Feedback Stabilization Based on Observer and Guaranteed Cost Control for Lipschitz Nonlinear Systems

A. Thabet, G. B. H. Frej, M. Boutayeb

Abstract—This paper presents a design of dynamic feedback control based on observer for a class of large scale Lipschitz nonlinear systems. The use of Differential Mean Value Theorem (DMVT) is to introduce a general condition on the nonlinear functions. To ensure asymptotic stability, sufficient conditions are expressed in terms of linear matrix inequalities (LMIs). High performances are shown through real time implementation with ARDUINO Duemilanove board to the one-link flexible joint robot.

Keywords—Feedback stabilization, DMVT, Lipschitz nonlinear systems, nonlinear observer, real time implementation.

I. Introduction

IN recent years, modern control and stabilization methods have found their way into feedback design of nonlinear systems leading to a wide variety of new concepts and results. For example, practical engineering systems are supervised for safety and reliability using state or/and output-feedback controllers [1], [2]. The study of stabilization problems on nonlinear continuous-time systems has achieved remarkable development, see [3]-[7] and the references. Many remarkable methods have been synthesized. They include, but not limited to, global input/output linearization techniques for SISO based on observers [3], [8] and MIMO systems [9], adaptive backstepping design [10], high-gain observers [11], [12], extended-state-observers with backstepping [7], [13], finite-time control [14], neural network control [15] and fuzzy-logic with non-fragile passive controller [16]. However, even with this important literature, some limitations exist:

- The consideration of a linear output (in the form $y = Cx$). However, a large range of sensors have provided nonlinear output signals (ultrasound sensors; Anisotropic Magneto Resistive sensor; Power-Transit sensor...). This adds more restrictive conditions to the synthesis of the state observer and control gains.

- The provided synthesis conditions are generally unfeasible for systems with large Lipschitz constants. Some recent results have been proposed to cope with this restriction [7], [12] but remain conservative.

These limitations motivate the design of a simple robust feedback stabilization based on observer for nonlinear systems with nonlinear (or linear) output and large Lipschitz constants. This approach can be easily generalized to nonlinear

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distributed and decentralized systems. The basic idea of this work is to use the DMVT which allows to write the dynamics of the estimation error using the nonlinear function terms as a class of Linear parameter varying (LPV) systems (based on the works of [17]). Stability of the global error is analyzed using the convexity principle and the Lyapunov stability theory with an optimization of simple quadratic cost performance (J). The observer/control gains guaranteeing the global convergence of the proposed scheme is computed by LMI. The idea behind the DMVT is to provide a non restrictive sufficient condition on nonlinear functions and to assure $\partial V/\partial t + J < 0$ for a standard Lyapunov function. The consequence is to ensure asymptotic convergence for large scale Lipschitz nonlinear systems.

This work is organized as follows: In Section II, the problem will be stated and a preliminary will be presented. Next, the method of synthesis of the feedback controller based on observer will be given in details. This method consists in LMIs feasibility conditions. The last section is devoted to the well known performance of the presented approach through a real time implementation using ARDUINO Duemilanove device with comparisons.

Notations: The following notations will be used throughout this paper.
- For a square matrix $S, S > 0$ ($S < 0$) means that this matrix is positive definite (negative definite);
- In a matrix, the notation ($\star$) is used for the blocks induced by symmetry;
- The set $Co(x, y) = \{\lambda x + (1 - \lambda)y, 0 \leq \lambda \leq 1\}$ is the convex hull of x, y;
- $e_s(j) = \left(\underbrace{0, ..., 0, \overbrace{1}^{j^{th}}, 0, ..., 0}_{s - components} \right)^T \in \mathbb{R}^s$, $s \geq 1$, is the vector of the canonical basis of $\mathbb{R}^s$.

II. Problem Statement and Preliminaries

Consider that the system described by:

$$\begin{cases} \dot{x} = Ax + Bu + f(t, x) \\ y = g(x, u) \end{cases} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are respectively the state, input and output vectors. A and B are constant matrices of adequate dimensions. $f(t, x) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $g(x, u) : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^p$ are nonlinear vectors fields (assumed to be differentiable with respect to x). The assumptions and proposals considered in this paper are as follows:

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:10, No:7, 2016

Assumption 1. Assume that the Jacobian matrices of f and g satisfy the following conditions [18]:

$$\begin{cases} -\infty < \underline{f}_{ij} \le \frac{\partial f_i(t,x)}{\partial x_j} \le \overline{f}_{ij} < +\infty \\ -\infty < \underline{g}_{ij} \le \frac{\partial g_i(x,u)}{\partial x_j} \le \overline{g}_{ij} < +\infty \end{cases} \qquad (2)$$

where

$$\begin{cases} \underline{f}_{ij} = \min\limits_{Z \in \mathbb{R}^{n \times \mathbb{R}^n}} \left(\frac{\partial f_i}{\partial x_j}(Z) \right) \\ \overline{f}_{ij} = \max\limits_{Z \in \mathbb{R}^{n \times \mathbb{R}^p}} \left(\frac{\partial f_i}{\partial x_j}(Z) \right) \\ \underline{g}_{ij} = \min\limits_{Z \in \mathbb{R}^{n \times \mathbb{R}^m}} \left(\frac{\partial g_i}{\partial x_j}(Z) \right) \\ \overline{g}_{ij} = \max\limits_{Z \in \mathbb{R}^{n \times \mathbb{R}^m}} \left(\frac{\partial g_i}{\partial x_j}(Z) \right) \end{cases}$$

Assumption 2. Assume that $f(t_0, 0) = 0$. The following proposals will be used for the synthesis of observer gain (transformation of the estimation error from a nonlinear form to an LPV form) and that of the control.

Proposition 1. Define the sets $\mathcal{M}_{n,n}$ and $\mathcal{F}_{p,n}$ as:

$$\begin{aligned} \mathcal{M}_{n,n} = \quad & \{ v = (v_{11}, ..., v_{1n}, ..., v_{nn}) \\ & : \underline{f}_{ij} \le v_{ij} \le \overline{f}_{ij}, i = 1, ..., n; j = 1, ..., n; \} \end{aligned} \qquad (3)$$

$$\begin{aligned} \mathcal{F}_{p,n} = \quad & \{ \rho = (\rho_{11}, ..., \rho_{1n}, ..., \rho_{pn}) \\ & : \underline{g}_{ij} \le \rho_{ij} \le \overline{g}_{ij}, i = 1, ..., p; j = 1, ..., n; \} \end{aligned} \qquad (4)$$

The sets $\mathcal{M}_{n,n}$ and $\mathcal{F}_{p,n}$ are a bounded convex domain of which the sets of vertices are defined by:

$$\begin{aligned} \mathcal{V}_{\mathcal{M}_{n,n}} = \quad & \{ \alpha = (\alpha_{11}, ..., \alpha_{1n}, ..., \alpha_{nn}) : \\ & \alpha_{ij} \in \{ \underline{f}_{ij}, \overline{f}_{ij} \} \} \end{aligned} \qquad (5)$$

$$\begin{aligned} \mathcal{W}_{\mathcal{F}_{p,n}} = \quad & \{ \beta = (\beta_{11}, ..., \beta_{1n}, ..., \beta_{nn}) : \\ & \beta_{ij} \in \{ \underline{g}_{ij}, \overline{g}_{ij} \} \} \end{aligned} \qquad (6)$$

The affine matrices function are:

$$\begin{aligned} \mathcal{A}_L(v) &= A + \sum_{i,j=1}^{n,n} v_{ij} e_n(i) e_n^T(j) \\ \mathcal{G}(\rho) &= \sum_{i,j=1}^{p,n} \rho_{ij} e_p(i) e_n^T(j) \end{aligned} \qquad (7)$$

where $v \in \mathcal{M}_{n,n}$ and $\rho \in \mathcal{F}_{p,n}$.

Proposition 2. (The DMVT for vector valued function [17]). Let $\Phi : \mathbb{R}^n \to \mathbb{R}^k$. Let $a, b \in \mathbb{R}^n$. We assume that Φ is differentiable on $Co(a, b)$. Then, there are constant vectors $z_1, ..., z_k \in Co(a, b)$, $z_i \ne a$, $z_i \ne b$ for $i = 1, ..., k$ where:

$$\Phi(a) - \Phi(b) = \left(\sum_{i,j=1}^{k,n} e_k(i) e_n^T(j) \frac{\partial \Phi_i}{\partial x_j}(z_i) \right) (a - b). \qquad (8)$$

The next section is meant to show that there is a simple solution, to the stabilization problem (the synthesis of an observer/control matrices gains), based on LMIs problem and using the principle of DMVT.

III. FEEDBACK CONTROL DESIGN

A. Synthesis of the Observer and Control Gains

The presented observer of the system (1) is given by:

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) + f(t, \hat{x}) \\ \hat{y} = g(\hat{x}, u) \end{cases} \qquad (9)$$

where $\hat{x}$ is the state of the observer system and L the observation gain matrix. Let $\varepsilon = x - \hat{x}$. Then, from

the observer (9) and the system (1), the dynamic of state estimation error is described by:

$$\dot{\varepsilon} = A\varepsilon - L\Delta g + \Delta f \qquad (10)$$

where $\Delta f = f(t, x) - f(t, \hat{x})$ and $\Delta g = g(x, u) - g(\hat{x}, u)$. In analogy to the approach of [18], and by applying Proposition 2 on the functions f and g, we deduce that there exists $(z_i(t), \overline{z}_i(t)) \in Co(x, \hat{x})$, for all $i = 1...n$, such that:

$$\begin{cases} \Delta f = f(t, x) - f(t, \hat{x}) = \left(\sum_{i,j=1}^{n,n} e_n(i) e_n^T(j) \frac{\partial f}{\partial x_j}(z_i) \right) \varepsilon \\ \Delta g = g(x, u) - g(\hat{x}, u) = \left(\sum_{i,j=1}^{p,n} e_p(i) e_n^T(j) \frac{\partial g}{\partial x_j}(\overline{z}_i) \right) \varepsilon \end{cases} \qquad (11)$$

For simplicity, we consider the notation:

$$\begin{aligned} h_{ij}(t) &= \frac{\partial f_i}{\partial x_j}(z_i(t)) \\ q_{ij}(t) &= \frac{\partial g_i}{\partial x_j}(\overline{z}_i(t), u) \end{aligned}$$

with

$$\begin{aligned} h(t) &= (h_{11}(t), .., h_{1n}(t), ..h_{nn}(t)) \\ q(t) &= (q_{11}(t), .., q_{1n}(t), ..q_{pn}(t)) \end{aligned}$$

From (7) and (11), the estimation error dynamics (10) becomes:

$$\dot{\varepsilon} = (\mathcal{A}_L(h(t)) - L\mathcal{G}(q(t)))\varepsilon \qquad (12)$$

Then, the observer design problem of the class of nonlinear systems (1) is transformed into a simple problem of stability of a class of LPV systems (12). This is one of the major advantages of using DMVT for the synthesis of the observer. A comparative study with solid works proving the advantages of the use of DMVT is given in [19], [20]. Now, the control law is given by:

$$u = -K\hat{x} \qquad (13)$$

where $K \in \mathbb{R}^{m \times n}$ is the control gain matrix of the system. The development of the nonlinear system, using the control law (13) leads to:

$$\begin{cases} \dot{x} = (A - BK)x + BK\varepsilon + f(t, x) \\ y = g(x, u) \end{cases} \qquad (14)$$

The term $f(t, x)$ is equivalent to $f(t, x) - f(t_0, 0)$ using Assumption 2. Now, using Proposition 2, then:

$$f(t, x) - f(t_0, 0) = \left(\sum_{i,j=1}^{n,n} e_n(i) e_n^T(j) \frac{\partial f}{\partial x_j}(r_i) \right) x$$

where $r_i \in Co(x, 0)$. This leads to: $Ax + f(t, x) = \mathcal{A}_C(h(t))x$. Therefore, the augmented system including the overall system (14) and the dynamics observation error system (12) is given by a state representation as:

$$\underbrace{\begin{bmatrix} \dot{x} \\ \dot{\varepsilon} \end{bmatrix}}_{\dot{\tilde{x}}} = \underbrace{\begin{bmatrix} \mathcal{A}_C(h(t)) - BK & BK \\ 0 & \mathcal{A}_L(h(t)) - L\mathcal{G}(q(t)) \end{bmatrix}}_{\tilde{\Phi}} \cdot \underbrace{\begin{bmatrix} x \\ \varepsilon \end{bmatrix}}_{\tilde{x}} \qquad (15)$$

The problem is to find a way to obtain the control matrix gain K and the observation matrix gain L which can achieve the stability of the system.

B. Stability Analysis

This section concerns the stability analysis with a guaranteed cost control of the closed loop system. To ensure

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:10, No:7, 2016

it, the following criteria (quadratic cost performance that considers a correlation between the state and the control) are optimized:

$$J = \int_0^\infty ([\begin{array}{cc} x & u \end{array}]^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} [\begin{array}{cc} x & u \end{array}] \, dt \quad (16)$$

where $Q = Q^T > 0$, $S > 0$ and $R = R^T > 0$ are given constant weighting matrices. Then, using the dynamic output feedback control $u = -K\hat{x}$, the cost function (16) can be rewritten as:

$$\tilde{J} = \int_0^\infty \left(\tilde{x}^T \underbrace{\begin{bmatrix} Q + K^TRK - SK - (SK)^T & -K^TRK + SK \\ -K^TRK + (SK)^T & K^TRK \end{bmatrix}}_{\tilde{Q}} \tilde{x} \right) dt \quad (17)$$

The control law based on state observer is said to be a quadratic guaranteed cost control with cost matrix $P > 0$ for the augmented system (15) and the cost function (16) if the closed loop system is quadratically stable [21]. The closed loop value of the cost function (17) satisfies the bound $J < \tilde{J}$ for all admissible nonlinearities. Initially, the candidate Lyapunov function $V(\tilde{x})$ is defined by:

$$V(\tilde{x}) = \tilde{x}^T P \tilde{x} \quad (18)$$

where Lyapunov matrix P is defined by: $P = \begin{bmatrix} P_c & 0 \\ 0 & P_0 \end{bmatrix}$.

where $P_c = P_c^T$ and $P_0 = P_0^T$ are Lyapunov positive definite symmetric matrices. The aim, in what follows, is to determine conditions for which:

$$\frac{d}{dt} V(\tilde{x}) + \tilde{x}^T \tilde{Q}\tilde{x} < 0 \quad (19)$$

From (18) and according to (19), we have:

$$(\tilde{\Phi}\tilde{x})^T P \tilde{x} + \tilde{x}^T P (\tilde{\Phi}\tilde{x}) + \tilde{x}^T \tilde{Q}\tilde{x} < 0 \quad (20)$$

Then, the condition for the asymptotic stability (using the assumption of [22]) with a guaranteed level of performance is given by:

$$\tilde{x}^T (\tilde{\Phi}^T P + P\tilde{\Phi} + \tilde{Q})\tilde{x} < 0 \quad (21)$$

Theorem 1. The system (1) is stable in the sense of Lyapunov and the cost performance (16) is guaranteed if there exists matrices $P = P^T$, L and K of appropriate dimensions where the following LMI is feasible:

$$\begin{aligned} & Block - diag(F(\alpha^1, \beta^1), ..., F(\alpha^{2^{qn}}, \beta^{2^{pn}})) < 0, \\ & \alpha^i \in \mathcal{V}_{\mathcal{M}_{q,n}} \; for \; i = 1, ..., 2^{qn} \\ & \beta^j \in \mathcal{W}_{\mathfrak{F}_{p,n}} \; for \; j = 1, ..., 2^{pn} \\ & F(\alpha^i, \beta^j) = \tilde{\Phi}(\alpha^i, \beta^j)^T P + P\tilde{\Phi}(\alpha^i, \beta^j) + \tilde{Q} \end{aligned} \quad (22)$$

To ensure it $(F(\alpha^i, \beta^j) < 0)$, (21) can be solved using the LMI optimization technique. The development of (21) leads to:

$$\begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} < 0 \quad (23)$$

where:

$$\begin{cases} X_{11} &= \mathcal{A}_C(\alpha)^T P_c + P_c \mathcal{A}_C(\alpha) - K^TB^TP_c - P_cBK + Q \\ & \quad + K^TRK - (SK)^T + KS \\ X_{12} &= P_cBK - K^TRK + KS \\ X_{21} &= X_{12}^T \\ X_{22} &= \mathcal{A}_L(\alpha)^T P_0 + P_0\mathcal{A}_L(\alpha) - \mathcal{G}(\beta)^TL^TP_0 - P_0L\mathcal{G}(\beta) \\ & \quad + K^TRK \end{cases}$$

Notice that there are no effective algorithms to solve simultaneously the control problem and the observer one. Thus, to solve it, we proceed in two steps. The first-step consists in multiplying the left-hand side and the right-hand side of (23) by:

$$\begin{pmatrix} W & 0 \\ 0 & I \end{pmatrix}, W = W^T = P_c^{-1} > 0 \quad (24)$$

The equation (23) becomes:

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} < 0 \quad (25)$$

Then, using the notations $Y = KW$ and $Z = P_0L$, we have:

$$\begin{cases} Y_{11} &= W\mathcal{A}_C(\alpha)^T + \mathcal{A}_C(\alpha)W - Y^TB^T - BY \\ & \quad + WQW + Y^TRY + Y^TS^TW + WSY \\ Y_{12} &= BK - Y^TRK + WSK \\ Y_{21} &= Y_{12}^T \\ Y_{22} &= \mathcal{A}_L(\alpha)^T P_0 + P_0\mathcal{A}_L(\alpha) - \mathcal{G}(\beta)^TZ^T - Z\mathcal{G}(\beta) \\ & \quad + K^TRK \end{cases}$$

Now, the determination of the control parameters (designed by the matrices W and Y) comes from the resolution of the matrix inequality $Y_{11} < 0$. Using the Schur complement formula [22], the inequality $(Y_{11} < 0)$ can be written as (26): Thereafter, the control gain matrix is given by:

$$K = YW^{-1} \quad (27)$$

The second step is devoted to find the observation gain L. Then, the determination of P_0 and Z is given by substituting the parameters obtained from the first step and solving the LMI (25). The observation gain is given by:

$$L = P_0^{-1}Z \quad (28)$$

Remark: The results remain valid with linear output $y = Cx$ ($\mathcal{G}(\beta)$ becomes C). In addition, this paper considers the observer dynamics in the closed loop system that can be generalized to the case of: Distributed systems with N sub-systems and decentralized systems with the elimination of the non-linear interconnection function in the synthesis of the observer.

IV. EXPERIMENTAL RESULTS

Studies are carried out on the real time implementation of the one-link flexible joint robot [23] to evaluate the performance of the presented approach with a Digital Signal Processing device ($ARDUINO^{®}$ Duemilanove board). The set of equations is given by:

$$\begin{cases} \dot{x}(t) = A_cx(t) + B_cu(t) + f_c(x(t)) \\ y(t) = g_c(x(t), u(t)) \end{cases} \quad (29)$$

with:

- $x = [\theta_m \; \omega_m \; \theta_l \; \omega_l]^T$, where θ_m and θ_l are, respectively, the angles of rotations of the motor and link. ω_m and ω_l are their angular velocities.

- $A_c = \begin{bmatrix} -10 & 1 & 0 & 0 \\ -48.6 & -1.26 & 48.6 & 0 \\ 0 & 0 & -22 & 1 \\ 1.95 & 0 & -19.5 & -6 \end{bmatrix}; B_c =$

World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:10, No:7, 2016

$$\begin{bmatrix} W\mathcal{A}_C(\alpha)^T + \mathcal{A}_C(\alpha)W - Y^TB^T - BY & W & Y^T & Y^T & W \\ \star & -Q^{-1} & 0 & 0 & 0 \\ \star & 0 & -R^{-1} & 0 & 0 \\ \star & 0 & 0 & -(S^T)^+ & 0 \\ \star & 0 & 0 & 0 & -S^+ \end{bmatrix} < 0 \qquad (26)$$

$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0.5 \end{bmatrix}; g_c(x,u) = \begin{bmatrix} x_1 \\ sin(2x_2) \end{bmatrix}; f_c(x(t)) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \gamma_f \sin(x_3) \end{bmatrix}$

The use of a sensor with nonlinear output as in this case is mainly due to the insertion of signal-conditioners (such as adding a oscillator to have a sinusoidal outputs using inductive or magnetic sensor for the measurement of the angular velocity). The initial conditions for the system and observer have been chosen as (with Lipschitz constant $\gamma_f = 3.33$): $x_k(0) = [0.5 \ 0.5 \ 0.5 \ 0.5]^T$ and $\hat{x}_k(0) = [-0.5 \ -0.5 \ -0.5 \ -0.5]^T$. Applying the DMVT approach, we obtain $\mathcal{M}_{1j} = 0$ for all $j = 1, 2, 3$ and $\mathcal{M}_{14} = \gamma_f cos(z_3)$. Then, the set of vertices $\mathcal{V}_{\mathcal{M}_{1,4}}$ can be reduced to $\mathcal{V}_{\mathcal{H}_{1,4}} = \{-\gamma_f, +\gamma_f\}$. Similarly for function $g(x,u)$: $\underline{g}_{2,2} = -2$ and $\overline{g}_{2,2} = 2$. Then, by solving LMIs of Theorem 1 we obtain: $K = \begin{bmatrix} -40.0473 & -17.8391 & -6.6076 & 42.08 \end{bmatrix}$

and $L = \begin{bmatrix} 0.0495 & -1.8578 \\ 0.0385 & 21.3563 \\ -0.0259 & -3.0453 \\ -0.0505 & -3.0309 \end{bmatrix}$ where $Q = 0.001I_4$,

$R = 0.01$ and $S = [0.1 \ 0.1 \ 0.1 \ 0.1]^T$. For the real time implementation on the "$ARDUINO^{\circledR}$ Duemilanove board", the mode "*ARDUINO Target interface mode*" is used. In this mode of programming, Arduino board becomes a target of the $Simulink^{\circledR}$ code of $MATLAB^{\circledR}$ compiled with the tool "Run on Target Hardware". It can also be managed online via the USB port of the PC (External Mode Enable). But, first we install Arduino IO library to $Simulink^{\circledR}$ Libraries. The reconstruction of signals (x, u or y) is provided by the sending of the desired data on the PWM outputs. These PWM outputs are, then, connected to low pass filters (with $R = 3.9K\Omega$ and $C = 33\mu F$). This implementation mode is used as a real time emulator of robot. The robot model is developed in Simulink using the Embedded MATLAB Function and then is transferred to the Arduino device as DSP target. First, a noise was added to the output of the system. The added signal is a sinusoidal signal with a frequency equal to $140Hz$ and also for an amplitude ($\pm10\%$ of y). Fig. 1 presents the real x_1 and its estimated $\hat{x}_1$.

As shown in Fig. 1, the state is very well estimated. In the second phase of implementation, two noises are added. The first is applied to the output of the system. The second is introduced on the system. The added signals are sinusoidal signals with frequencies equal to $240Hz$ and variables amplitudes ($\pm30\%$ of y and f). Fig. 2 presents the real x_3 and its estimated $\hat{x}_3$.

Fig. 2 shows clearly the contribution added by the method proposed in this paper (during transitional regime and at the moment where the frequency and amplitude of noises varies). Now, the same noise signals (with $\pm45\%$ of y and f) are used but with varying frequencies (between $550Hz$ and $3800Hz$).
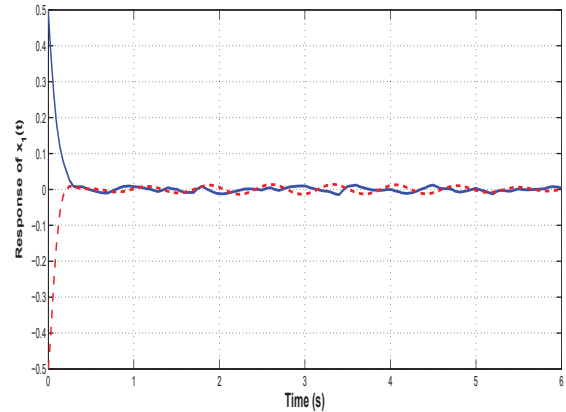

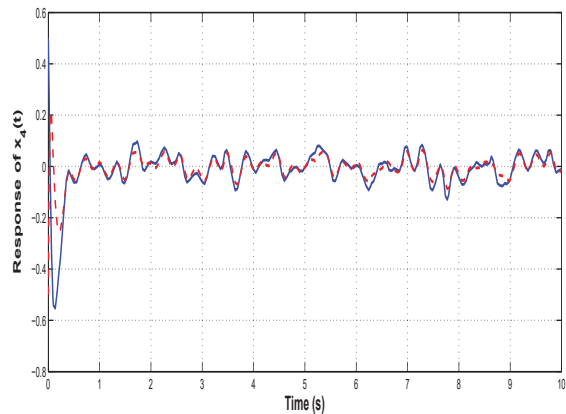Fig. 1 Response of $x_1(t)$ (solid line) and its estimated $\hat{x}_1(t)$ (dashed line)

Fig. 2 Response of $x_3(t)$ (solid line) and its estimated $\hat{x}_3(t)$ (dashed line)

Fig. 3 presents the real x_2 and its estimated ($\hat{x}_2$).

Fig. 3 shows that the presented approach converges despite the different frequencies and amplitudes of the noise (this can simulate extreme industrial conditions). Now, regarding the point of nonlinear systems with a large value of the Lipschitz constant and in order to prove the contribution acquired in the convergence, let us consider the same system with a linear output: $y = C_c x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. Then, increasing the Lipschitz constant leads to the comparative Table I (where **C** there is a solution ensuring convergence and in **D** there is no solution):

From Table I, it is clear that the presented method ensures convergence with all high values of Lipschitz constant compared to the recent method of [13].

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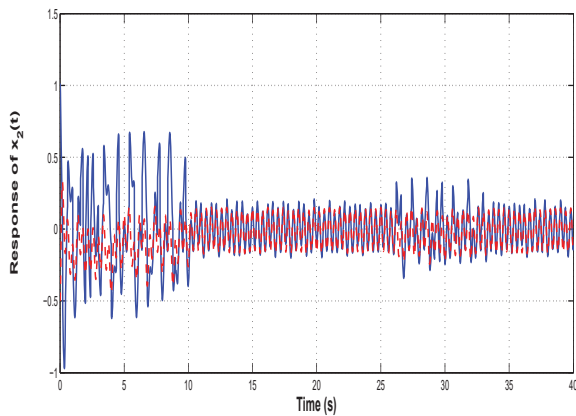International Journal of Computer and Information Engineering
Vol:10, No:7, 2016

Fig. 3 Response of $x_2(t)$ (solid line) and its estimate $\hat{x}_2(t)$ (dashed line)

TABLE I
RESULT OF CONVERGENCE WITH VARIABLES LIPSCHITZ CONSTANTS

γ_f	33.3	333	3330	33300	333300
Proposed Approach	C	C	C	C	C
Method of [13]	C	C	C	D	D

V. CONCLUSION

Efficient feedback stabilization for a class of large-scale lipschitz nonlinear systems is presented. The use of the DMVT has ensured that the stability analysis is performed with non restrictive sufficient conditions on nonlinear functions. Real time implementation with Arduino Duemilanove board in *"Target Mode"* has confirmed the high quality of estimation and control offered by the proposed method with the presence of noises where the amplitudes and frequencies are variable. The remaining open question is to reduce the conservatism of the presented solution by avoiding the constraint on the Jacobian matrix. This issue will be investigated in the near future.

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