A Type-2 Fuzzy Model for Link Prediction in Social Network

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Abstract—Predicting links that may occur in the future and missing links in social networks is an attractive problem in social network analysis. Granular computing can help us to model the relationships between human-based system and social sciences in this field. In this paper, we present a model based on granular computing approach and Type-2 fuzzy logic to predict links regarding nodes' activity and the relationship between two nodes. Our model is tested on collaboration networks. It is found that the accuracy of prediction is significantly higher than the Type-1 fuzzy and crisp approach.

Keywords—Social Network, link prediction, granular computing, Type-2 fuzzy sets.

I. INTRODUCTION

NOWADAYS, social network analysis is one of the main research subjects in computational intelligence, computer science, and sociology. Its importance is growing every day with the expansion of social media, networks, and technological advancements [1]. In fact, social network analysis (SNA) is a significant technique in sociology, geography, economy, organization, and terrorism study, etc. [2], [3].

Social network analysis provides a precise way to define important social concepts, a theoretical alternative to the assumption of independent social actors, and a framework for testing theories about structured social relationships [4].

"Social networks are often divided into groups or communities, and it has recently been suggested that this division could account for the observed clustering" [5]. There has been a considerable growth of interest in the potential that is offered by the relatively new techniques of social network analysis [6].

Prediction of links and network growth are two most prominent tasks in SNA. The problems that are relevant to link prediction are concerned with the missing links in the network and links that will be formed in the future [7].

Several indices such as the number of common neighbors (CN) and local clustering coefficient (CC) were proposed in the preceding studies [8]. These indices can be classified into three categories: Local, quasi-local, and global indices. Local indices include the direct neighbors of the nodes, and quasi-local indices include broader range, such as friends of friends, and global indices include the entire network [8].

The link prediction based on the graphs' structure does not take into account real-world factors, such as users' characteristics and the root of relations [7]. For example, if two people meet each other in a party, it means that they are likely of the same age, sex, and educational level. However, to model the networks, no extra information about the nodes will be used in the structural view [1].

To date based on our knowledge; all proposed models for predicting links are concerned with the relation between two nodes [9]. It is worth mentioning that nodes are often inclined to communicate with more active nodes. Active nodes are always looking for new connections. Addressing this issue could be closer to the real world; for example, the more articles a scientist has, there be more willing to communicate with him. This paper tries to present a model based on granular computing and Type-2 fuzzy systems to predict links regarding the node's activity in addition to the relationship between two nodes.

Type-1 fuzzy sets are not able to directly model uncertainties well because their membership functions are totally crisp [10]. On the other hand, Type-2 fuzzy sets are able to model such uncertainties because their membership functions are themselves fuzzy. Membership functions of Type-1 fuzzy sets are two-dimensional, whereas membership functions of Type-2 fuzzy sets are three-dimensional. It is the new third-dimension of Type-2 fuzzy sets that provide additional degrees of freedom that makes it possible to directly model uncertainties [10].

In this study, Section II presents an introduction to fuzzy graphs and granular computing. The link-prediction problem and the ways of checking the accuracy of the model are discussed in Section III. Section IV argues about the fuzzy link-prediction problem and its extension to Type-2 fuzzy link prediction. A new model of link prediction with respect to Type-2 fuzzy logic is presented in Section IV. Section V delineates the results of the models, and Section VI gives the conclusion and further researches.

II. FUZZY GRAPHS AND GRANULAR COMPUTING

Fuzzy logic was considered by Zadeh in 1965 [11]. Fuzzy logic may model the systems with vagueness and imprecision that helped scientists to address the realistic cases more effectively. Fuzzy logic deals with linguistic terms that are known as linguistic variables [11]. Linguistic terms have different meanings for different people in related problems [12]. The concepts such as close, weak and strong have

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different meanings for different people. These concepts are understandable for machines with fuzzy logic. It is necessary to review graph concepts and fuzzy graphs and recognize the Type-2 fuzzy concepts before considering proposed model.

Type-2 fuzzy sets are used to model uncertainty and imprecision; originally they were proposed by Zadeh [13] and they are essentially ''fuzzy–fuzzy'' sets in which the membership degrees are Type-1 fuzzy sets [14]. Type-2 fuzzy systems (T2 FSs) are described by membership functions (MFs) that are characterized by more parameters than are MFs forType-1 fuzzy systems (T1 FSs). Hence, T2FSs provide us with more design degrees of freedom; so using T2 FSs has the potential to outperform using T1 FSs, especially when we are in an uncertain environment [2], [15].

A. Graph Concepts

Each graph is shown with a pair set G= (V, E), V as a set of nodes and E as a set of edges, in which the nodes show objects and the edges show a connection between two nodes. If $x_i x_j$ is an edge of graph G; therefore, these two nodes (x_i and x_j) are adjacent and $R(x_i, x_j)=1$[16].

The path between nodes x_i and x_j in a graph is a consequence of the edges that start from x_i and end in x_j. If there is a path with the length of k between two nodes, then $R^k(x_i, x_j) =1$.A graph is complete if every pair of vertices is connected to each other by a link. Clique is a complete subgraph of the graph. A noteworthy definition in graphs is the degree of a node. The degree of a node is the number of nodes that are adjacent to the considered node [16].

B. Fuzzy Graphs

Without considering fuzzy concepts, the relations between some possible related sets such as X_1, X_2,…, X_n can be modeled using $X_1 \times X_2 \times ... \times X_n$ which can be assumed as a function given by (1)[9]:

$$R(x_1, x_2,...,x_n) = \begin{cases} 1 & if \ < x_1, x_2, ...x_n >\in R \\ 0 & otherwise \end{cases} \quad (1)$$

The strength of relations between the sets can be shown by the MF in fuzzy logic which shows the degree of the relationship between the members. For example, the relationship between two sets x_i and x_j can be modeled with [9]:

$$\mu_R(x_i, x_j) = \begin{cases} 1 & if \ x_i \ has \ the \ stongest \ relationship \ with \ x_j \\ \in [0,1] & if \ x_i \ is \ related \ to \ x_j \ to \ a \ certain \ extent \\ 0 & if \ x_i \ is \ not \ related \ to \ x_j \end{cases} \quad (2)$$

$\mu_R(x_i, x_j)$ is the strength relationship between two nodes or members of the sets.

It is worth mentioning that the reflexivity and symmetry are being assumed. These two conditions guarantee the un-directedness of the social networks graph.

C. Granular Computing

Despite the fact that for the first time, granular computing was introduced by Zadeh in 1979, it had not been studied seriously prior to the publication of his paper in 1997 [17]. Granular computing breaks a system into its components which are the granules of the system. For example, eyes, lips, and nose are some granular of parts of the human face [18].

In crisp granulation, the system should be separated into some well identifiable components. However, in the real world, it is not easy to define the boundary of the granularity. Thus, the fuzzy granular computing is considered to solve the problem.

III. LINK PREDICTION

Recently, predicting missing links in the network and links that will be formed in the future has attracted the attention of many scientists. According to a structural view toward the networks, efforts were made to find the most similar people based on the homophily theory [19]. By considering this theory, people try to form stable relationships with others by means of their similar attitudes. It means people who have more friends have more chances to form a relationship with each other [20]. Kleinberg and Liben–Nowell have studied many unsupervised methods that employed a proximity measure between nodes for link prediction [21]. There are some studies that are based on the random walks in the network. These studies try to find the ways with the maximum probability and generate a connection based on it [21]-[23].

Studies in social network are not limited to unweighted networks. There are some scientists who have studied the weighted networks [20]-[24]. Their results lead to poor predictions in the weighted networks. It might be due to disregarding the weak ties theory [25]. Using the weak ties can lead to small groups and generating larger networks. This theory connects the micro and macro levels of SNA.

Determining and predicting communications within a network is the main interest of social networks scientists and researchers. In the real world, the communications are not usually defined crisply. In other words, the human communications usually encounter with imprecision and vagueness. Fuzzy theory, especially Type-2 fuzzy logic, is a very powerful approach to model social networks and analysis different ties (strong or weak) between nodes of the graphs.

Type-2 fuzzy logic can handle and minimize the effects of uncertainties that provide additional degrees of freedom that make it possible to model uncertainties directly. Also, if all the uncertainties disappear, Type-2 fuzzy logic reduces to Type-1 fuzzy logic, in the same way, that, if the randomness disappears, the probability is reduced to the determinism [15], [26], [27].

A. Checking Accuracy

To check the accuracy of the measures, the area under the receiver operating characteristic curve (AUC) is used [28]. This model generates a score for all the non-existing links in each step and makes a relationship between two nodes with the highest score. Assume that there is a graph G= (V, E), that

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V is the set of vertices and E is the set of edges. Some of the edges are removed to create the EP set, which is the set of non-observed links. Fig. 1 shows a graph that it has six vertices (V= 1, 2, 3, 4, 5, 6) and five edges (E= 1-4, 3-4, 3-5, 3-2, 6-2) and edges like 3-6 or 2-4 can be members of EP set.
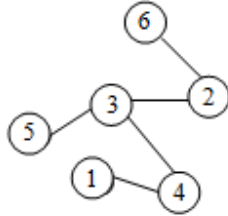

Fig. 1 Graph consists of nodes and edges

Therefore, the score of these links is compared with the score of nonexisting links. In the implementation of link prediction models, since making two lists in each step is time-consuming, only some of the non-observed and some of the non-existing links are chosen, and their scores are compared. If in n independent comparison, n' counts the times that the score of non-observed links is higher, and n'' counts the times that the score of non-observed links is equal, then AUC will be computed as [28]:

$$AUC = (n' + n'') / n \qquad (3)$$

Recently, many researchers studied the domain of fuzzy social network community detection. For example, Palla has proposed a model based on the clique percolation model that the overlapping communities can be found [29]. Zhang et al. propose a new model based on the fuzzy C-mean clustering model [30]. Mishra et al. also propose a model showing that the clusters can be found when there is a gap between the internal density and external sparsity [31]. Mukherjee and Holder have investigated the idea of graph-based data mining on the social networks to find the structure and clusters of the networks [24]. Brunelli and Fedrizzi propose a model for finding the M-ary adjacency matrix with maximum similarity. It means that by employing this model, it would be possible to find the groups with maximum similarity [32]. Yager proposes a paradigm for SNA, based on granular computing [33]. Bastani et al. have also proposed a model based on granular computing and fuzzy Type-1 for link prediction in social networks [9].

Although there are some worthwhile studies in the area of community detection, no study is reported employing the Type-2 fuzzy approach for link prediction in social networks. In the current study, Yager's and Bastani et al.'s paradigm has been used to develop a Type-2 fuzzy model of link prediction in networks [33], [9]. In this paradigm, one should start with human-focused concepts related to SNA and then formulate them using fuzzy Type-1 and Type-2 concepts.

IV. FUZZY LINK PREDICTION

Like the crisp modeling of networks, there are common concepts and models with graph theory in modeling the networks or predicting the links using fuzzy logic.

In the following section, some basic concepts of graph theory using in modeling the system with fuzzy Type-1 and Type-2 logic are presented.

A. CC

An important concept that shows how much the neighbors of a node are related to each other is the CC. This measure calculates the number of triangles over the number of possible triangles related to the node [34]. CC is based on the clique concept. Recently, Yager proposed softer definitions for CC [33]. If S shows a clique in the graph, then the following criteria can define the clique:

C_1 : "Most of the elements in S are closely connected."

C_2 : "None of the elements in S are too far from the others."

C_3 : "No element not on the clique is better connected to the members of a clique than any element in the clique."

In the above-mentioned criteria, there are some concepts that should be defined in fuzzy terms.

1. First Criterion

In the first criterion, the first fuzzy term is the concept of close, which means how much two nodes are closely connected. The close concept can be defined as a path with a minimum length that connects two nodes to each other. Yager proposes some prototypes for the close function, such as ramp function [33], but according to the small world phenomenon in social networks, every two nodes on average meet each other with the length of 6 [34], [35]. Therefore, it means that the closeness in social networks decreases exponentially. Thus, the close function for undirected and unweighted social networks is as follows [9]:

$$close(x_j, x_i) = q_{ij} / (2 \times 10^{q_{ij}-2}) \qquad (4)$$

In the above formula, q_{ij} is the length of a path that relates x_i to x_j if the considered path is the shortest path that relates these two nodes.

Bastani et al.'s function for weighted fuzzy social networks can also be generalized. In a weighted fuzzy social network, Bastani et al.'s close function were defined as follows [9]:

$$close(i, j) = \begin{cases} 1 & q_{ij} < 2 \\ (w(i,z) + w(z,j)) / 2 \times 10^{q_{ij}-2} & q_{ij} = 2 \\ (w(i,z) + w(z,e) + w(e,j)) / 2 \times 10^{q_{ij}-2} & q_{ij} = 3 \\ 0 & q_{ij} > 3 \end{cases} \qquad (5)$$

This function considers only the strength of the relationship between two nodes to predict a link. Nodes are often inclined to communicate with the more active nodes. Active nodes are always looking for new connections. Addressing this issue

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could be closer to the real world; for example, the more articles a scientist has, there be more willing to communicate with him.

In this case, the Type-1 fuzzy membership values are not able to determine the degree of node's activity. However, the Type-2 fuzzy number is able to express the node's activity base on the general Type-2 fuzzy sets. In Type-1 fuzzy, the membership values are between zero and one, whereas the Type-2 fuzzy membership values are considered as Type-1 fuzzy membership values themselves, $\tilde{A}$ as a general Type-2 fuzzy set is described as follows [36]:

$$\tilde{A} = \int_X \mu_{\tilde{A}}(x)/x = \int_X [\int_{J_x^u} f_x(u)/u]/x$$

$$J_x^u = \{(x,u): u \in [\underline{\mu}_{\tilde{A}}(x), \overline{\mu}_{\tilde{A}}(x)]\} \subseteq [0,1]$$

(6)

The present study tries to propose a model based on granular computing and T2 FSs to predict links regarding the node's activity and the relationship between two nodes. Therefore, the relationship between two nodes is considered as a primary MF. Also, node's activity is considered as a secondary MF. Proposed close function for two nodes i and j are as follows:

$$close(i,j) = \begin{cases} 1 & q_{ij} < 2 \\ (\hat{u}_{iz} \times \hat{\hat{u}}_z + \hat{u}_{zj} \times \hat{\hat{u}}_j)/2 \times 10^{q_{ij}-2} & q_{ij} = 2 \\ (\hat{u}_{iz} \times \hat{\hat{u}}_z + \hat{u}_{ze} \times \hat{\hat{u}}_e + \hat{u}_{ej} \times \hat{\hat{u}}_j)/2 \times 10^{q_{ij}-2} & q_{ij} = 3 \\ 0 & q_{ij} > 3 \end{cases}$$

(7)

In this function, $\hat{u}_{iz}$ is a primary membership value that denotes the strength of the relationship between two nodes i and z.

$\hat{\hat{u}}_{iz}$ is a secondary membership value that denotes node's activity in a network. In this paper, the degree of a node is assumed as a node's activity. In Fig. 2, this issue is illustrated:
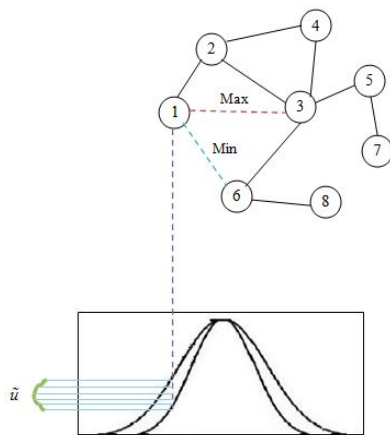

Fig. 2 Connections in a social network

As it is shown in Fig. 2, node 1 has max and min

relationship with nodes 3 and 6 respectively. In this Figure, the strength of the relationship between each pairs of nodes is shown with the primary MF. Also, the degree of each node is shown with the secondary MF. Where $\tilde{u}$ is a set $\{[\hat{u}_{ik}, \hat{\hat{u}}_k]\}$.

Another important concept that is used in the clustering definition is the Most concept. Most can also be defined as a fuzzy function like M(p), which indicates that the proportion p satisfies the Most. Fig. 3 shows the schematic function that is proposed by Yager for the function Most [33].

$$M(p) = \begin{cases} 0 & p \leq \alpha \\ (\beta - p)/(\beta - \alpha) & \alpha \leq p \leq \beta \\ M(p) = 1 & p \geq \beta \end{cases}$$
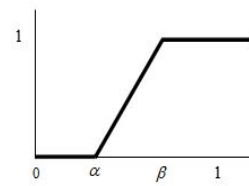
(8)

Fig. 3 Concept Most as a fuzzy set

The value of p can be computed for any node in the network as follows [33]:

$$p_i = \sum_{\substack{j=1 \text{ to } n_s \\ i \neq j}} close(x_i, x_j)/(n_s - 1)$$

(9)

Obviously, C_1 criteria can be calculated by using the amount of $M(p_i)$.

2. Second Criterion

In the second criterion of the cluster, there are Far and Not Far concepts. First, the concept of Far is needed [33]. A basic form for the fuzzy subset Far corresponding to the concept Far is shown in Fig. 4.
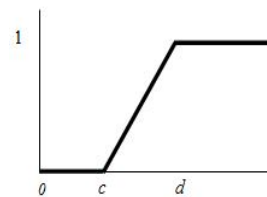

Fig. 4 Fuzzy set Far

The concept of not far is as the negation of F thus $\overline{F}(k) = 1 - F(k)$ that F(K) is the degree to which a shortest distance of K links is Far. Not Far is calculated by the following equation [33]:

$$NotFar(x,y) = \underset{k=1 \text{ to } n}{Max} [R^k(x,y) \wedge \overline{F}(k)]$$

(10)

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After defining the Not Far function, (11) is calculated for all pairs of nodes in the cluster that u represents a pair of nodes in the cluster:

$$C_2(s) = \underset{u \in U_s}{Min} \left[Not\ Far(u) \right] \qquad (11)$$

This equation, therefore, finds the furthest pair of vertices in S and calculates the degree to which they are not far.

3. Third Criterion

To calculate third criterion that every node out of the cluster of considered nodes should not be close to most of the nodes in the cluster, the following functions are used which y is a node out of the cluster and x_i is the node within the cluster [33]:

$$M(y/s) = Most\left(\sum_{j=1}^{n_s} close(y, x_j)/n_s \right) \qquad (12)$$

$$M(x_i/s) = Most\left(\sum_{\substack{j=1\ to\ n_s \\ i \neq j}}^{n_s} close(x_i, x_j)/(n_s - 1) \right) \qquad (13)$$

The third criterion can be calculated after defining (12) and (13). If M(y/s) is less than $M(x_i/S)$ for all of the nodes in the cluster then $C_3=1$, otherwise $C_3=0$.

After calculating all of the criterions, the links' score is calculated by $C(x_i)+C(x_j)$ which [33]:

$$S(x_i, x_j) = C(x_i) + C(x_j) \qquad (14)$$

$$C(x_i) = \underset{j}{Min} \left[C_j(x_i) \right] \qquad (15)$$

To make better predictions, the parameters in the prediction model are tuned by trial and error. The best values for the parameters α and β have been calculated 0.4 and 0.75 respectively.

Using the F2CC model, because of the better expression of FCC and CC and other related concepts such as closeness, it is possible to make a more accurate prediction.

V. RESULTS

The performance of the proposed algorithm is studied in this section. A collaboration network amid scientists that has been generated by Newman is employed [37]. The weights are calculated by the number of works, which is done by two researchers in common. In every step, about 10% of the data were chosen for the test and for every model about 10 experiments were done. Comparing the F2CC model (Type-2 fuzzy model) with the FCC model (Fuzzy Type-1 model) shows that the F2CC model predicts the links generation more accurately. The results of the prediction are shown in Fig. 5.

The comparison also shows that the results of the F2CC model are much better than the weighted clustering coefficient (WCC) model. The results are shown in Fig. 6.
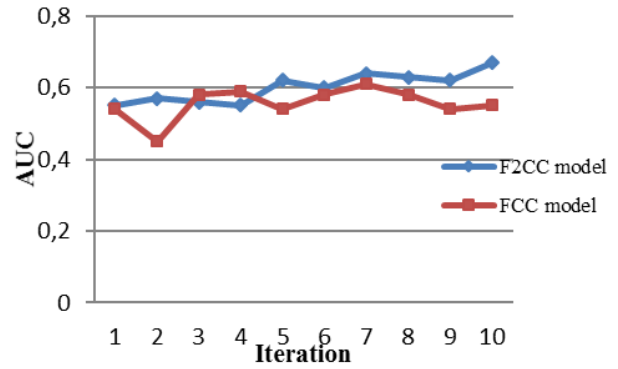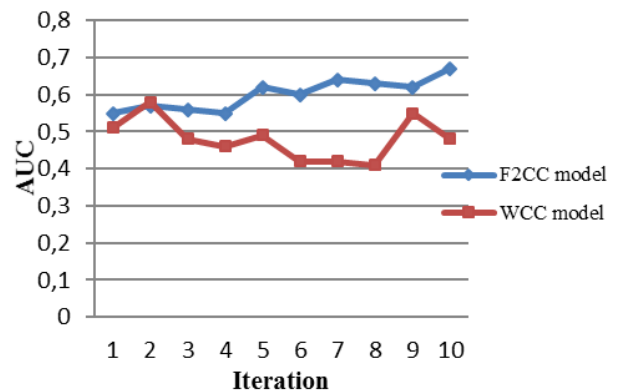

Fig. 5 Results of F2CC model compared to FCC model

Fig. 6 Results of F2CC model compared to WCC model

VI. CONCLUSION AND FUTURE WORK

This paper considered Type-2 fuzzy technology in SNA. The results show that using Type-2 fuzzy logic can make better predictions because of the better definition of networks characteristics and the concepts related to the links and nodes in social networks. The type-1 fuzzy set can be interpreted as a Type-2 fuzzy set all of whose secondary grades equal unity. In fact, a Type-1 fuzzy set is an instance of a Type-2 fuzzy set. It is a crisp version of a Type-2 fuzzy set. In other words, the accuracy of the predictions using Type-2 fuzzy link prediction models is higher as compared to the results of the considered crisp models and Type-1 fuzzy models.

Because the current study is the first attempt in the domain of Type-2 fuzzy link prediction, more studies must be carried out in the future. In fact, further studies can be done in this domain. Weak ties theory which plays an important role in social networks can model the strength of the ties very well. Using the weak ties can lead to generating larger networks and predicting links more accurately.

In this paper, it is demonstrated that Type-2 fuzzy technology can make a significant improvement in link prediction models. It is worth mentioning that employing the fuzzy probability-based models can modify the methods for finding the strength of the links or predicting the evolution of the social networks.

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