesized based on a desired mechanism response, such as, velocity, acceleration and bodies deformations. Moreover, knowing the work space, for a robot, and mechanism response allow defining optimal parameters mechanism handling with the desired target response. To this end, evolutionary genetic algorithm has been deployed. A demonstrative example for imperfect mechanism has been treated, mainly, a slider crank mechanism with a flexible connecting rod. The transversal deflection of the connecting rod has been chosen as response to identify the mechanism design parameters.

**Keywords**—Dynamic response, flexible bodies, optimization, evolutionary genetic algorithm.

## I. INTRODUCTION

THE dynamic behavior of multi-body systems has been widely treated with a tremendous numbers of work [1]-[5]. It has been shown that imperfections, such as, joint clearance and bodies' flexibility, considerably affect the mechanism response. These imperfections are unavoidable and mainly due to manufacturing process. Despite these unavoidable imperfections, in some application fields, the mechanism accuracy is highly required. The imperfection effects should be avoided along the mechanism response. Thus, imperfections' impacts on dynamic response are alleviated. Consequently, the mechanism exhibits more steady behavior.

For the last few decades, dimensional synthesis of multibody systems has been object of many works. Lariibi [6] has treated a trajectory synthesis of four bar mechanism. An evolutionary genetic algorithm has been used for this aim. The problem was modeled as an optimization problem. Hence, an objective function is necessary for the resolution. The evolutionary genetic algorithm, for a desired trajectory, proposes optimal geometrical design variable for the four bar mechanism in order to obtain the desired path. An enhanced algorithm coupling fuzzy logic to genetic algorithm has been performed.

Erkaya [7] proposed kinematic optimization for a planar slider crank mechanism with a clearance. Using artificial

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neural network, the transmission angle of a slider crank mechanism has been optimized.

Erkaya [8] has treated the optimization of a four bar mechanism with clearance. Genetic algorithm has been used in order to minimize clearance effects on dynamic response through optimal link parameters. Genetic algorithm and artificial neural network are combined together, in order to guarantee more efficient synthesis algorithm [9]. The algorithm conceived for the optimization of a mechanism with joint clearance.

Erkaya [10] reduced effects of the clearance in a slider crank mechanism with clearance. The presented algorithm reduces the clearance effects on the mechanism response. Thus, impacts involved in the revolute joint contribute to create volatilities and sharps increase in the crank torque. Using genetic algorithm, optimal design variable minimizing the clearance effects are determined. Consequently, mechanism response is more accurate and clearance effects are significantly alleviated.

Varedi presented a dynamic synthesis for a slider crank mechanism using particle swarm optimization [11], [12]. The proposed design parameters relieve the clearance effects.

In this work, a genetic algorithm for dynamic synthesis of a flexible slider crank mechanism based on a desired dynamic response is presented. Moreover, efficiency and robustness of the proposed algorithm have been investigated.

## II. EQUATION OF MOTION

Dynamic resolution of the equations of motion for multibody systems is one of the stiffest problems. This is mainly because of the highly nonlinear equations system describing its dynamic response. In order to solve the governing differential equations of motion, several resolution schemes, algorithms and methods were developed.

Based on parameters depicted in Fig. 1, analytical study has been established for the flexible slider crank mechanism with perfect joints. The Lagrange's equations applied for the mechanism yields:

$$\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = Q_e + Q_v \quad (1)$$

For the slider crank mechanism three parameters are required in order to establish the mechanism equations of motion. Thus, the generalized coordinate vector yields:

$$q = \{q_1, q_2, \dots, q_n\}^T \quad (2)$$

$$T = T_1 + T_2 + T_3 = \left( \frac{1}{2} I_2 \dot{\theta}_2^2 + \frac{1}{8} m_2 r_2^2 \dot{\theta}_2^2 \right) + \frac{1}{2} \int_0^{x_5} \rho A \dot{r}(x,t)^2 dx + \frac{1}{2} m_c V_c^2 \quad Q_{vi} = \sum_{j=1}^2 \lambda_j \frac{\partial \varphi_j}{\partial q_i} \quad (4)$$

$$(3)$$

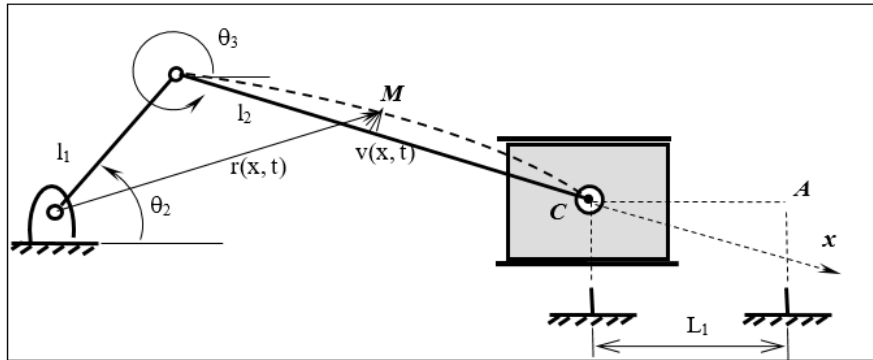


Fig. 1 Flexible slider crank

wherein, T represents the total kinetic energy of the system.  $Q_s$  and  $Q_v$  represent respectively the total applied forces and the total constrained forces.

The constraint equation for the slider crank mechanism, which is a system of one degree of freedom, with holonomic constraints based on general coordinates, is:

$$\varphi(q,t) = \begin{pmatrix} \varphi_1(q,t) \\ \varphi_2(q,t) \end{pmatrix} = \begin{pmatrix} l_1 \cos \theta_1 + l_2 \cos \theta_2 - x \\ l_1 \sin \theta_1 + l_2 \sin \theta_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (5)$$

In the presented work, mechanism parameters, mainly, the crank length, as well as, the connecting rod length, for a desired response, are identified using evolutionary genetic algorithm optimization. The connecting rod transversal deflection (Fig. 2) is the dynamic response used to identify the mechanism parameters.

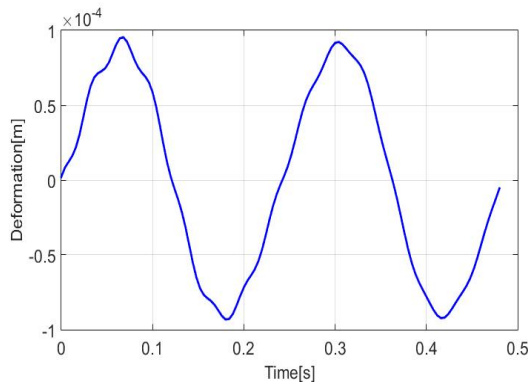


Fig. 2 Transversal deflection of the connecting rod center

### III. EVOLUTIONARY GENETIC ALGORITHM

Evolutionary Genetic Algorithm is one of the most prominent optimization tools used for the last few decades. It is mainly inspired from natural phenomena. In fact, it is based on nature law, which assigns best probability or chance for the strongest individual to survive. However, low quality or weak

individuals have a high death probability. Evolutionary Genetic Algorithm process is mainly divided in the following steps.

#### A. Initial Population Choice

In this step, an initial population is chosen with a defined numbers of chromosomes. Every chromosome contains its own allele. For the presented optimization problem, the number of alleles depends on the mechanism parameters required. In this work, we are limited to two design variables optimization of the slider crank mechanism, mainly, the crank length and the connecting rod length. Mass and inertia will be optimized consequently due to their relation to the length.

#### B. Evaluation

In order to classify the chromosomes, the evaluation of each one, in the initial population, is necessary. This evaluation will foster the selection of the best individual for next steps of the evolutionary genetic algorithm process.

The evaluation of each chromosome performance is made by means of the following objective function:

$$F = \min(\text{error}) \quad (6)$$

$$\text{error} = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_{iDV} - X_{i\text{target}})^2} \quad (7)$$

$$\text{Subject to: } \begin{cases} lb_1 \leq l_1 \leq ub_1 \\ lb_2 \leq l_2 \leq ub_2 \end{cases} \quad (8)$$

with  $lb_1, ub_1$  are respectively lower and upper bound of the search interval for the crank length,  $lb_2, ub_2$  are respectively lower and upper bound of the search interval for the connecting rod length,  $X_{iDV}$  is the design variable response at instant i,  $X_{i\text{target}}$ : the target response at instant i,  $n$ : total number of point.

**C. Selection**

Throughout this step, a selection probability will be assigned to each chromosome. Then, the selection operation will be similar to a casino wheel operation, wherein, best individual evaluation has higher probability to be retained. Nevertheless, the chromosomes selection with low performance remains always possible.

**D. Crossover**

In the crossover process, two selected chromosomes exchange a set of alleles to give birth to two children as referred in Fig. 3. These children will be later incorporated in a mutation process. Usually, crossover is favored to occur, this leads to have a new population with better performance. The crossover probability is 0.9 along algorithm execution.

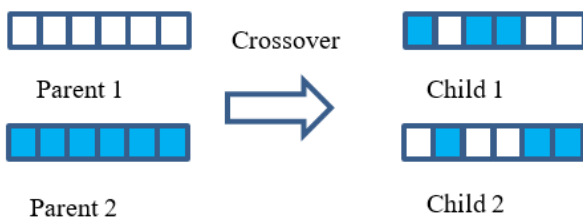


Fig. 3 Crossover operation

**E. Mutation**

Throughout the mutation operation, a randomly chosen allele of the chromosome is substituted with another allele as witnessed in Fig. 4. This allows to considerably improve the chromosome performance in some cases, and to investigate a broader search space for the parameters.

In this work, mutation probability is 0.3 along algorithm execution.



Fig. 4 Mutation operation

**IV. RESULTS AND DISCUSSION**

Initially, parameters chromosomes are constructed with two alleles, mainly,  $X=[l_1 \ l_2]$  which corresponds respectively to the crank and the connecting rod length. A set of chromosomes are chosen randomly. The evaluation of all these chromosomes is established. For this work the mechanism objective parameters are illustrated in Table I.

The search interval for each parameter is fixed. A short search interval is in accordance with a well informed designer about workspace and manufacturing tolerance. High interval for design variable is significant for a completely ignorance of both workspace and manufacturing tolerance.

A set of 10 chromosomes (C1 to C10) are chosen from Table I, as well as, iteration numbers is fixed to 50 iterations. In about 10 generations, algorithm start convergence and

objective function is minimized (Fig. 5). In about 50 iterations, convergence is established for the algorithm. The couple of design variable results are 48.5 mm and 351.9 mm respectively for crank and connecting rod length. The required calculation time is 179.6 sec using an Intel processor I7 3.4 Ghz for an estimated error of  $1.40904 \cdot 10^{-3}$ . The proposed length response is illustrated in Fig. 6.

TABLE I  
 REDUCED INTERVAL

Chromosomes	$l_1$ (mm)	$l_2$ (mm)
Interval	[45 55]	[345 355]
References	50	350
C1	45.8	354.4
C2	54.1	345.6
C3	53.6	352.9
C4	45.8	354.9
C5	47.4	348.1
C6	46.2	354.6
C7	52.8	345.3
C8	46.4	347.2
C9	52.2	346.1
C10	45.5	352.8
C11	54.6	348.5
C12	50.5	347
C13	46.4	347.5
C14	46.5	351.2
C15	47.6	349.7
C16	53.4	348.5
C17	47.5	353.3
C18	53.1	350.9
C19	47.4	350.5
C20	54.3	354.2

**A. Effects of Generations Numbers**

This part is focused on generations' number effects on the proposed solution for the design variable by means of evolutionary genetic algorithm. For a 250 generations, the algorithm provides a better solution giving more accurate response. However, better solution needs a longer CPU time calculation. After 250 iterations, the algorithm proposes a couple of 49.7 mm and 350.7 mm respectively for crank and connecting rod length. The calculation time is estimated to 788 sec. This infers that increasing the generations' number instigates the optimization algorithm to explore more interesting search space.

Analyzing the objective function evolution depicted in Figs. 5 and 6 to Figs. 7 and 8, confirms the efficiency of the generations number increase in spite of longer CPU calculation time.

**B. Effects of Chromosomes Numbers**

This section is devoted to initial population choice, thus chromosomes numbers of the initial population.

All the chromosomes C1 to C20 are involved in the initial population. Fig. 9 depicts the effects of population size on design variable dimension proposed. For 20 chromosomes, the algorithm proposes a couple of (50.5, 347) respectively for an error of  $1.02 \cdot 10^{-3}$  reached in just 20 iterations. However, it

needs 30 iterations to converge for the same crossover and mutation probabilities. Obviously, for 20 chromosomes CPU time is about 208 sec, calculation lasts for longer time but for a better design variable response accuracy.

In order to get better results, generation number is extended to 250 generations. As witnessed in Fig. 10, for higher number of iterations the algorithm reaches better the solutions with lower error. For a 250 iterations, the design variable couple is (50.1, 350.3) respectively for  $l_1$  and  $l_2$  with an error estimated about  $2.5 \cdot 10^{-4}$  and a CPU time of 208 sec. A higher iteration number offers a better solution, whereas, it needs much more CPU time.

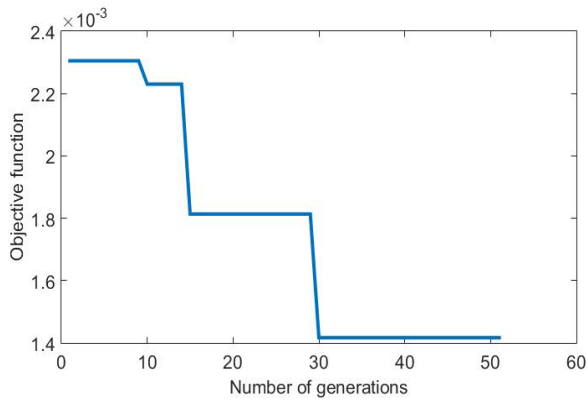


Fig. 5 Error evolution with 10 chromosomes

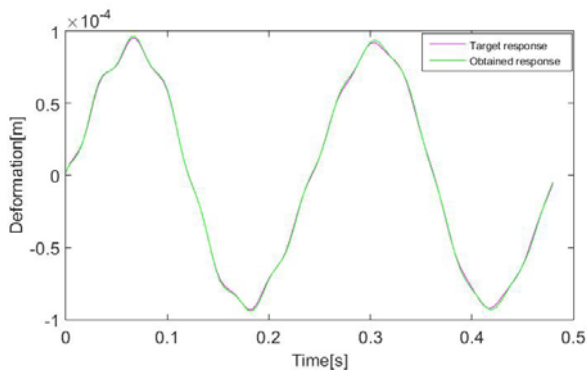


Fig. 6 Response after 50 iteration

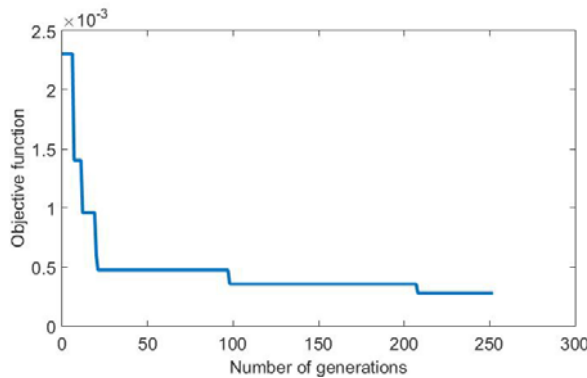


Fig. 7 Error evolution with 10 chromosomes

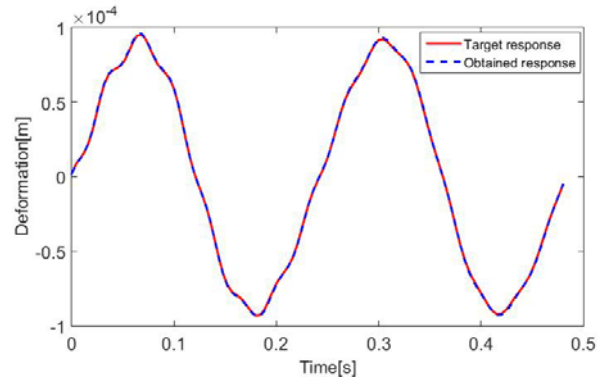


Fig. 8 Response after 250 iterations

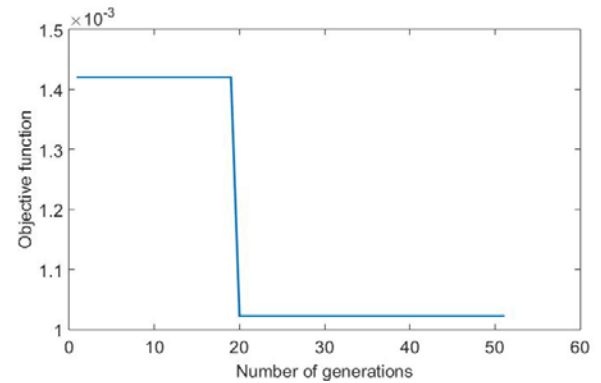


Fig. 9 Error evolution with 20 chromosomes

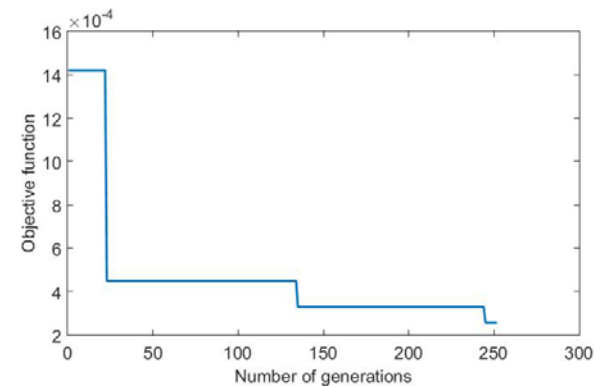


Fig. 10 Error evolution with 20 chromosomes

## V. INTERVAL SEARCH EFFECTS

A larger interval of search is adopted: [30 80] for the crank length and [300 400] for the connecting rod length.

### A. Iteration Numbers Effects

In this section, the efficiency of the proposed algorithm is investigated. Thus, a larger interval of search for design variables is proposed in order to test the algorithm efficiency to converge for the target design variable. The new interval of search is [30 80] and [300 400] for, respectively, the crank and the connecting rod length. In the first step only the 10<sup>th</sup> first chromosomes are considered.

As depicted in Fig. 11, a large interval of search make convergence for the desired design variable a heavy task for the algorithm. For a 50 iterations, the proposed values are 45 and 367.7 mm respectively for  $l_1$  and  $l_2$  with a CPU time of 206 sec and an estimated error of  $4.7 \cdot 10^{-3}$ . For 10 chromosomes and 50 iterations with a larger search interval of variable, the algorithm is not able to reach the desired parameters. This represents a real issue for some application wherein the design is not well known. Moreover, with 250 iterations, as it is shown in Fig. 12, the proposed solution by the evolutionary genetic algorithm remain inaccurate with a proposed couple of 45.2 and 365.9 mm, an error of  $3.6 \cdot 10^{-4}$  and a CPU time of 807 sec.

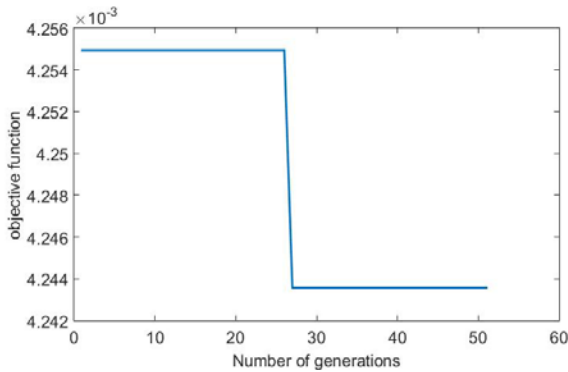


Fig. 11 Error evolution with 10 chromosomes from enlarged interval

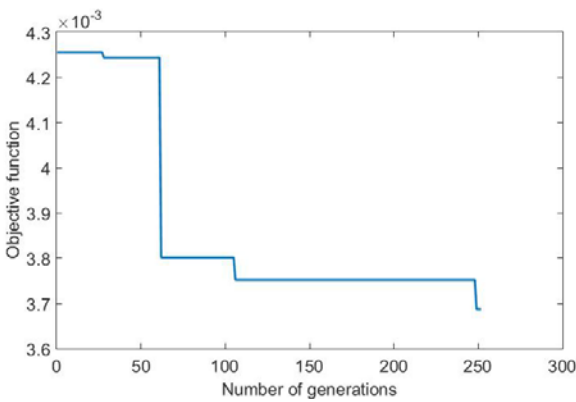


Fig. 12 Error evolution with 10 chromosomes from enlarged interval

The effects of iterations numbers for the enlarged interval, with an initial population composed of 10 chromosomes are not pronounced. In sharp contrast of the results presented in Figs. 5 and 7 to Figs. 11 and 12, the objective function evolution error is not significantly improved. Moreover, increasing generations number is not really efficient, thus, improvement of accuracy for the proposed couple of design variable for 250 relative to 50 iterations is not really significant, as well as, CPU time is about 3 times more.

#### B. Effects of Chromosomes Numbers

In this section, a comparative study of the initial population size effects for the previous interval of search [20 80] [300 400] is presented.

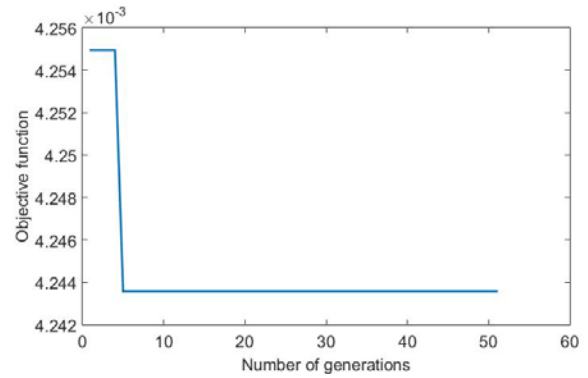


Fig. 13 Error evolution with 20 chromosomes from enlarged interval

The initial population containing 20 chromosomes (C1 to C20) is illustrated in Table II.

Chromosomes	$l_1$ (mm)	$l_2$ (mm)
Interval	[20 80]	[300 400]
References	50	350
C1	67.7	349.4
C2	75.5	384.9
C3	48	367.8
C4	56.8	344.6
C5	44.6	315.6
C6	78.2	349.7
C7	59.2	369.8
C8	66.3	320.5
C9	44.9	367.7
C10	28.9	312.5
C11	59.3	315.8
C12	22.1	397.1
C13	70.9	395.7
C14	76	348.5
C15	60.7	380
C16	65.5	314.2
C17	64.6	342.2
C18	43.5	391.6
C19	59.3	379.2
C20	30.3	395.9

As illustrated in Fig. 13, for 50 iterations, the presented solution remains always inaccurate and mechanism response is far away to the desired response. The same solution is proposed for both populations with 10 or 20 chromosomes for almost the same CPU time. Consequently, for low number of iterations (50 iterations) effects of population size are not significant.

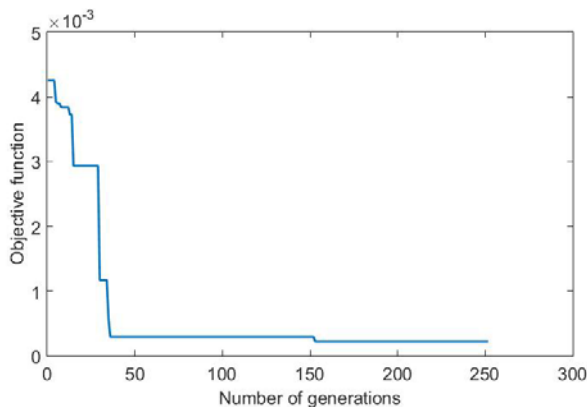


Fig. 14 Error evolution with 20 chromosomes from enlarged interval

Increasing the generations' number, as shown in Fig. 14, emphasizes its effects as proposed dimension of the genetic algorithm. A proposed couple of 50.2, 349.4 respectively for  $l_1$  and  $l_2$ , crank and connecting rod length with an error estimated at  $2e-4$ . The proposed couple response handle well with acceptable accuracy required. The mechanism response with the proposed crank and connecting rod length after 250 iterations for different initial population size is presented in Fig. 15. It can be seen that the proposed solution for a 20 chromosomes initial population correlate well with the required system reliability. However, mechanism response for the proposed length with only 10 chromosomes is far away to the desired mechanism response.

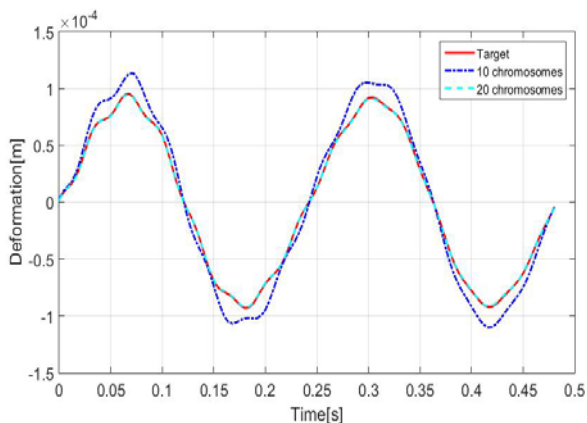


Fig. 15 Mechanism response for different initial population

## VI. CONCLUSION

In this paper, a dynamic synthesis of a flexible slider crank mechanism with evolutionary genetic algorithm has been presented. A comparative study of initial population size, number of generations as well as the search interval of design variable has been carried out.

It has been shown that for a small interval of search, the algorithm convergence is well ensured even for low iterations number. Subconsciously, increasing generation's numbers guarantees low error for the design variable proposed by genetic algorithm.

If population initial values are chosen from a large interval, the proposed couple response is far away from target response, this is mainly for low iteration number (about 50 iterations). However, for high number of iterations and initial population size, the convergence of the algorithm is ensured and the proposed design variable responses almost overlap the desired response.

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