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Thermal Analysis of Extrusion Process in Plastic Making

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***Abstract*—**Plastic extrusion has been an important process of plastic production since 19th century. Meanwhile, in plastic extrusion process, wide variation in temperature along the extrudate usually leads to scraps formation on the side of finished products. To avoid this situation, there is a need to deeply understand temperature distribution along the extrudate in plastic extrusion process. This work developed an analytical model that predicts the temperature distribution over the billet (the polymers melt) along the extrudate during extrusion process with the limitation that the polymer in question does not cover biopolymer such as DNA. The model was solved and simulated. Results for two different plastic materials (polyvinylchloride and polycarbonate) using self-developed MATLAB code and a commercially developed software (ANSYS) were generated and ultimately compared. It was observed that there is a thermodynamic heat transfer from the entry level of the billet into the die down to the end of it. The graph plots indicate a natural exponential decay of temperature with time and along the die length, with the temperature being 413 K and 474 K for polyvinylchloride and polycarbonate respectively at the entry level and 299.3 K and 328.8 K at the exit when the temperature of the surrounding was 298 K. The extrusion model was validated by comparison of MATLAB code simulation with a commercially available ANSYS simulation and the results favourably agree. This work concludes that the developed mathematical model and the self-generated MATLAB code are reliable tools in predicting temperature distribution along the extrudate in plastic extrusion process.

***Keywords*—**ANSYS, extrusion process, MATLAB, plastic making, thermal analysis.

I. Introduction

PLASTICS in the 20th century have become the most widely used amongst other materials. It offers many unique advantages when used in product design. From cost perspective, plastics offer not only a low cost per unit volume of material, but also low manufacturing and assembly costs due to their ability to be easily formed into net shape products containing assembly features [7]. Their light weight and ease of recyclability also often result in the lowest possible life cycle costs compared to product designs based on other types [1]. Concurrent with their cost advantages, plastics offer an extremely wide range of color, chemical, electrical, mechanical, thermal, wear, and other properties. Given the potential cost benefits, thermoplastics can be well utilized for many components in a complex product design. The primary challenge in their use is the optimal decomposition of the overall system into a set of components that best utilize the capabilities of the constitutive materials. Primary design considerations in engineering design often include structural performance, end-use temperature, electrical and thermal conductivity, manufacturability, assembly time, material costs, and others [9]. It is versatile as it can easily be manufactured and modified at a suitable temperature and pressure. Moreover, since the daily activity of man requires that durable materials are to be used not only to carry out manufacturing, but also for domestic uses. It is therefore necessary to know the characteristics and behavior of such materials and to have detailed information about such materials by means of analyses. Many engineering systems may be analysed by subdividing them into components or elements [8]. This process of analyses is carried out clearly and understandably to know why such characteristics or properties are inherent in such materials. Therefore, in order to make proper designs with any material, plastics inclusive, it is necessary to know certain physical (thermal, chemical, electrical and mechanical) properties of the material. References [5] and [10] introduced the stream function to express three-dimensional flow in die and analyzed the force of extrusion from round billet to elliptical bars. Reference [2] made an upper bound analysis on drawing of square sections from round billets by using triangular element at entry and exit of the die. A method employing a discontinuous velocity field was proposed by [4]. This method is based on discretizing the deformation zone into elementary rigid regions. In such a scenario, the rigid regions have a constant internal velocity and the deformation is assumed to occur at the interfaces of these regions. The rigid element assumption limits the use of this technique to problems with flat boundaries. Further, Gatto and Giarda's [4] formulation appears suitable for problems where billet and product sections are similar. Reference [6] modified this technique to solve problems with dissimilar billet and product sections. However, their formulation was also limited to problems with flat boundaries and, as such as the analysis of extrusion from round billets is excluded from their formulation. Reference [12] used the reformulated spatial elementary rigid region (SERR) technique for the analysis of round-to-square extrusion process and [11] simulated extrusion dies dimensions with a view to designing an optimal die mode. In order to gain a better understanding into thermal characteristics of plastic extrusion process, this work developed and simulated a mathematical model to predict temperature distribution along the extrudate of plastic extrusion process within a given time range.

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II. Method of the Analysis

A. Modeling of the Temperature Distribution Process

The thermal analysis stems from numerical modeling of temperature distribution and flow pattern. The thermo-mechanical response of the billet during extrusion is governed by a set of nonlinear, partial differential equations given by heat transfer given by the below equations [3].

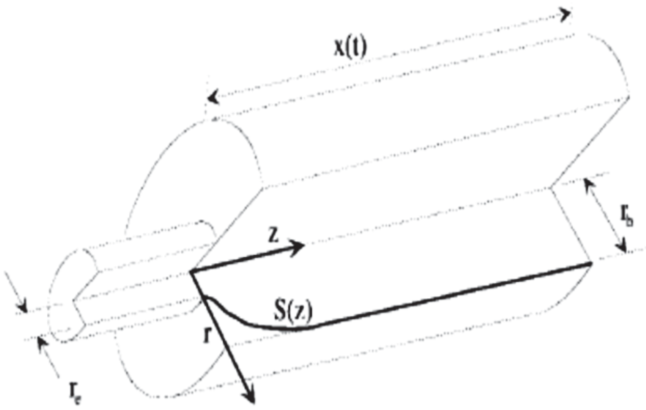


Fig. 1 Problem Coordinate

By meshing the cylindrical figure to small finite elements, the heat transfer radially and axially are given by $q'_r and\, q'_z$ respectively hence, total heat transferred $q'_{(r,z)}$ is:

$$q'_{(r,z)} = q'_r + q'_z = \lim_{\delta r\to 0,\delta z\to 0,\delta t\to 0} -(k_r \frac{\delta T}{\delta r} A_r + k_z \frac{\delta T}{\delta z} A_z) \quad (1)$$

As elements becomes infinitesimal,

$$q'_{(r,z)} = q'_r + q'_z = -(k_r \frac{\partial T}{\partial r} A_r + k_z \frac{\partial T}{\partial z} A_z) \quad (2)$$

where A_r and A_z represents the radial and axial area respectively.

When the r and z changes by dr and dz respectively, generation of heat adds up with $q'_{(r,z)}$ to give

$$q'_{(r+dr,z+dz)} = q'_r + q'_z + \frac{\partial q}{\partial r} dr + \frac{\partial q}{\partial z} dz \quad (3)$$

Substituting (2) into (3), we have:

$$q'_{(r+dr,z+dz)} = q'_{generated} V + \frac{\partial}{\partial r}\left(k_r A_r \frac{\partial T}{\partial r}\right) dr + \frac{\partial}{\partial z}\left(k_z A_z \frac{\partial T}{\partial z}\right) dz = q'_{stored} \quad (4)$$

where;

$$q'_{stored} = \rho c_p \frac{\partial T}{\partial t} V \quad (5)$$

Since the system is thin radially, it implies that heat transferred along r-axis is negligible. Hence q'_{stored} is a function of time t, this implies that T= T(z,t) Where $k_z = k$ this is due the isotropic properties of plastics, hence the equation becomes

$$\rho c_p \frac{\partial T}{\partial t} V = k\left(\frac{\partial^2 T}{\partial z^2}\right) V + q_{generated} V \quad (6)$$

Now, $q'_{generated} V = H(z,t)$. Then, the equation becomes

$$\frac{\partial T(z,t)}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T(z,t)}{\partial z^2}\right) + \frac{H(z,t)}{\rho c_p} \quad (7)$$

Then,

$$\frac{\partial T(z,t)}{\partial t} = \alpha \nabla^2 T(z,t) + \frac{H(z,t)}{\rho c_p} \quad (8)$$

From this equation, T was solved at various instances of (z, t). Direct analytical solution of such equation is practically impossible. For simplicity, the following assumptions are made;

1. The same deformation field.
2. Stress is taken constant for all fields of deformation by not considering the temperature dependence of the stress flow.
3. There is no external heat source to the die during the extrusion process.

B. Developing the Heat Transfer Model

The model was developed as;

$$\frac{\partial T(z,t)}{\partial t} = \alpha \nabla^2 T(z,t) + \frac{H(z,t)}{\rho c_p}$$

is equivalent to

$$\frac{\partial T(z,t)}{\partial t} = \alpha \frac{\partial^2 T(z,t)}{\partial z^2} + \frac{H(z,t)}{\rho c_p}$$

as r-axis is neglected. More so no heat is generated; therefore,

$$\frac{H(z,t)}{\rho c_p} = 0.$$

Thus,

$$\frac{\partial T(z,t)}{\partial t} = \alpha \frac{\partial^2 T(z,t)}{\partial z^2}$$

Subject to the initial condition

$$T(z,0) = T_0$$

and the boundary conditions;

$$T(0,t) = T_0;$$

and

$$298K \le T(L,t) < T(z)\, for\; t > 0$$

Considering first the solution of the problem by separating variables, $T(z,t) = T(x)T(t)$ and proceeding in the usual manner leads to the separated equations

$$\frac{T'(t)}{T(t)} = \alpha \frac{T''(z)}{T(z)}$$

Introducing a separation constant $-\lambda$ with $-\lambda > 0$, where the negative sign is chosen to make the solution satisfy the physical requirement that it decays with time, we arrive at the two separated ordinary differential equations

$$\frac{dT(t)}{dt} = -\lambda T(t) \; and \; \frac{d^2T(z)}{dz^2} + \alpha\lambda T(z) = 0$$

To satisfy the boundary conditions on the temperature $T(z,t)$, T_0 must satisfy the boundary conditions $T(0,t) = T(z,0)$.

The general solution for $T(z)$ and $T(t)$ are;

$$T(z) = A\cos\sqrt{\alpha\lambda}\,z + B\sin\sqrt{\alpha\lambda}\,z$$

and $T(t) = C\exp[-\lambda t]$ respectively. Therefore,

$$T(z,t) = T(x)T(t) = C\left(A\cos\sqrt{\alpha\lambda}\,z + B\sin\sqrt{\alpha\lambda}\,z\right)\exp[-\lambda t]$$
$$= (P\cos\sqrt{\alpha\lambda}\,z + Q\sin\sqrt{\alpha\lambda}\,z)\exp[-\lambda t]$$

Appling the boundary conditions, the boundary conditions will only be satisfied when $\alpha\lambda = [n\pi/L]^2$ and $A = 0$, so the eigenvalues are $\lambda = [n\pi/L]^2$ and the associated Eigen functions can be taken to be

$$T_n(z) = \sin[n\pi/L]^2\text{, with } n = 1, 2 \ldots$$

Integrating the equation for the time variation $T(t)$ with $\lambda = \lambda_n$ gives $T_n(t) = \exp[-\lambda n\alpha t]$, so the elementary solutions for this problem are

$$T_n(z,t) = \exp(-\left((\frac{n\pi}{L})^2\alpha t\right)\sin\left(\frac{n\pi z}{L}\right)\text{, with } n = 1,2 \ldots$$

It follows from this that the solution for the temperature distribution will be of the form

$$T(z,t) = \sum_{n=1}^{\infty} T_n(z,t) = \sum_{n=1}^{\infty} a_n \exp(-\left((\frac{n\pi}{L})^2\alpha t\right))\sin\left(\frac{n\pi z}{L}\right)$$

The coefficients a_n follow in the usual manner by setting $t = 0$ and using the initial condition that $T(z,0) = T(z)$, when we find that

$$T_0 = \sum_{n=1}^{\infty} a_n \sin\left(\frac{n\pi z}{L}\right)$$

Multiplying this result by $\sin\left(\frac{n\pi z}{L}\right)$ and integrating over the interval $0 \le x \le L$ according to Fourier series,

$$a_n = \frac{2}{L}\int_0^L T_0 \sin\left(\frac{n\pi z}{L}\right)dz, \quad for\; n = 1,2 \ldots$$

We arrive at the required solution

$$T(z,t) = \sum_{n=1}^{\infty} a_n \exp\left(-\left((\frac{n\pi}{L})^2\alpha t\right)\right)\sin\left(\frac{n\pi z}{L}\right)$$

Substituting

$$a_n = \frac{2}{L}\int_0^L T(z)\sin\left(\frac{n\pi z}{L}\right)dz, \quad for\; n = 1,2$$

into the above equation results in the final equation

$$T(z,t) = \sum_{n=1}^{\infty}\frac{2}{L}\int_0^L (T_0 \sin\left(\frac{n\pi z}{L}\right)dz)\exp\left(-\left((\frac{n\pi}{L})^2\alpha t\right)\right) \quad (9)$$

The summation equation above is the solution of the homogeneous equation where, T_0 is the inlet temperature to the die and α is the thermal diffusivity.

C. Problem Solution

The problem was simplified to one dimensional transient state problem and this was solved analytically. The boundary conditions are the inlet temperature (T_0), outlet temperature (which is greater than the ambient temperature but less than the inlet temperature) and thermal diffusivity of the plastics considered. The Plastics considered are: Polyvinylchloride; and Polycarbonate. The respective thermo-physical properties were used as the input parameters in the solution of the governing equation.

- Length of the die: 1.14m
- Diameter of the die: 0.038m
- Length-to-diameter ratio: 30
- Maximum time: 20s
- Thermal diffusivity
- Polyvinylchloride: 0.000000116m^2/s
- Polycarbonate: 0.000000147m^2/s
- Inlet temperature (T_0)
- Polyvinylchloride: 413K
- Polycarbonate: 474K

The basic parameters were obtained from the thermo-physical property table of plastics. The temperature at different time intervals were solved for in the governing equation using a computer program.

III. Results and Discussion

A. Computer Implementation

Distribution pattern of temperature was solved for in the developed governing equation by running the MATLAB codes on HP Intel 1.6 GHz processor with 1 GB RAM and 160 GB hard drive space. The program code was written such that when the values of time, length, initial entry temperature and the plastic's thermal diffusivity were imputed, the temperature distribution were displayed (See Figs. 2 and 3). The program ran twice for the two types of plastics (polyvinylchloride and polycarbonate) employed. The various results obtained were plotted on a graph for both plastic materials.

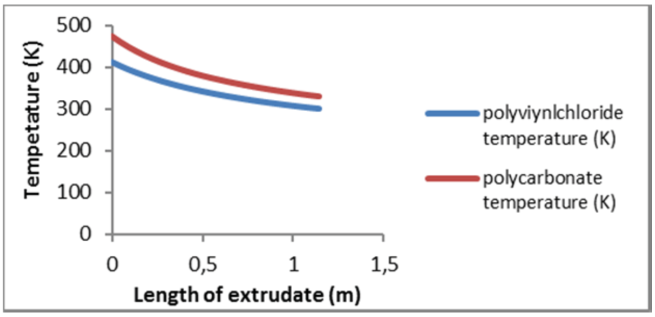
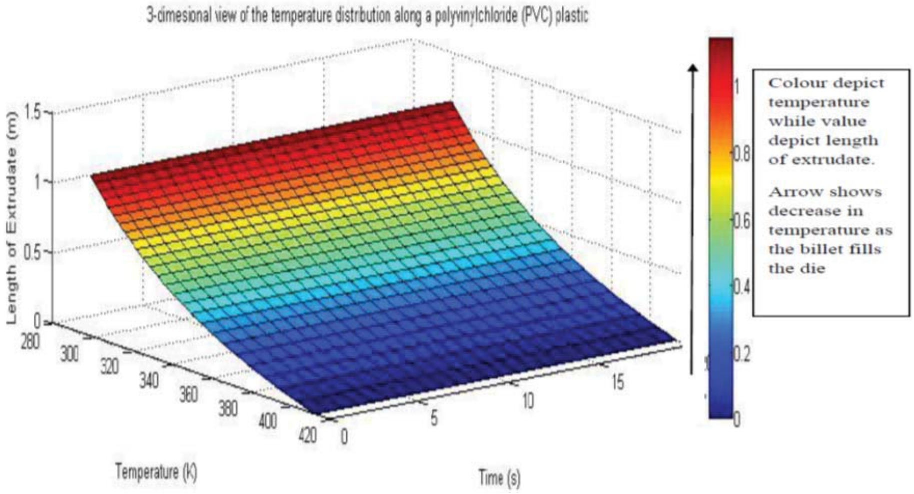
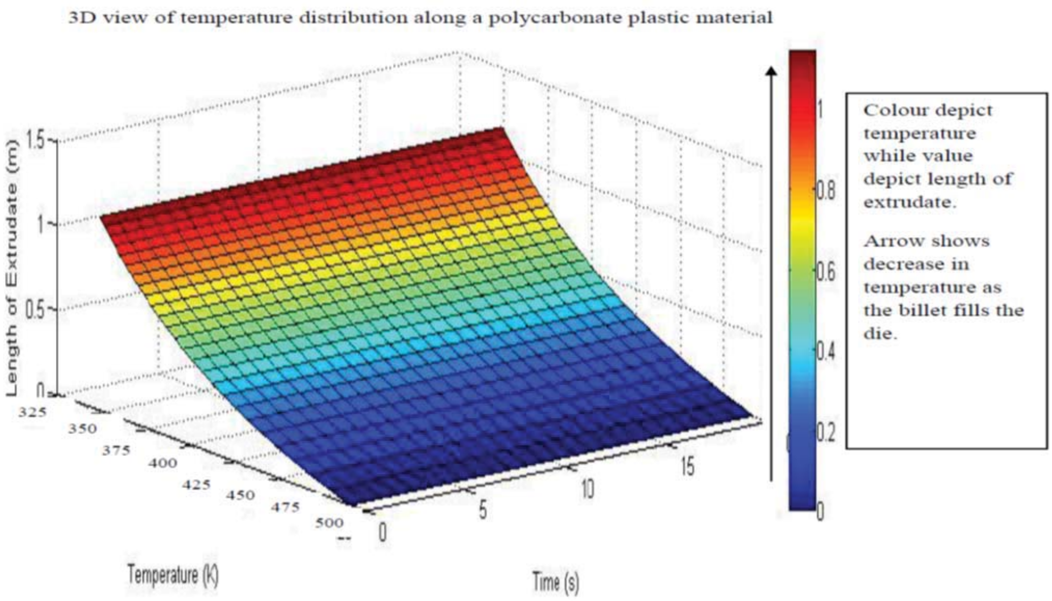


Fig. 2 Temperature distribution and extrudate length of the plastics employed

From the result, it is observed that there is a thermodynamic heat transfer from the entry of the billet into the die to the end of the die. Figs. 2 and 3 indicate a natural exponential decay of temperature with time and along the die length. The temperature at the entry being 413 K and 474 K for polyvinylchloride and polycarbonate respectively, the temperature of the surrounding being fixed at 298 K, there was a flow of heat from the entry to exit such that 299.3 K and 328.8 K were attained. The temperature attained by both plastics at the exit is sufficiently low to attain rigidity in preventing them from deformation outside the die, which is the sole aim of every manufacturer. From the result, thus in about 20s the plastic is being extruded to attain an optimum quality.



(a) Polyvinylchloride plastic material



(b) Polycarbonate plastic material

Fig. 3 Temperature distributions for Polycarbonate and Polyvinylchloride plastic material

B. Model Validation

The ANSYS software was employed in simulating the process so as to validate the mathematical model in order to establish if the solution is accurate for its intended use (see Figs. 4 and 5). The extrusion model is validated by comparison of simulation and numerical predictions of the geometry of the extruded product, especially when simulating the extrusion of shapes and temperature distribution across it. Also the ANSYS simulation further reveals that the solution of the mathematical model on temperature distribution along the extrudate is within an acceptable limit.

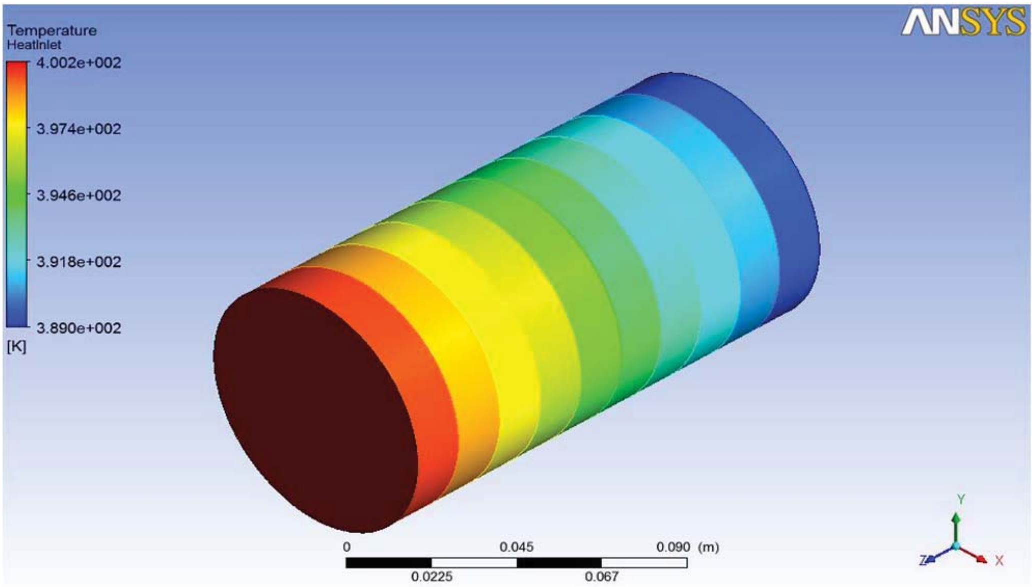


Fig. 4 Temperature distribution along a discreet Extrudate in 3-dimension

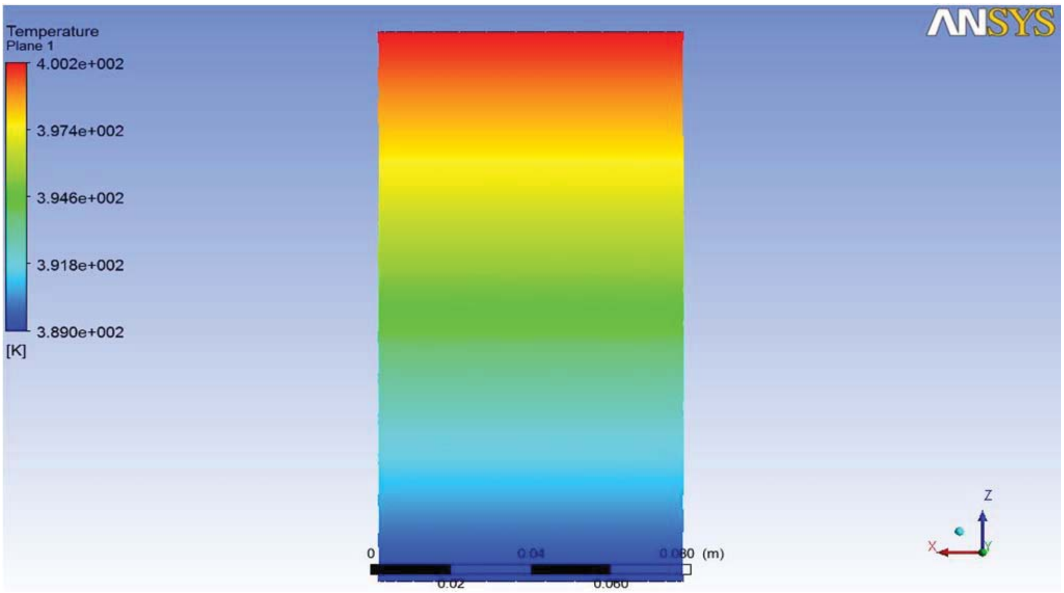


Fig. 5 Temperature distribution along an Extrudate in 2-dimension

IV. Conclusion

This study has developed and simulated a mathematical model to predict temperature distribution along the extrudate in a die within a given range of time. The summary of our findings are:

(i) Polycarbonate material dissipates more heat than polyvinylchloride along the same die length within the same time frame. Both plastics dissipate enough heat to attain rigidity at the exit of the die.
(ii) Plastic deformation temperatures depend on the initial extrusion temperature of the billet and die.
(iii) The exit temperature has a direct effect on the flow stress of the plastic.
(iv) Surface quality is also strongly influenced by the temperature at the exit of extrusion die.

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