

# Primary School Teachers' Conceptual and Procedural Knowledge of Rational Number and Its Effects on Pupils' Achievement in Rational Numbers

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**Abstract**—The study investigated primary school teachers' conceptual and procedural knowledge of rational numbers and its effects on pupil's achievement in rational numbers. Specifically, primary school teachers' level of conceptual knowledge about rational numbers, primary school teachers' level of procedural knowledge about rational numbers, and the effects of teachers conceptual and procedural knowledge on their pupils understanding of rational numbers in primary schools is investigated. The study was carried out in Bauchi metropolis in the Bauchi state of Nigeria. The design of the study was a multi-stage design. The first stage was a descriptive design. The second stage involves a pre-test, post-test only quasi-experimental design. Two instruments were used for the data collection in the study. These were Conceptual and Procedural knowledge test (CPKT) and Rational number achievement test (RAT), the population of the study comprises of three (3) mathematics teachers' holders of Nigerian Certificate in Education (NCE) teaching primary six and 210 pupils in their intact classes were used for the study. The data collected were analyzed using mean, standard deviation, analysis of variance, analysis of covariance and t- test. The findings indicated that the pupils taught rational number by a teacher that has high conceptual and procedural knowledge understand and perform better than the pupil taught by a teacher who has low conceptual and procedural knowledge of rational number. It is, therefore, recommended that teachers in primary schools should be encouraged to enrich their conceptual knowledge of rational numbers. Also, the superiority performance of teachers in procedural knowledge in rational number should not become an obstruction of understanding. Teachers Conceptual and procedural knowledge of rational numbers should be balanced so that primary school pupils will have a view of better teaching and learning of rational number in our contemporary schools.

**Keywords**—Achievement, conceptual knowledge, procedural knowledge, rational numbers.

## I. INTRODUCTION

IN primary Mathematics curriculum, much emphasis is placed on the development of number and number sense. (Whole numbers, rational numbers and number operation) The most difficult number concept is rational numbers.

Researchers have shown and continue to show that teaching and learning of rational number have been problematic [1]. A rational number is one of the major concepts which were introduced to pupils in primary schools in Nigeria. This concept is continuous throughout their mathematical learning from primary to secondary school. Rational numbers are

important because they enhance pupils' abilities to solve real-world problems that are necessary for an increased mathematical understanding and provide a foundation for Algebraic thinking [2], [3]. The abilities will, in turn, become the intellectual and mathematical cornerstone of much of what is to come in the secondary school years. Pupils in primary school do not create appropriate meanings of rational numbers due to insufficient instructions made by their teachers. Reference [4] posits that understanding of rational numbers has not only been frustrated by the nature of the concept but also by the clumsy method and instructional materials used.

Several theories of learning and cognition posit that our behavior is shaped by at least two different kinds of knowledge. One provides an abstract understanding of the principles and relations between pieces of knowledge in a certain domain and another enabling us to solve problems quickly and efficiently. In recent empirical research in Mathematics, learning the former is frequently named conceptual knowledge, while the latter is labeled procedural knowledge [5]. For instance [6]-[8], identified conceptual and procedural knowledge as the two kinds of knowledge that builds up our understanding of a topic. The teaching and learning of this form of knowledge go on hand in hand. For example, conceptual knowledge goes alongside with procedural knowledge, and procedural knowledge goes alongside with conceptual knowledge. For learning and teaching to be effective, a teacher has to teach with an understanding of both conceptual and procedural knowledge. Also, these two types of knowledge are intertwined with each other. Conceptual knowledge is the knowledge used to understand mathematical concepts by being able to interpret and apply them correctly to variety of situation as well as to translate the concept into verbal statement and their equivalent mathematical knowledge while Procedural knowledge is the knowledge used to solve a problem through manipulating of mathematical skills such as procedures, rules, formulae, algorithm ad symbol used in mathematics.

It was also evident from the literature reviewed that mathematics teachers at elementary level face difficulty in interpreting problems in such a way that learners would be able to relate the mathematics they already know [9]. This makes it difficult for the learners to learn with understanding. Teachers do not know how to interpret the question in ways that enable learners to relate the mathematics to what they already know, they will not learn with understanding and the foundation of the children success lies in the teachers'

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knowledge of the topic can this problem be attributed to the teachers lack of possession of the required knowledge base on mathematics for effective teaching? Also, there appeared to be very little or no much information in the literature about teachers' possession of adequate knowledge of rational number concepts in Nigerian school, particularly at the primary school level. Hence, there is a need to fill in this gap. The present study is an attempt to explore primary school teachers conceptual and procedural knowledge of rational numbers and its effects on pupils' achievement on rational numbers.

## II. STATEMENT OF THE PROBLEM

Given the central role of rational number in primary school mathematics, students widely acknowledge difficulties in learning this topic and the crucial role that teachers' knowledge of a particular school topic plays in the instructional process [10], [11]. The importance of mathematics at all level of the education sector cannot be over-emphasized. It has been observed by the researcher that pupils have problems of learning and understanding rational number concepts. It could be that this is caused probably by their teacher's lack of conceptual and procedural knowledge in a rational number. When a teacher has conceptual and procedural knowledge of mathematics it influences classroom instruction in a positive way, if teachers themselves have difficulty with rational numbers, they are not likely to facilitate the construction of meaning of rational numbers, or to recognize related errors the pupils make, teachers with limited knowledge of rational number may feel insecure when teaching. Coupled with the fact that pupils learn rational number through classroom experience and solving problems given by their teacher. It, therefore, becomes imperative to investigate the teachers' conceptual and procedural knowledge of rational number.

### A. Purpose of the Study

The main purpose of the study is to explore the primary school teachers' conceptual and procedural knowledge of rational numbers and how it affects the conceptual and procedural knowledge of pupils in a rational number. Specifically, the study will seek to achieve the following objectives.

- To determine the primary school teachers' level of conceptual knowledge about rational numbers
- To determine the primary school teachers' level of procedural knowledge about rational numbers
- To determine the effects of teacher' conceptual and procedural knowledge of their pupils' understanding of rational numbers in primary school

### B. Research Questions

The following questions were formulated to guide the study:

- What is the level of primary school teachers' conceptual knowledge of rational number?

- What is the level of primary school teachers' procedural knowledge of rational number?
- What is the effect of primary school teachers' conceptual and procedural knowledge of their pupils' understanding of rational numbers?

### C. Hypotheses

The following hypotheses were investigated

- There is no significant difference between the achievement of pupils taught rational number by a teacher with high conceptual and procedural knowledge of rational number and those taught by any other teacher.
- There is no significant difference between the achievement of the pupils taught rational numbers by a teacher with high conceptual knowledge of rational numbers and those pupils taught by a teacher with low conceptual knowledge of rational numbers.
- There is no significant difference between the achievement of the pupils taught rational numbers by a teacher with high procedural knowledge of rational numbers and those pupils taught by a teacher with low procedural knowledge of rational numbers.

## III. METHODOLOGY

The design of the study was a multi-stage design. The first stage was a descriptive design, while the second stage involves a pre-test post-test only quasi- experimental design.

The area of study was in Bauchi metropolis in Bauchi state where the primary school teachers were selected for the study are located. The population for the study comprised of all primary school teachers with National Certificate of Education (N.C.E) qualification teaching mathematics to class six, in all Primary schools in Bauchi metropolis.

### A. Sample and Sampling Techniques

From the design, the experiment requires three groups. Each group consists of a teacher, and each teacher has an intact class consisting of 70 pupils. The total number of teachers' sample is three and 210 pupils. The samples of the teachers were selected from the primary schools in Bauchi metropolis. Four (4) schools were randomly selected from the schools using simple random sampling. Then, the teachers that have N.C.E teaching mathematics class six in these schools were given a test, where they were divided into two 2 groups which are those with high conceptual and procedural knowledge of rational numbers ( $G_1$ ) and those teachers with low conceptual and procedural knowledge of rational numbers ( $G_2$ ). According to their result and from each group, a teacher was randomly selected as a sample for the experiment and another teacher was selected randomly from a different school that was not tested and was used as a control group ( $G_3$ ). The intact classes of the teachers were used for the experiments.

### B. Instrument for Data Collection

Two instruments were used for data collection in this study; these instruments are the Conceptual and Procedural Knowledge Test (CPKT) and the Rational Number

Achievement Test (RAT). The CPKT about rational numbers is a six open-ended question test that was adopted from [12]. The CPKT is further sub-divided into 17 item question. A particular set of these questions measure procedural knowledge and another set measure conceptual knowledge, this test was used to measure the conceptual and procedural knowledge of rational numbers of teachers in primary six that have NCE qualification and teaches mathematics. Each item was given a score of 0-4 according to [21] marking scheme. The second instrument is a 24 item multiple choice test RAT. RAT was developed by the researcher who was administered to only primary six pupils; that is the intact class of the teachers selected. The pupils' performance gave an indication of the effect of teacher's possession of conceptual and procedural knowledge of the rational number. The RAT questions have response option A to D. It was constructed based on the contents of primary school 1-6 mathematics curriculum in Nigeria. The content of the test covers the five sub-construct of rational number, namely, part-whole, quotient, ratio, operator and measure. The test was to the pupils as both pre and post-test to confirm further effects of the teachers' conceptual and procedural knowledge of the pupils understanding.

The Conceptual and Procedural Knowledge Test (CPKT) and Rational Number Achievement Test (RAT) were validated by six experts. Three experts in mathematics education from Abubakar Tafawa Balewa University Bauchi and one expert in mathematics education and measurement and evaluation from Abubakar Tatar Ali polytechnic Bauchi, and two mathematics teachers with over five years of teaching experiences from a primary school in Bauchi metropolis. These experts carried out both face and content validation of the two tests.

Reliability of instrument: The reliability coefficients of both CPKT and RAT were determined after the test had been trial tested in a pilot study.

The split half method was used to determine the reliability of RAT which was found to be 0.76, while the inter-scorer method was used to determine the reliability value of the CPKT and was found to be 0.97. Data was collected by the researcher with the help of research assistants using, CPKT and RAT. The research assistants were trained on how to administer the two instruments. The first instrument which is the Conceptual and Procedural Knowledge Test (CPKT) was administered to those teachers with NCE qualification teaching mathematics in primary six, while the second instrument which is the Rational Number Achievement Test (RAT) was administered to their pupils in primary six, (the intact classes of each of the teachers selected in the study)

### C. Treatment

The teachers in the two experiment groups which are G<sub>1</sub> and G<sub>2</sub> and another teacher from the control group G<sub>3</sub> were given lesson plan, on what they are to teach during their intact classes for 3 weeks before the post-test.

The lesson notes comprised of note on the different sub-construct of rational number, which is part-whole quotient, ratio, operator, and measure, in each sub-construct, a topic was

selected after a careful study of the primary school mathematics curriculum. The teachers covered these topics in three weeks that is in 15 periods following the normal time table of the school since each intact class must have at least one period for mathematics a day. The researcher did not disclose the classification of the teachers and did not participate in the teaching process to avoid experimental bias. The pupils were given a posttest with the RAT instrument on the topics taught by the teachers. The test then was scored, recorded and analyzed. Was administered to their pupils in primary six (the intact class of each of the teachers selected in the study)

### D. Method of Data Analysis

Descriptive statistics of mean and standard deviation were used to describe the level of the primary school teachers conceptual and procedural knowledge of rational number and also the effects of primary school teachers conceptual and procedural knowledge of rational numbers on their pupils understanding of rational numbers The Vassar statistics online software was used to calculate the mean score and the standard deviation. The result of Pre-Test of RAT was analyzed using analysis of variance (ANOVA). The post test result of RAT was analyzed using analysis of covariance (ANCOVA) with the help of Vassar statistics online software. Then, hypothesis two and three were tested using t- test to determine the significant difference at 0.05 level of significance. The data was presented taking each research question one after the other.

## IV. RESULTS

### A. Research Question 1

What is the level of primary school teachers' conceptual knowledge of rational number?

From the result presented in Table I mean and standard deviation were calculated. The cutoff point of mean 2.00 was used. Fifty percent of the teachers have a high level of conceptual knowledge. These are T<sub>1</sub> (3.11) T<sub>3</sub> (2.33) and T<sub>5</sub> (2.67). The remaining fifty percent have low level of conceptual knowledge which are T<sub>2</sub> (1.22), T<sub>4</sub> (1.22) and T<sub>6</sub> (1.33)

TABLE I  
 MEAN AND STANDARD DEVIATION OF TEACHERS' PERFORMANCE ON  
 CONCEPTUAL ITEMS IN THE CONCEPTUAL AND PROCEDURAL KNOWLEDGE  
 TEST ON RATIONAL NUMBERS

Teachers	Mean	Sd	Performance
T1	3.11	0.53	High
T2	1.22	0.63	Low
T3	2.33	0.67	High
T4	1.22	0.63	Low
T5	2.67	0.47	High
T6	1.33	0.67	Low

Note: Sd-Standard deviation

### B. Research Question 2

What is the level of primary school teacher's procedural knowledge of rational number?

Form the result presented in Table II, mean and standard deviation were calculated, and the cutoff point of mean 2.00 was used 83.33% have a high level of procedure knowledge. These are T<sub>1</sub> (3.75), T<sub>2</sub> (2.25) T<sub>3</sub> (3.5), T<sub>5</sub> (3.625) and T<sub>6</sub> (2.25) meanwhile only 16.66%, has a low level of procedural knowledge which is T<sub>4</sub> (1.875).

TABLE II  
MEAN AND STANDARD DEVIATION OF TEACHER'S PERFORMANCE ON PROCEDURAL ITEMS IN THE CONCEPTUAL AND PROCEDURAL KNOWLEDGE TEST OF RATIONAL NUMBERS

Teachers	Mean	Sd	Performance
T1	3.75	0.43	High
T2	2.25	0.06	High
T3	3.50	0.50	High
T4	1.88	0.78	Low
T5	3.63	0.70	High
T6	2.25	0.66	High

Note: Sd-Standard deviation

### C. Research Question 3

What is the effect of primary school teachers' conceptual and procedural knowledge of pupils understanding of rational numbers?

To answer this research question, rational number achievement test (RAT) was developed and administered to the three intact classes of the teachers (G<sub>a</sub>, G<sub>b</sub>, G<sub>c</sub>). The mean achievement and standard deviation of the pre-test result of the rational number achievement test of the three intact classes of the teachers (G<sub>a</sub>, G<sub>b</sub>, G<sub>c</sub>) were computed in Table III, which shows that mean achievement and standard deviation of the pupils taught by a teacher with a high conceptual and procedural knowledge of rational number (G<sub>a</sub>) were 59.45 and standard deviation of 17.25 similarly the mean achievement scores of the pupils taught by the teacher with low conceptual and procedural knowledge of rational number (G<sub>b</sub>) was 24.64 and standard deviation of 16.66 and for those taught by a teacher in the control group (G<sub>c</sub>) the mean achievement is 32.57 with standard deviation of 18.88. the Pre-test result of the three intact classes of the teachers was tested for significant differences using the analysis of variance ANOVA as shown in the Table IV, which indicates that the pre-test result of these intact classes was significantly different after the ANOVA. Therefore, the pre-test result were used as covariance in testing the significance of difference of the post-test of the intact classes from the results in Table V, which shows the mean achievement and standard deviation of the pupils taught by a teacher with high conceptual and procedural knowledge were 78.93 and standard deviation of 13.34, similarly the mean achievement and standard deviation of the pupils taught by a teacher with low conceptual and procedural knowledge of rational number were 29 and 19.96, and for those who were taught by a teacher in the rational group mean achievement is 45.14 and standard deviation 20.44. The research question will be answered after testing hypothesis one. The result of hypothesis one will determine the answer to research question three.

TABLE III  
MEAN ACHIEVEMENT AND STANDARD DEVIATION OF PRE-TEST RESULT OF THE THREE INTACT CLASSES OF THE TEACHERS IN RATIONAL NUMBER ACHIEVEMENT TEST

Intact classes of the Teachers	Mean	Sd
G <sub>a</sub>	59.43	17.25
G <sub>b</sub>	24.64	16.67
G <sub>c</sub>	32.57	18.59

Note: Sd-Standard deviation

TABLE IV  
SUMMARY OF THE ANOVA OF THE PRETEST RESULT OF THE INTACT CLASSES IN THE RATIONAL NUMBER ACHIEVEMENT TEST (ONE WAY ANALYSIS OF VARIABLES FOR INDEPENDENT SAMPLES)

Source	Ss	df	Ms	F <sub>cal</sub>	F <sub>crit</sub>	P
Treatment (between Group)	46531.67	2	23265.83	75.81	3.04	<0.0001
Error	63530.36	207	306.91			
S/B						
Total	110061.02	209				

TABLE V  
MEAN ACHIEVEMENT AND STANDARD DEVIATION OF POST TEST RESULT OF THE THREE INTACT CLASSES OF THE TEACHERS IN RATIONAL NUMBER ACHIEVEMENT TEST

Intact Classes of the Teachers	Mean	Sd
G <sub>a</sub>	78.93	13.34
G <sub>b</sub>	29.00	19.96
G <sub>c</sub>	45.14	20.44

Note: Sd-Standard deviation

### D. Hypothesis 1

H0.1. There is no significant difference between the achievement of the pupils taught rational numbers by a teacher with high conceptual and procedural knowledge of rational number and those taught by any other teacher.

To test hypothesis one, analysis of covariance ANCOVA was used on the post-test results of the three intact classes of the teachers in rational number achievement test with their pre-test as covariance, from the result presented in Table VI, it shows that the F calculated is 24.22 which is greater than the F critical (3.04) at P>0.001 the null hypothesis (H<sub>01</sub>) was therefore rejected, while the alternative (H<sub>a 1</sub>) was upheld. It was therefore concluded that there is a significant difference between the mean achievement of the three intact classes of the teachers in rational numbers achievement test (G<sub>a</sub>, G<sub>b</sub> G<sub>c</sub>) with respect to their teachers conceptual and procedural knowledge of rational numbers.

The answer to research question three is that primary school teachers' conceptual and procedural knowledge has effects on improving pupils' understanding of rational number

TABLE VI  
SUMMARY OF ANCOVA RESULTS OF THE POST-TEST OF THE 3 INTACT CLASSES OF THE RAT

Sources of variance	SS	df	MS	F <sub>cal</sub>	F <sub>crit</sub>	p
Adjusted Means	4791.69	2	2395.85	24.22	3.04	<0.0001
Adjusted error	20375.88	206	98.91			
Adjusted total	25167.57	208				

*E. Hypothesis 2*

H0.2. There is no significant difference between the achievement of the pupils taught rational numbers by a teacher with high conceptual knowledge of rational numbers and the pupils taught by a teacher with low conceptual knowledge of rational numbers.

The hypothesis was analyzed using an independent sample t-test to compare the mean achievement of the two intact classes of the teachers, which are the class of pupils' taught by the teacher with high conceptual and procedural knowledge of rational number ( $G_a$ ) and the class of pupils' taught by the teacher with low conceptual and procedural knowledge of rational number ( $G_b$ ) on the pupils' performance on conceptual items in the Rational number achievement test (RAT). The results presented in Table VII show that t-calculated value 17.09 is greater than the t-critical 1.97, this led to the rejection of the null hypothesis and the alternate hypothesis ( $H_{a2}$ ) was upheld. It indicates that there is a significant difference between the pupils taught by a teacher that has high conceptual knowledge in rational number and those pupils taught by a teacher with low conceptual knowledge of rational numbers.

TABLE VII

T- TEST OF THE DIFFERENCE BETWEEN THE MEAN OF SCORES OF THE INTACT CLASSES ( $G_A$  AND  $G_B$ ) PERFORMANCE ON CONCEPTUAL ITEMS IN RATIONAL NUMBERS ACHIEVEMENT TEST

Intact classes of pupils	$\bar{X}$	S. D	N	df	$t_{cal}$	$t_{crit}$	P	Decision
Ga	33.0714	6.2274	70	138	17.09	1.9799	>0.0001	SD
Gb	10.05	9.0297	70					

Note: SD = Significant Difference

*F. Hypothesis 3*

H0.3. There is no significant difference between the achievement of pupils taught rational numbers by a teacher with high procedural knowledge of rational numbers and the pupils taught by a teacher with low procedural knowledge of rational numbers.

The hypothesis was analyzed using an independent sample t-test to compare the mean achievement of the two intact classes of the teacher, which are the class of pupils' taught by teachers with high conceptual and procedural knowledge of rational number ( $G_a$ ) and the class of pupils' taught by a teacher with low conceptual and procedural knowledge of rational number ( $G_b$ ) on the pupils' achievement on procedural items in the rational number achievement test (RAT). The result presented in Table VIII shows that t-calculated value 12.74 is greater than the t-critical 1.97 ( $t_{12.74} > t_{1.9799}$  at  $p > 0.0001$ ). This led to the rejection of the null hypothesis while the alternative hypothesis was upheld.

Table VIII indicates that there is a significant difference between the pupils taught by a teacher that has high procedural knowledge of Rational number and those pupils' taught by a teacher with low procedural knowledge of rational number.

TABLE VIII

T- TEST OF THE DIFFERENCE BETWEEN THE MEAN SCORES OF THE INTACT CLASSES ( $G_A$  AND  $G_B$ ) PERFORMANCE ON PROCEDURAL ITEMS IN RATIONAL NUMBERS ACHIEVEMENT TEST

Intact classes of pupils	$\bar{X}$	S. D	N	df	$t_{cal}$	$t_{crit}$	P	Decision
Ga	41.8571	8.8306	70	138	12.74	1.9799	>0.0001	SD
Gb	18.7143	12.2524	70					

Note: SD = Significant Difference

*G. Summary of Findings*

1. The primary school teachers' level of conceptual knowledge was found to be average (50%)
2. The primary school teachers' level of procedural knowledge was found to be high (83.33%)
3. The primary school teachers' conceptual and procedural knowledge in rational number had effects on improving pupils' understanding of rational number.
4. The achievement of the pupils in the three intact classes of the teachers was in accordance with the level of their teachers' conceptual and procedural knowledge in a rational number.
5. The achievement of the pupils on conceptual items, in the rational number achievement test (RAT), was according to their teachers' level of conceptual knowledge in a rational number.
6. The achievement of the pupils on procedural items, in the RAT, was in accordance with their teachers' level of procedural knowledge in Rational number.

*H. Discussion of Findings*

Based on the result obtained and presented the following major findings were discussed.

Table I presented the result of the primary school teachers' level of conceptual knowledge which shows that 50% of the primary school teachers have a high level of conceptual knowledge of rational number, and 50% have a low level of conceptual knowledge of rational number. This indicates that the teachers' conceptual knowledge on the rational number is average. This remains a serious problem since all teachers of mathematics at primary school level are expected to have conceptual knowledge of all topics to be taught in their classes. The findings in Table I agree with the demonstrations of [13], who reported that teachers have difficulties with the content of fraction and meaning of division of fraction. Also in another study by [14] it was found that teachers had mastered procedural knowledge more thoroughly than they have mastered conceptual knowledge, teachers perform well on familiar items that required applications of procedural knowledge, they had difficulty in relating different part of their schemata to solve problems that required application or analysis, lack of conceptual knowledge make learning and teaching more difficult and ineffective. This finding is consistent with those of [15] who state the problems in conceptual knowledge given to respondents consisted of mathematical knowledge that should be learned in primary and secondary schools. Teachers also did not utilize or exhibit expertise in conceptual knowledge related to these topics. The

teachers were supposed to master conceptual knowledge in order to avoid teaching misconceptions. It was also found that answers given by teachers demonstrated insufficient conceptual knowledge, which was often illogical and even confusing.

It was found out in the study that teachers have a high level of procedural knowledge as indicated by the result in Table II, 83.33% of the teachers have a high level of procedural knowledge, and 16.66% have a low level of procedural knowledge. This result implied that teachers know how to compute with rational numbers but did not understand the procedure rationale behind the procedure used in solving rational number problems. Also, that teacher with high-level performance can teach the concept of rational number effectively. This finding agrees with that of [16] which states that teachers with high procedural knowledge can solve the problem quickly and effectively because it is some extent automated by automation it means execution and activated.

The study found out that Primary school teacher's conceptual and procedural knowledge had an effect on pupils understanding of rational number. Table III indicates that there was a significant difference between the three intact classes of the teachers ( $G_a$ ,  $G_b$ ,  $G_c$ ), which implies that teachers' knowledge that is use in teaching in the class of the pupils are being impacted (transfer) to the pupil in respective of the teachers' level of knowledge, either high conceptual and procedural knowledge and low conceptual and procedural knowledge of rational number. This result agrees with the findings of [17], [18], stated that teachers' knowledge leads to improve pupils' achievement which reveals a positive effect of knowledge of rational number competence on pupils understanding. [13], [19], [20] pointed out that teachers' effects on pupils' achievement are driven by teachers' ability to understand and use subject matter knowledge to carry out the task of teaching. This agrees with [21] who said that pupils' performances could only be enhanced when teachers have mastery of the contents. Pupils that have low or no deep understanding of rational number are taught by teachers that have no deep understanding of Rational number. These findings agree with [12] who pointed out that teachers' experiences difficulties in representing a rational number as a ratio and as a point of the region to be divided into equal pieces. In another study [22] stated that the obstacle to effective instructions is that "either teachers do not have enough conceptual and procedural knowledge or what they do know is not the right content knowledge."

The result obtained from the analysis of covariance (ANCOVA) in Table VI shows that there was a significant difference between the intact classes of the pupils ( $G_a$   $G_b$   $G_c$ ) taught by the three different teachers ( $G_1$   $G_2$   $G_3$ )  $P < 0.0001$  indicating that pupils' achievement was in accordance with the level of their teachers conceptual and procedural knowledge in rational number. That is the class of the pupils taught by a teacher with high conceptual and procedural knowledge achieved better than any other class. This finding was supported by the views of [23] who maintained that teacher's qualification and knowledge of the subjects count best in

impacting pupils' knowledge. The finding also agrees with that of [24] who reported that teachers who have deep understanding are able to create productively, rational number learning environment.

The second hypothesis tested using an independent sample t-test revealed that there is a significant difference between the pupils taught by a teacher that has high conceptual knowledge of rational number and those pupils taught by a teacher with low conceptual knowledge of rational number. The results presented in Table VII signify that pupils in the class of a teacher with high conceptual knowledge of rational number has deep understanding of conceptual knowledge, while the pupils taught by a teacher with low conceptual knowledge have low understanding or no deep understanding of conceptual knowledge in rational number, which means that primary six pupils understand conceptual knowledge according to their teachers' level of conceptual knowledge in rational number. This agrees with the findings of [25] who pointed out that the pupils need to understand the meaning of concept before learning the related algorithm order to internalize the conceptual knowledge of the topic. In order words the meaning of the principle embedded in the procedure of solving mathematical problems in a logical way, rather than by rote. Therefore, it can be inferred that conceptual knowledge is important in helping to construct an understanding of a topic and also related procedure. Inconsistent with this study [26] asserted that pupils without conceptual knowledge could not understand the meaning of mathematics concept and related procedure.

The third hypothesis tested using an independent sample t-test revealed that there was a significant difference between the pupils taught by a teacher with high Procedural knowledge and those pupils taught by a teacher with low procedural knowledge. This implies that pupils in the class of a teacher that has high procedural knowledge understand better procedural knowledge while the pupils in the class of a teacher with low procedural knowledge have no deep understanding of rational number procedurally. This indicates that a pupil understanding of rational number procedurally is in accordance with their teachers' level of procedural knowledge of rational number. This finding consists of those of [27] which shows that procedural knowledge in rational number teaching is significantly related to pupils' achievement gains in their class.

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