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Estimating Bridge Deterioration for Small Data Sets Using Regression and Markov Models

Yina F. Muñoz, Alexander Paz, Hanns De La Fuente-Mella, Joaquin V. Fariña, Guilherme M. Sales

***Abstract*—**The primary approach for estimating bridge deterioration uses Markov-chain models and regression analysis. Traditional Markov models have problems in estimating the required transition probabilities when a small sample size is used. Often, reliable bridge data have not been taken over large periods, thus large data sets may not be available. This study presents an important change to the traditional approach by using the Small Data Method to estimate transition probabilities. The results illustrate that the Small Data Method and traditional approach both provide similar estimates; however, the former method provides results that are more conservative. That is, Small Data Method provided slightly lower than expected bridge condition ratings compared with the traditional approach. Considering that bridges are critical infrastructures, the Small Data Method, which uses more information and provides more conservative estimates, may be more appropriate when the available sample size is small. In addition, regression analysis was used to calculate bridge deterioration. Condition ratings were determined for bridge groups, and the best regression model was selected for each group. The results obtained were very similar to those obtained when using Markov chains; however, it is desirable to use more data for better results.

***Keywords*—**Concrete bridges, deterioration, Markov chains, probability matrix.

I. Introduction

BRIDGE Management Systems (BMS) are used by Departments of Transportation (DOTs) to monitor and make decisions regarding maintenance, preservation, and repair, subject to budget constraints [1]. In order to collect consistent data for BMS, DOTs use the *Manual for Bridge Element Inspection* [2], developed by the American Association of State Highway and Transportation Officials (AASHTO). This manual provides guidelines for data collection as well as how to qualify defects in order to determine structural conditions.

The Nevada Department of Transportation (NDOT) uses this manual to collect and provide data for the National Bridge Inventory (NBI) [3], including overall conditions ratings (CR) that represent the overall conditions for the deck, superstructure, substructure, and culvert of a bridge. Developing plans for maintenance, rehabilitation, and replacement requires estimating the bridge conditions. A number of methodologies for forecasting bridge deterioration involves dynamic response sensors [4], Markov chain models [5], and regression-based methods. The dynamic response sensors methodology requires instrumentation which is expensive for a wide coverage [4]. Deterministic models use regression-based methods to handle the randomness present during structural deterioration [6], [7]. Stochastic processes and deterministic models have contributed to modelling infrastructure deterioration. The most common stochastic technique used for a deterioration model has been the Markov chain model, and regression analysis for deterministic models. For example, Lijun and Ning [5] used Markov chain models to determine the deterioration of urban bridges at the network and individual levels [5]. Their study included three deterioration models, Natural Decay (ND), Conventional Recoverable Decay (CRD), and Enhanced Recoverable Decay (ERD).

Markov models primarily are used to estimate the deterioration of bridge components. Using hazard models, Kobayashi et al. [8] developed Markov models to estimate the transition probabilities after characterizing the deterioration process of each road section. This approach considers various possible states; at each time step, each state could move to another state according to the transition probabilities [8]. Tolliver and Lu [9] developed a model that predicted bridge deterioration rates over time without explicitly considering the history of bridge deterioration in previous periods; in addition, it predicted the effects of individual factors.

Other researchers have evaluated prediction models when using two bridge-management software developed by AASHTO, Pontis [10] and BRIDGIT [11]. These models use discrete condition states and constant inspection periods for all bridge components. In addition, these models assume that the condition of a bridge component depends only on its present condition [12]. Ranjith et al. [13] proposed Markov-chain models and a discrete condition-rating regime to predict bridge deterioration for both concrete and timber bridges. Ahmad [1] developed a Markov model to estimate deterioration rates for subsets or classes of bridges, categorized by material, design, operating rating, and average daily traffic. Tolliver and Lu [9] used nonlinear optimization and Markov models developed by Ranjith et al. [13] to study the deterioration of timber bridges. The percentage distribution of condition ratings on a network level in any year, as well as the deterioration trend of single bridges within

M. F. Yina is with the Universidade de Brasília, Brasília, DF 70910-900, Brasil (phone: +17023747619; fax: +556131070990; e-mail: yina.munoz@unlv.edu).
P. Alexander is with the University of Nevada Las Vegas, Las Vegas, NV 89154-4015, USA. (e-mail: apaz@unlv.edu).
D.L.F.M. Hanns is with the Pontificia Universidad Católica de Valparaíso, Valparaíso, Av. Brasil 2950, 2340025, Chile. (e-mail: hanns.delafuente@ucv.cl).
F. V. E. Joaquin is with the École Centrale Nantes, Nantes, 44321, France. (e-mail: jefarina@eleves.ec-nantes.fr).
S. M. Guilherme is with the Universidade de Brasília, Brasília, DF 70910-900, Brasil (e-mail: melog@ unb.br).

any state, was evaluated by Morcous [12]. Islam et al. [4] used Markov-chain models to predict the future conditions of bridge components, systems, and networks [4]. In general, Markov-chain models are the preferred tool to estimate bridge deterioration with a 95% level of confidence, assuming state independence [12].

Most of the existing literature developed bridge deterioration models by using more than 10 years of data. In many cases, however, state DOTs have not collected reliable data for as many years as required by the existing literature. In this study, only four years of data (2011-2014) were used to develop the Markov-chain models and regression analysis for bridge deterioration. In order to obtain reliable model estimates when using a relative small sample size, all transition periods were considered simultaneously rather than one at a time, as is done typically. A linear approximation was used to estimate the transition probabilities required by the Markov model. After that, the condition ratings were estimated for each bridge group by means of regression analysis.

II. Markov-Chain Model

A Markov process involves states and corresponding matrices of transition probabilities. In the context of modelling bridge deterioration, the states represent bridge condition ratings. Hence, the elements in the transition matrix represent the probability of bridges changing their condition rating at a determined and fixed period of time [6]. Markov-chain models are based on the concept of probabilistic cumulative damage, which estimates changes on component conditions over multiple transition periods [14].

A Markov chain is a special case of the Markov process. Its development can be considered as a series of transitions between certain condition states. When the probability of a future state in the process depends only on the present state, but not the past states, the Markov chain becomes a stochastic process, referred to as a first-order Markov process [12].

Markov chains are used to describe the evolution of a system represented by states $\{S = S_1, \ldots S_m\}$. The system undergoes transitions from one state to another during a defined transition period. If the system is in state S_i, then it will move into a future state S_j with transition probability $p_{s_{ij}}$ [15], as illustrated conceptually in Fig. 1. Markov processes are based on the hypothesis that future and past states are independent from each other; in other words, future states of the system only depend on the current state.

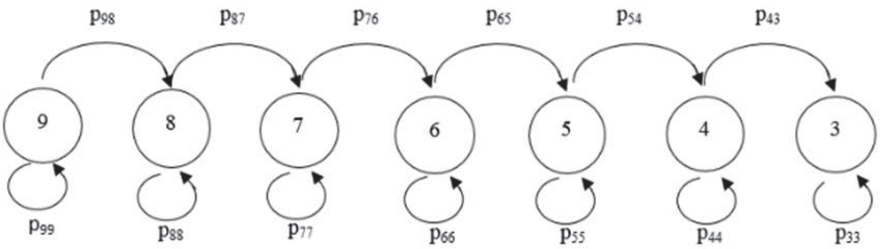

Fig. 1 Conceptual illustration of a Markov-chain process

When the probability of moving across states remains constant over transitions, independent of time, the process is stationary. That is, the system is completely described by one-step transition probabilities, which are grouped into a transition probability matrix (TPM), as depicted by (1). For purposes of simplicity, state S_i is denoted as State i.

$$P = \left[p_{ij} \right] = \begin{bmatrix} p_{11} & p_{12} & . & . & . & p_{1m} \\ p_{21} & p_{22} & & & & p_{2m} \\ . & . & . & & & . \\ . & . & & . & & . \\ . & . & & & . & . \\ p_{m1} & p_{m2} & . & . & . & p_{mm} \end{bmatrix} \qquad (1)$$

where, p_{ij} represents the probability of moving from state i to state j in a single transition period for all $i, j = 1, 2 \ldots m$; and m denoting the total number of states that the system can experience. The ith row represents the probability distribution of State i. The following relationship holds:

$$\sum_{j=1}^{m} p_{ij} = 1 \qquad (2)$$

If vector C_0 describes the initial condition of the system, the condition vector after t transition periods, $C(t)$, is given by [14]:

$$C(t) = C_0 . P^t \qquad (3)$$

where each element ij of matrix P^t provides the probability of moving from State i to State j after t transition periods. Generally, the TPM is estimated by statistical inference, using available data [5].

III. Analysis Regressions

Deterministic models are dependent on a mathematical or statistical formula for the relationship between the factors affecting bridge deterioration and the measure of a bridge's condition. The models can be developed as using straight-line extrapolation as well as by regression and curve-fitting methods [16].

Regression models are used to establish an empirical relationship between two or more variables: one dependent variable and one or more independent variables. Interpreting these models is complex because the models are nonlinear. In this study, deterioration functions were developed based on regression analysis of time series with NDOT data.

Regression analysis generates a coefficient for the imitation variable, using:

$$Y = a + bX + cZ\,, \tag{4}$$

where Y = the rating for the deck condition; X = the age of the bridge, and Z = a dummy variable for the structure type; a, b, and c are parameters estimated by using regression analysis.

The statistical significance of the coefficient estimated by regression analysis for the dummy variable represents a measure of the extent to which the potential determinant is influenced the deterioration of the bridge superstructure [17].

IV. Methodology

A. Markov Chains for Bridge Structures

In the context of bridge deterioration and Markov-chain models, for this study, the states for bridge structures were the condition ratings as set by FHWA guidelines [1]. These CR states range from 3 (serious condition) to 9 (excellent condition); the condition of a bridge is unlikely to go below 3 [8]. A transition period of one year was used because the corresponding data was collected on an annual basis. In this study, one assumption was that CRs do not change more than one level during a single transition period. This is a common assumption for this type of analysis [6], [12], and is consistent with actual data used in this study; that is, natural decay or bridge maintenance do not affect CRs significantly in one year.

In this study, only the natural decay process was considered, and bridges were excluded that experienced more than a medium amount of maintenance. Hence, the TPMs only contained probabilities about a structure with CRs that persisted or worsened during a period of one year. Therefore, each row of the TPM only had two values, one representing the probability of the structure staying in its current condition and the other one representing the probability of the structure moving to the next worse condition [13]. The TPMs were expressed in the following form:

$$P = \begin{bmatrix} p_{99} & 1-p_{99} & 0 & 0 & 0 & 0 & 0 \\ 0 & p_{88} & 1-p_{88} & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{77} & 1-p_{77} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{66} & 1-p_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & 1-p_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & p_{44} & 1-p_{44} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

where p_{ij} for all $i \in \{9,\dots,3\}$ represents the probability that a structure currently in state i remained in the same state over a single transition period, and $p_{ii} + 1 = 1 - p_{ii}$ corresponds to the probability of a structure in state i moving to state $i+1$ in one period [18]. The remaining elements of the TPMs are zero, because it is not possible to improve CRs or to degrade CRs by more than level.

1.Estimation of the TPM

Two methods were proposed and tested to estimate the elements of the transaction probability matrix. Given that, often times, the sample size that is available to calculate the transition probabilities is small, this study used the Small Data Method to obtain more reliable estimates.

Even though all transition periods were considered at once, a small sample size can result in transition probabilities equal to zero. In practice, however, this is not realistic. When a transaction probability is estimated to be zero, a linear approximation is used to estimate the corresponding transition probability [15]:

$$\frac{p_{ii}}{p_{i-1,i-1}} = \frac{p_{i+1,i}}{p_{i,i-1}}, i \in \{8,....,4\} \tag{6}$$

State i ranges from 4 to 8 because the lowest CR is 3; the analysis in this paper only considered natural decay.

2.Traditional Method

The traditional method corresponds to the traditional approach to estimate the transition matrix of a Markov process. For each one of the two transition periods, the following ratios were calculated using:

$$p_{ii}{}^{(period)} = \frac{n_{ii}}{n_i} \tag{7}$$

where n_{ii} is the number of bridges that started and stayed in state i for that particular transaction period and n_i is the total number of bridges in state i at the beginning of the same transaction period. The transaction probabilities, p_{ii}, are the average of the ratios included in interval:

$$\left[\overline{p_{ii}{}^{(period)}} - sd(p_{ii}{}^{(period)}), \overline{p_{ii}{}^{(period)}} + sd(p_{ii}{}^{(period)})\right],$$

where $\overline{p_{ii}^{(period)}}$ and $sd(p_{ii}^{(period)})$ are, respectively, the average and standard deviation of ratios $p_{ii}^{(period)}$. This interval tries to improve the estimate of the transition probabilities by removing ratios that are at least one standard deviation from the mean. The remaining ratios, $p_{i,i+1}$, are equal to $1-p_{ii}$.

3.Small Data Method

When the sample size is small, removing ratios $p_{ii}^{(period)}$, as in the traditional method, is very problematic because there is insufficient information to obtain an estimate. In contrast, the Small Data Method considers all the transition periods at once, that is, it directly obtains the TPM ratios, p_{ii}, using:

$$p_{ii} = \frac{N_{ii}}{N_i} \tag{8}$$

where N_{ii} is the number of bridges in state i before and after any transaction period and N_i is the total number of bridges that started with state i at any transition period. Equation (8) uses large numbers, which is expected to reduce the negative effect of potential outliers. This proposed approach is expected to provide more reliable estimates as it uses more information compared to the traditional method. The remaining ratios $p_{i,\,i+1}$ are equal to 1 - p_{ii}.

B.Regression Analysis – Functional Forms

Regression analysis is used to estimate parameters in order to describe the relationship between one variable dependent and one or more variable independents. For example, the parameters can be calculated with:

$$Y = ab^x \tag{9}$$

$$Y = \ln(a)X + b \tag{10}$$

$$Y = b(e)^{a(x)} \tag{11}$$

$$Y = aX^d + bX + c \tag{12}$$

where Y is the dependent variable; X is the independent variable; and a, b, c, and d are the parameters that describe the functional relationship. Used to calculate the deterioration prediction, (9) represents power regression, (10) is a logarithmic regression, (11) is an exponential regression, and (12) represents a polynomial regression.

C.Condition Rating Forecast

The TPM enables calculating forecast of CRs for each bridge group using:

$$E(CR) = C(t).CR \tag{13}$$

where *CR* is the vector of conditions ratings. Equation (13) gives the expected *CR*, *E*(*CR*), after *t* periods (in years) [19]. Table I [1] provides the meaning and corresponding description for each of the CRs [1].

According to FHWA, maintenance is recommended when a structure reaches a CR of 5. If a score of 4 or less is reached due to lack of maintenance, and it is declared structurally deficient, this means that this structure either had a very low load capacity or the bridge is subject to overtopping with significant or severe traffic delays [1]; thus, CR = [9, 8,…, 3]. For this reason, this study focused on determining when the structure could reach Ratings 4 and 3, because a bridge with condition rating 3 is far more eligible for replacement than a bridge with a condition rating of 5 [7].

A.Bridge Data

There are two major types of deterioration behavior, natural decay (ND) and convectional recoverable decay (CRD). ND behavior represents bridges under routine maintenance or having minor repairs, while CRD behavior stands for bridges having medium or major repairs as well as reconstruction. This study focused exclusively on ND bridges. Fig. 2 shows the distribution of the bridges without reconstruction and the corresponding condition ratings.

Graphical representation of the data revealed that few data points had an age less than 20 years and condition ratings of 4, 5, or 6 as well as data points of 60 years or older and condition ratings of 7 or 8. Most bridges deck considered in this study had a condition rating between 8 and 6, and were between 20 and 60 years in age. Prediction calculations were made for the bridges with condition ratings of 8.

TABLE I
GENERAL CONDITION RATING

CR	Meaning	Description
9	Excellent	New bridges
8	Very Good	No problems noted
7	Good	Some minor problems
6	Satisfactory	Structural elements show some minor deterioration
5	Fair	All primary structural elements are sound but may have minor deterioration
4	Poor	Major deterioration is occurring
3	Serious	Deterioration has seriously affected the primary structural components of the bridge. Local failures are possible
2	Critical	Advanced deterioration of the primary structural elements is evident. Unless closely monitored it may be necessary to close the bridge until corrective action
1	Imminent Failure	Major deterioration is affecting the stability of the bridge. The bridge is closed to traffic but corrective action may allow it to be out back in light service
0	Failed	The bridges are out of service and beyond corrective action

This study used four years of bridge data (2011-2014) from NDOT. Specifically, the source data consisted of 1613 bridges with ND behavior. These bridges were classified by material and/or design, as illustrated in Table II.

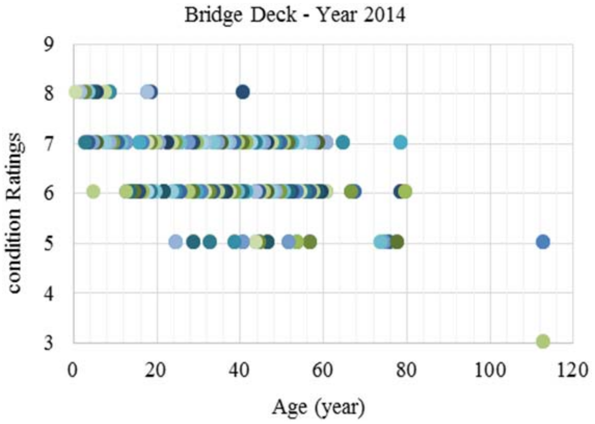

Fig. 2 Age versus condition ratings for a bridge deck at Year 2014

TABLE II
ND BRIDGES GROUPED BY MATERIAL AND/OR DESIGN

Kind of Material and/or Design	No. of Bridges
Concrete	775
Concrete Continuous	208
Steel	107
Steel Continuous	90
Pre-stressed Concrete	202
Pre-stressed Concrete Continuous	222
Wood or Timber	9
Total	1613

In the database, bridges are composed of up to five structures: Deck, Superstructure, Substructure, Culvert, and Channel. Most of the time, a single bridge includes two to three of these structures. The most common combinations include (i) Deck, Superstructure and Substructure (DSS); and (ii) Culvert and Channel (CC). Using this information, further classification was conducted in order to have the groups as homogeneous as possible. For each material type and common combination of structures, 14 groups were created. Table III lists all material types as well as the two most common combinations of structures. Thus, each of the 14 groups resulted from taking a material type and a structure combination from Table III.

TABLE III
ND BRIDGES GROUPED BY MATERIAL AND/OR DESIGN

Material and/or Design	Structure
Concrete	Deck Superstructure Substructure
Concrete Continuous	
Steel	
Steel Continuous	
Pre-stressed Concrete	Culvert Channel
Pre-stressed Concrete Continuous	
Wood or Timber	

For each inspected element, the CR represents its structural condition based upon the severity of observed defects. The CR scale includes 10 possible (integer) grades from 0 to 9, where 0 represents a structure that has failed and is completely out of service while 9 represents the best possible condition, usually attributed only to new structures [1].

V. RESULTS

The following examples provide the expected CRs for decks in general, using regression analysis, and using Markov-chain models for decks belonging to the concrete group and decks belonging to the steel group. Interest was focused on decks whose initial states were 8 in 2014.

A. Using Regression Analysis

Decks with condition ratings equal to 8 had a distribution from 2011 to 2014, as shown in Table IV; they formed four groups of possible distributions (A, B, C, and D), with initial condition ratings. For each one of these groups, analysis regressions were conducted, with the following results for the interval from 2011 to 2014. A forecast over 10 years was common for this type of analysis [5]. Tables V-VIII provide the results for these calculations.

TABLE IV
CONDITION RATING (2011-2014) -DECK WITH CD EQUAL TO 8 FOR 2014

Years	Deck Condition Ratings			
	A	B	C	D
2011	9.00	9.00	9.00	8.00
2012	9.00	9.00	8.00	8.00
2013	9.00	8.00	8.00	8.00
2014	8.00	8.00	8.00	8.00

TABLE V
EXPECTED CONDITION RATINGS A BY ANALYSIS REGRESSIONS

Regression A	Years					
	2014	2015	2016	2017	2018	2019
Linear	8	8	8	7	7	7
Logarithmic	8	8	8	8	8	8
Exponential	8	8	8	7	7	7
Polynomial	8	7	5	3	0	-3
Power	8	8	8	8	8	8
Regression A	Years					
	2020	2021	2022	2023	2024	
Linear	7	6	6	6	5	
Logarithmic	8	8	8	8	8	
Exponential	7	6	6	6	6	
Polynomial	-7	-12	-16	-22	-27	
Power	8	8	8	8	8	

TABLE VI
EXPECTED CONDITION RATINGS B BY ANALYSIS REGRESSIONS

Regression B	Years					
	2014	2015	2016	2017	2018	2019
Linear	8	8	7	7	6	6
Logarithmic	8	8	8	8	7	7
Exponential	8	8	7	7	7	6
Polynomial	8	7	7	6	6	5
Power	8	8	8	8	7	7
Regression B	Years					
	2020	2021	2022	2023	2024	
Linear	6	5	5	4	4	
Logarithmic	7	7	7	7	7	
Exponential	6	6	5	5	5	
Polynomial	5	5	4	4	3	
Power	7	7	7	7	7	

TABLE VII
EXPECTED CONDITION RATINGS C BY ANALYSIS REGRESSIONS

Regression C	Years					
	2014	2015	2016	2017	2018	2019
Linear	8	8	7	7	7	6
Logarithmic	8	8	8	7	7	7
Exponential	8	8	7	7	7	7
Polynomial	8	9	10	12	14	17
Power	8	8	8	7	7	7
Regression C	Years					
	2020	2021	2022	2023	2024	
Linear	6	6	5	5	5	
Logarithmic	7	7	7	7	7	
Exponential	6	6	6	6	6	
Polynomial	20	23	28	32	38	
Power	7	7	7	7	7	

TABLE VIII
EXPECTED CONDITION RATINGS D BY ANALYSIS REGRESSIONS

Regression D	Years					
	2014	2015	2016	2017	2018	2019
Linear	8	8	8	8	7	7
Logarithmic	8	8	8	8	8	8
Exponential	8	8	8	8	7	7
Polynomial	8	8	8	7	7	6
Power	8	8	8	8	8	8
Regression D	Years					
	2020	2021	2022	2023	2024	
Linear	7	7	7	7	7	
Logarithmic	8	8	8	8	7	
Exponential	7	7	7	7	7	
Polynomial	5	4	3	2	1	
Power	8	8	8	7	7	

To select the best-forecast model, results were compared using goodness-of-fit measures, the Root Mean Square Error (RMSE) (14), and the Mean Absolute Percentage Error (MAPE) (15). The model having the least MAPE and RMSE values was considered to be the best model and the representative curve model.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(F_t - A_t)^2}{n}} \quad \ldots \tag{14}$$

where *RMSE* = Root Mean Square Error, A_t = the actual value, F_t = the forecast value, and n = the number of fitted points

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{A_t - F_t}{A_t}\right| \tag{15}$$

where *MAPE* = Mean Absolute Percentage Error, A_t = the actual value, F_t = the forecast value, and n = the number of fitted points

With calculations made by using regression analysis for each of the five regression's types – linear, logarithmic, exponential, polynomial, and power – and calculations made for RMSE and MAPE, better condition rating curves were determined for each initial class (A, B, C, and D). Each regression class was selected for the lesser value for RMSE and MAPE, as shown in Table IX. The lesser value for each regression class is shown, suggesting curves that have better approximations.

TABLE IX
SELECTION BY RMSE AND MAPE

Regression	RMSE	MAPE
A - Logarithmic	0.29	2.86%
B - Power	0.28	2.94%
C – Power	0.32	3.49%
D - Exponential	0.28	2.98%

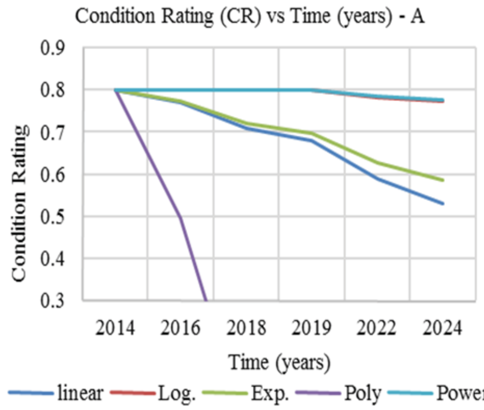

Fig. 3 Age versus condition ratings A for bridge deck at Year 2014

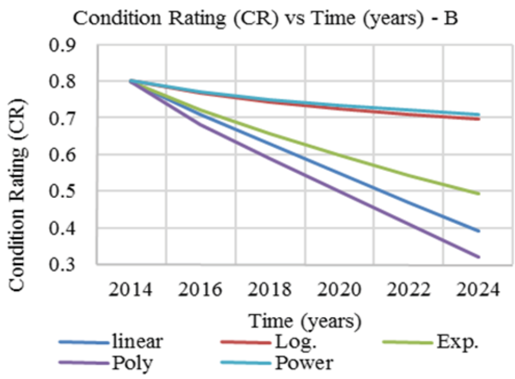

Fig. 4 Age versus condition ratings B for bridge deck at Year 2014

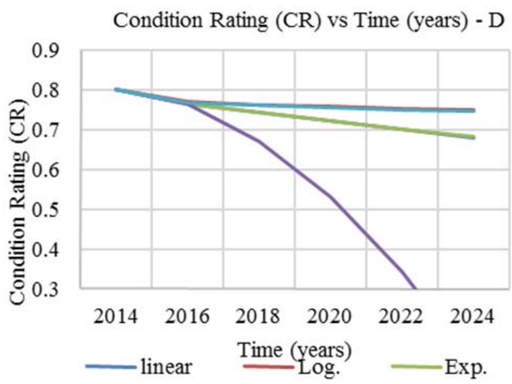

Fig. 5 Age versus condition ratings C for bridge deck at Year 2014

Figs. 3-6 show the expected condition ratings for each initial condition and for each regression type. According to these results, for Year 2024, the condition rating for decks will be equal to 8 for initial condition A and 7 for conditions B, C, and D.

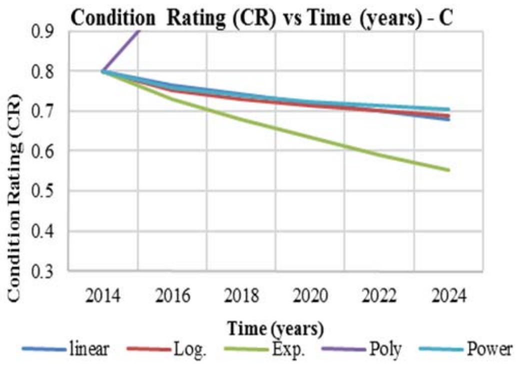

Fig. 6 Age versus condition ratings D for bridge deck at Year 2014

B.Using Markov-Chain Models

Equation (13) was used for each year of the prediction interval from 2014 through 2024. Tables X and XI provide the corresponding results.

TABLE X
EXPECTED CONDITION RATINGS - CONCRETE DECK

Method	Years					
	2014	2015	2016	2017	2018	2019
Traditional	8	7	7	7	7	7
Small Data	8	7	7	7	7	7
Method	Years					
	2014	2015	2016	2017	2018	2019
Traditional	7	7	7	7	7	
Small Data	7	6	6	6	6	

TABLE XI
EXPECTED CONDITION RATINGS – STEEL DECK

Method	Years					
	2014	2015	2016	2017	2018	2019
Traditional	8	8	8	8	7	7
Small Data	8	7	7	7	7	7
Method	Years					
	2014	2015	2016	2017	2018	2019
Traditional	7	7	7	7	7	
Small Data	7	7	7	7	7	

The calculation of the expected CRs is shown in Fig. 7 for the Small Data Method and Year 2014. In this particular case, there were no culverts and decks with an initial condition rating higher than 8. Fig. 7 shows the results for calculations used for concrete culverts, keeping in mind that each material has its own the probability matrix. Both Figs. 7 and 8 provide the plots of the expected CRs from Tables X and XI, respectively.

As expected, the Small Data Method provided more conservative estimates. That is, concrete culverts, concrete decks, and steel decks are expected to deteriorate faster than estimated by the traditional method; therefore, faster maintenance is recommended. Large data sets, which are not available in Nevada, are required to corroborate the superiority of any of these two methods. However, from a numeric standpoint, the Small Data Method is expected to be superior to the traditional method for small data sets, which often are the only resource available to estimate bridge deterioration.

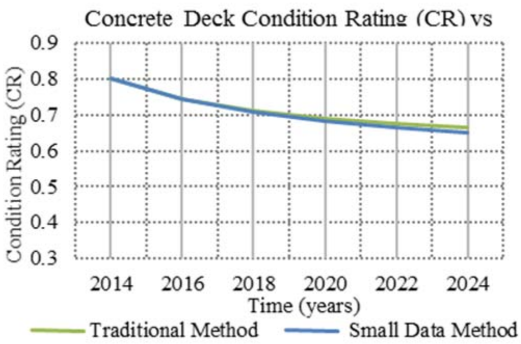

Fig. 7 Expected ratings for deck conditions.

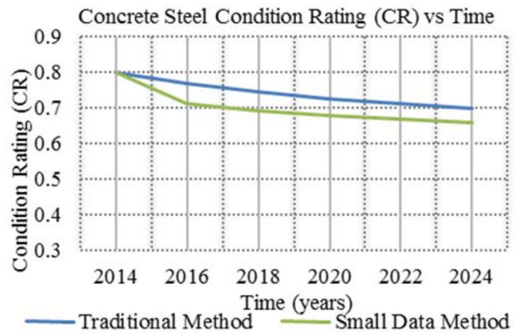

Fig. 8 Expected ratings for steel conditions

VI.CONCLUSION

This paper illustrates how to use Markov-chain models and regression analysis to estimate bridge deterioration and their expected condition ratings. Markov chains analyses were conducted using two similar but distinct methods to estimate the required transition probabilities. The results obtained using analysis regression were very similar to those obtained with the Markov chains.

The motivation for using the second method – the Small Data Method – was the limited number of sample measurements available in this study. Limited sample size is a common problem, given cost, changes in data collection methods, and various issues associated with data quality and legacy systems. The results illustrate that the proposed and traditional approach provided similar estimates. However, the proposed approach provided more estimates that were conservative. This means that maintenance should be performed to the structure earlier than what is estimated when using the traditional method, thus ensuring an extension in the useful life of the bridge. Considering that bridges are critical infrastructures, and that the traditional approach for estimating bridge deterioration has issues when only small sample sizes are available, this study recommends using the Small Data

Method and Regression Analysis, which uses more information and provides more conservative estimates.

For future studies, it is recommended that such variables as the average daily traffic (ADT), wearing surface, type of service, and other factors that interfere with the optimal performance of bridges over time be added to the analysis. In addition, it would be interesting to study bridge behavior under different environmental conditions.

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