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# Reliability of Slender Reinforced Concrete Columns: Part 1

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***Abstract*—**The main objective of structural design is to ensure safety and functional performance requirements of a structural system for its target reliability levels. In this study, the reliability index for the reinforcement concrete slender columns with rectangular cross section is studied. The variable parameters studied include the loads, the concrete compressive strength, the steel yield strength, the dimensions of concrete cross-section, the reinforcement ratio, and the location of steel placement. Risk analysis program was used to perform the analytical study. The effect of load eccentricity on the reliability index of reinforced concrete slender column was studied and presented. The results of this study indicate that the good quality control improve the performance of slender reinforced columns through increasing the reliability index β.

***Keywords*—**Reliability, reinforced concrete, safety, slender column.

## I. INTRODUCTION

THE importance of reliability theory in the area of civil engineering has received greater attention during the last two decades. This is due to the importance of studying the uncertainties of the existing loads, reinforced concrete material property strength and the member dimension. As a result, the nominal reinforced concrete material property strength and nominal existing loads computed by the engineering designer are different from the actual values. Besides, there are many uncertainties in the models used for determination of effects of loads and resistances of structures.

The reliability of structure is its ability to achieve its design purpose during a specified time. The reliability index is equivalent to the probability of safety. There are many sources of uncertainties existing in structural design such as loads, material strength, concrete dimension, reinforced steel area and error in the model.

Frangopol et al. [1] carried out many investigations for columns and bridges, where the reliability is shown to be affected by load correlation and loading paths. In this work, the reliability analysis is performed by using Monte Carlo simulations, using an appropriate nonlinear stress-strain relationship and a strongly simplified elastic-plastic buckling model. Frangopol et al. [2] showed the reliability of reinforced concrete columns. For short columns, the fiber model is used for generating failure surfaces, strain and stress of both steel and concrete fibers under proportional and sequential loads. Two failure criteria: One based on the collection of peak-load points, and the other based on prescribed maximum concrete strains were presented. Jiang et al. [3] used the Monte Carlo method to analyze the reliability of RC columns with moment and axial force in random correlation. Jiang et al. [3] proposed an improved Monte Carlo method based on the complex characteristics of limit state function. Hong and Zhou [4] proposed an improved approach to deal with the uncertainty in eccentricity. This method introduces an extra random variable correlated with eccentricity, which simplifies the reliability analysis process. For slender columns, failure surfaces are generated using a method proposed by Bazant [5]. The reliability estimation of short and slender columns under random loads is formulated by Monte Carlo simulation in load space. In this space, iso-reliability contours for both deterministic and nondeterministic columns under different load path and load correlations were plotted. It was demonstrated that these factors may have substantial effects on the reliability of reinforced columns. Therefore, the results can be used to support the consideration of load path and load correlation in the development of improved evaluation and design specifications for reinforced concrete columns. The mechanical behavior of concrete is supposed to remain linearly elastic for the whole load history.

Holicky and Vrouwenvelder [6] have analyzed 12 reinforced concrete columns, where the reliability index was studied in terms of time. The study was conducted by applying long and short-term actions, for which the reinforced concrete element behavior was represented by a bending moment-normal force relationship; the FORM method is applied to approximate the failure probability.

A. Mohamed et al. [7] presented a partial safety factor format proposed to ensure a uniform reliability for RC columns. Knowing the high computation cost in nonlinear analysis, the reliability scheme is built by coupling the finite element model with adaptive Response Surface Method (RSM). This is calculated by fitting the ultimate state of structure using quadratic polynomials. To represent the nonlinear behavior of concrete structures at the failure stage, local models based on axial stress-strain relationships are adopted to describe the concrete behavior in tension and in compression. The steel is assumed to follow an elastic-plastic law with constant strain hardening. The parametric analysis of columns allows us to identify the influence of each parameter, and to define the safety factor format. The correctness of the

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proposed format is verified by the mean of RC structural design examples, and compared with the Eurocode2. In this study, a new format for concrete and steel safety factors is proposed for reinforced concrete columns. These factors are related to the design parameters: Cross-section aspect, concrete quality, slenderness ratio, reinforcement ratio and applied loading eccentricity. Comparing with the Eurocode2 specifications, the proposed factors ensure regular reliability for a wide range of design configurations. The reliability analysis takes into account material and geometrical nonlinearities in order to evaluate the ultimate capacity of reinforced concrete structures. The coupling between mechanical and reliability models is carried out by the response surface methods, leading to robust procedures with reduced computation cost.

The long-term reliability of eccentrically loaded slender HSC columns under sustained loads was evaluated by Sofia and Frangopol [8]. Since most of the variables involved in column design (material properties, geometric characteristics, load) are random, probabilistic method is used in the analysis. The effects of the concrete compressive strength, amount of longitudinal steel, load eccentricity, and slenderness ratio on the column reliability were investigated. It was found that all these factors have a considerable impact on the column reliability. The results presented that for small eccentricities the ACI procedure for slender column design is more conservative for HSC than for NSC columns.

Considering the random distribution of eccentricity, Youba Jiang and Weijun Yang [9], proposed a practical reliability analysis approach for RC column based on the theorem of total probability. The main conclusion was that the proposed approach is accurate for reliability analysis of RC column with random eccentricity, and can explicitly give the ratio of the conditional failure probability. Mostafa [10] presented an experimental work aimed to study the effect of changing the percentages of reinforcement and the volume of stirrups in the concrete on the probability of failure of reinforced concrete short columns under axial loads. The experimental program consists of testing of twenty RC column divided into five groups of columns under axial loads. In [11], the probabilistic finite element method was presented to analyze reinforced concrete columns. Also, a simple design formula to predict the resisting capacity of slender RC columns was proposed by Kwak and Kim [12]. Therefore, there is a lake of study on the reliability index of slender reinforced concrete columns. The objective of this study is to introduce a risk analysis technique via the risk analysis program model to identify and assess the effect of load eccentricity on the reliability index of slender reinforced concrete column designed according to the Egyptian code for design and construction of concrete structures (ECP 203).

## II. PARAMETRIC STUDY

In this study, the risk analysis simulation program called @Risk (Version 6.0) is used. This software uses Monte Carlo simulation. The conducted parametric study consists of input variables of six groups of slender reinforced concrete columns as shown in Table I.

TABLE I
STUDIED PARAMETERS OF GROUPS

| Group ID | Concrete dimensions ($mm^2$) | μ% | $F_{cu}$ (MPa) | $P_{ult}$ (KN) | $M_{ult}$ (KN.m) |
|---|---|---|---|---|---|
| 1- A and 1-B | 500×500 | 1.6 | 25 | 0.15 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 2.8 | 30 | 0.20 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 4 | 35 | 0.25 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| 2- A and 2-B | 500×1000 | 1.6 | 25 | 0.15 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 2.8 | 30 | 0.20 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 4 | 35 | 0.25 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| 3-A and 3-B | 500×1500 | 1.6 | 25 | 0.15 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 2.8 | 30 | 0.20 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |
| | | 4 | 35 | 0.25 $P_c$ | 100 |
| | | | | | 200 |
| | | | | | 300 |
| | | | | | 400 |

Fy = 360 MPa for group A and 420 MPa for group B

where μ is the steel reinforcement ratio; fcu is the concrete characteristic strength; fy is the steel reinforcement yield strength; Pc is the column capacity in case of pure compression; pult is the ultimate applied load ( the values of pult are equal to 0.15, 0.2 and 0.25 Pc); e is the loading eccentricity Where e = Mult/pult,

### *A. Statistics of the Basic Variables*

The basic statistics of variables related to column strength and load were taken as values used in [8].

### *B. Interaction Diagram Depicts All Combinations of Axial Loads and Bending Moments*

For each cross section and each case of loading, the interaction diagram was plotted using first principals according to the Egyptian code of reinforced concrete design, ECP203, with some simplifications as shown in Fig. 1 and therefore, the mode failure could be specified.

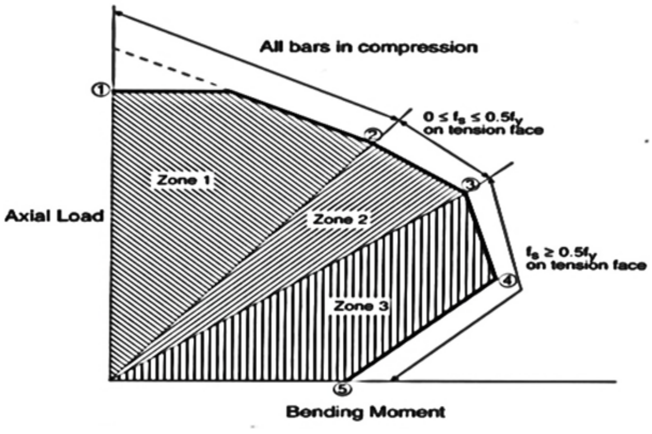

Fig. 1 Modes of failure of a section subjected to eccentric forces

### C. Reliability Formulation (Verification of the Structural Safety)

Structure is safe when the resistance is more than the external loads. The structural safety and the reliability index were estimated using the method described in details in [8]. Fig. 2 expresses that, at given eccentricity e/t, the reliability of a RC column shall be measured as the distance from the point representing load effects to the boundary of the resisting domain, i.e., the interaction diagram. This failure criterion assumes perfect correlation between the axial load and moment acting on the column. Although this formulation does not establish an upper bound of the "true" probability of failure of the column, it is a good measure for comparing different design performances.

## III. RESULTS AND DISCUSSIONS

There are many factors that affect the reliability index of R.C long column such as load eccentricity, the reinforcement ratio and the cross section dimensions. In the present study, the effect of load eccentricity on the reliability index was considered.

The reliability indexes for the studied reinforcement concrete columns are listed in Tables II-X, also, for good comparison, the relation between the reliability index β and eccentricity (e/t) was plotted. Figs. 3 and 4 show the relation between the reliability index β and eccentricity (e/t) for 500*500 mm$^2$ cross section, reinforcement ratio (μ= 1.6%) and yield stress of 360 MPa and 420 MPa respectively for different concrete compressive strength. Figs. 5 and 6 show this comparison for different reinforcement yield stress. The comparison of reinforcement ratio (μ= 2.8%) was shown in Figs. 7 and 8 while Fig. 9 show the comparison of reinforcement ratio (μ= 4.0%). Figs. 10 and 11 show this relation for 500*1000 mm$^2$ and 500*1500 mm$^2$ cross sections respectively. From these results, it can be noticed that:

- Reliability index decreased when the eccentricity (e/t) increased but there is a slight effect on reliability index value when increasing the eccentricity in bigger concrete cross sections.
- Increasing the concrete compression strength from 25MPa to 30MPa will cause an increase in reliability index by range of (20% to 35%) in most cases.
- Increasing the concrete compression strength from 30MPa to 35MPa will cause an increase in reliability index by range of (10% to 25%) in most cases.
- Increasing the reinforcement yield stress from 360 MPa to 420 MPa will cause an increase in reliability index by range of (10% to 30%) for group 1 and range of (4% to 6%) for group 2, but group 3 range was (1% to 5%).

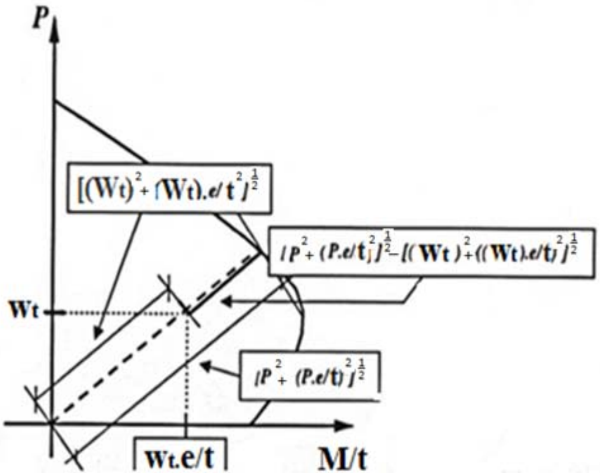

Fig. 2 Graphical representation of the failure criterion [8]

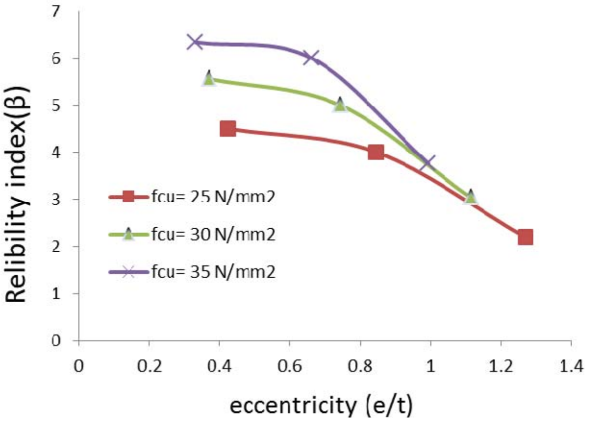

Fig. 3 Relation between reliability index (β) and eccentricity (e/t) for 500*500 mm$^2$ Concrete cross section, μ = 1.6% and fy=360 MPa

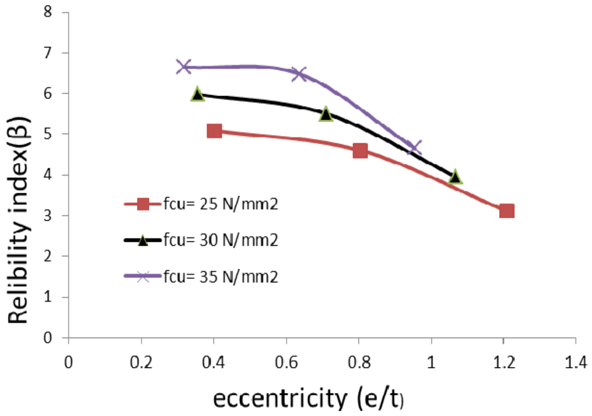

Fig. 4 Relation between reliability index (β) and eccentricity (e/t) for 500*500 $mm^2$ Concrete cross section, μ = 1.6% and fy=420 MPa

TABLE II
RELIABILITY INDEX (Β) FOR 500*500 MM R.C. CROSS SECTION, μ= 1.6%

| *Fcu $N/mm^2$* | *fy $N/mm^2$* | *P= 0.15 Pult KN* | | *P= 0.20 Pult KN* | | *P= 0.25 Pult KN* | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | β |
| 25 | 360 | 0.422 | 4.50 | 0.31 | 4.48 | 0.25 | 4.12 |
| | | 0.84 | 4.00 | 0.63 | 3.50 | 0.50 | 3.27 |
| | | 1.26 | 2.19 | 0.95 | 1.80 | 0.76 | 0.96 |
| | | --- | ---- | --- | ---- | --- | ---- |
| 30 | | 0.37 | 5.55 | 0.27 | 5.19 | 0.22 | 4.78 |
| | | 0.74 | 5.00 | 0.55 | 4.73 | 0.44 | 3.96 |
| | | 1.11 | 3.03 | 0.83 | 2.46 | 0.66 | 1.42 |
| | | --- | ---- | --- | ---- | --- | ---- |
| 35 | | 0.33 | 6.34 | 0.24 | 5.89 | 0.19 | 5.45 |
| | | 0.66 | 6.00 | 0.49 | 5.35 | 0.39 | 4.61 |
| | | 0.99 | 3.76 | 0.74 | 3 | 0.59 | 1.85 |
| | | --- | ---- | --- | ---- | --- | ---- |
| 25 | 420 | 0.40 | 5.08 | 0.30 | 4.72 | 0.24 | 4.33 |
| | | 0.80 | 4.60 | 0.60 | 4.00 | 0.48 | 3.57 |
| | | 1.20 | 3.11 | 0.90 | 2.76 | 0.72 | 2.63 |
| | | --- | ---- | --- | ---- | --- | ---- |
| 30 | | 0.35 | 5.97 | 0.26 | 5.41 | 0.21 | 5.04 |
| | | 0.71 | 5.50 | 0.53 | 4.70 | 0.42 | 4.20 |
| | | 1.06 | 3.96 | 0.79 | 3.49 | 0.63 | 3.07 |
| | | --- | ---- | --- | ---- | --- | ---- |
| 35 | | 0.31 | 6.64 | 0.23 | 6.14 | 0.19 | 5.60 |
| | | 0.63 | 6.47 | 0.47 | 5.45 | 0.38 | 4.92 |
| | | 0.95 | 4.65 | 0.71 | 4.15 | 0.57 | 3.55 |
| | | --- | ---- | --- | ---- | --- | ---- |

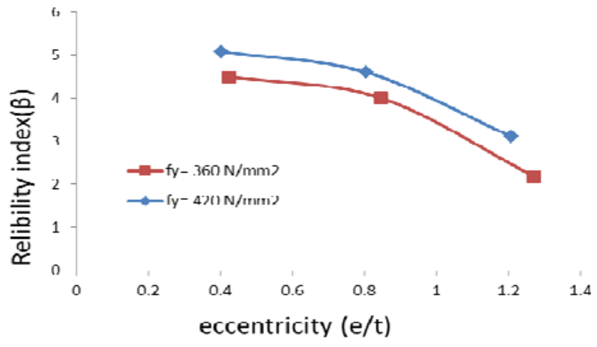

Fig. 5 Relation between reliability index (β) and eccentricity (e/t) for 500*500 $mm^2$ Concrete cross section, μ = 1.6% and $f_{cu}$=25 MPa

TABLE III
RELIABILITY INDEX (Β) FOR 500*500 MM R.C. CROSS SECTION, μ= 2.8%

| Fcu $N/mm^2$ | fy $N/mm^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | β |
| 25 | 360 | 0.34 | 5.30 | 0.25 | 5.00 | 0.20 | 4.80 |
| | | 0.68 | 5.00 | 0.51 | 4.74 | 0.41 | 4.27 |
| | | 1.03 | 4.00 | 0.77 | 3.89 | 0.61 | 3.37 |
| | | 1.37 | 2.95 | 1.03 | 2.66 | 0.82 | 2.65 |
| 30 | | 0.30 | 6.00 | 0.23 | 5.96 | 0.18 | 5.46 |
| | | 0.61 | 5.60 | 0.46 | 5.40 | 0.37 | 4.95 |
| | | 0.92 | 5.00 | 0.69 | 4.40 | 0.55 | 4.17 |
| | | 1.23 | 3.52 | 0.92 | 3.50 | 0.74 | 3.24 |
| 35 | | 0.28 | 7.27 | 0.21 | 6.62 | 0.16 | 6.10 |
| | | 0.56 | 6.69 | 0.42 | 6.05 | 0.33 | 5.58 |
| | | 0.84 | 5.85 | 0.63 | 5.27 | 0.50 | 4.78 |
| | | 1.12 | 4.50 | 0.84 | 4.15 | 0.67 | 3.89 |
| 25 | 420 | 0.32 | 6.28 | 0.24 | 5.67 | 0.19 | 5.33 |
| | | 0.64 | 5.80 | 0.48 | 5.50 | 0.38 | 4.69 |
| | | 0.96 | 5.00 | 0.72 | 4.80 | 0.57 | 3.94 |
| | | 1.28 | 4.00 | 0.96 | 4.00 | 0.76 | 3.00 |
| 30 | | 0.29 | 6.88 | 0.21 | 6.12 | 0.17 | 5.89 |
| | | 0.58 | 6.41 | 0.43 | 5.71 | 0.34 | 5.35 |
| | | 0.87 | 5.74 | 0.65 | 4.60 | 0.52 | 4.63 |
| | | 1.16 | 4.82 | 0.87 | 4.13 | 0.69 | 3.69 |
| 35 | | 0.26 | 7.68 | 0.19 | 7.05 | 0.15 | 6.58 |
| | | 0.52 | 7.22 | 0.39 | 6.57 | 0.31 | 6.01 |
| | | 0.79 | 6.72 | 0.59 | 5.79 | 0.47 | 5.31 |
| | | 1.05 | 5.80 | 0.79 | 5.00 | 0.63 | 4.32 |

TABLE IV
RELIABILITY INDEX (Β) FOR 500*500 MM R.C. CROSS SECTION, μ= 4.0%

| Fcu $N/mm^2$ | fy $N/mm^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | B |
| 25 | 360 | 0.28 | 6.56 | 0.21 | 5.93 | 0.17 | 5.63 |
| | | 0.57 | 5.86 | 0.43 | 5.52 | 0.34 | 5.02 |
| | | 0.86 | 5.19 | 0.65 | 4.74 | 0.52 | 4.20 |
| | | 1.15 | 4.50 | 0.86 | 4.00 | 0.69 | 3.51 |
| 30 | | 0.26 | 7.28 | 0.19 | 6.80 | 0.15 | 6.32 |
| | | 0.52 | 6.52 | 0.39 | 6.12 | 0.31 | 5.53 |
| | | 0.79 | 6.09 | 0.59 | 5.43 | 0.47 | 5.03 |
| | | 1.05 | 5.12 | 0.79 | 4.67 | 0.63 | 4.19 |
| 35 | | 0.24 | 7.67 | 0.18 | 7.15 | 0.14 | 6.84 |
| | | 0.48 | 7.36 | 0.36 | 6.75 | 0.29 | 6.25 |
| | | 0.73 | 6.80 | 0.54 | 6.20 | 0.43 | 5.49 |
| | | 0.97 | 5.66 | 0.73 | 5.26 | 0.58 | 4.80 |
| 25 | 420 | 0.26 | 6.91 | 0.19 | 6.59 | 0.15 | 6.14 |
| | | 0.53 | 6.67 | 0.39 | 6.08 | 0.31 | 5.64 |
| | | 0.79 | 5.83 | 0.59 | 5.34 | 0.47 | 4.86 |
| | | 1.06 | 5.22 | 0.79 | 4.66 | 0.63 | 4.00 |
| 30 | | 0.24 | 7.81 | 0.18 | 7.21 | 0.14 | 6.91 |
| | | 0.49 | 7.39 | 0.36 | 6.72 | 0.29 | 6.17 |
| | | 0.73 | 6.90 | 0.55 | 6.08 | 0.44 | 5.50 |
| | | 0.98 | 5.90 | 0.73 | 5.26 | 0.58 | 4.88 |
| 35 | | 0.22 | 8.33 | 0.17 | 7.78 | 0.13 | 7.39 |
| | | 0.45 | 8.12 | 0.34 | 7.41 | 0.27 | 6.76 |
| | | 0.68 | 7.50 | 0.51 | 6.66 | 0.40 | 6.19 |
| | | 0.90 | 6.55 | 0.68 | 5.97 | 0.54 | 5.38 |

TABLE V
RELIABILITY INDEX (Β) FOR 500*1000 MM R.C. CROSS SECTION, μ= 1.6%

| Fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | Β | e/t | β | e/t | β |
| 25 | 360 | 0.10 | 5.22 | 0.07 | 4.94 | 0.06 | 4.65 |
| | | 0.21 | 5.12 | 0.15 | 4.75 | 0.12 | 4.49 |
| | | 0.31 | 5.11 | 0.23 | 4.75 | 0.19 | 4.36 |
| | | 0.42 | 4.99 | 0.31 | 4.55 | 0.25 | 4.21 |
| 30 | | 0.09 | 5.94 | 0.06 | 5.58 | 0.05 | 5.31 |
| | | 0.18 | 5.84 | 0.13 | 5.45 | 0.11 | 5.12 |
| | | 0.27 | 5.83 | 0.20 | 5.42 | 0.16 | 4.94 |
| | | 0.37 | 5.78 | 0.27 | 5.25 | 0.22 | 4.90 |
| 35 | | 0.08 | 6.73 | 0.06 | 6.24 | 0.04 | 5.99 |
| | | 0.16 | 6.39 | 0.12 | 6.19 | 0.09 | 5.72 |
| | | 0.24 | 6.30 | 0.18 | 6.02 | 0.14 | 5.62 |
| | | 0.33 | 6.25 | 0.24 | 5.98 | 0.19 | 5.45 |
| 25 | 420 | 0.10 | 5.47 | 0.07 | 5.20 | 0.06 | 4.88 |
| | | 0.20 | 5.24 | 0.15 | 4.99 | 0.12 | 4.72 |
| | | 0.30 | 5.18 | 0.22 | 4.98 | 0.18 | 4.55 |
| | | 0.40 | 5.12 | 0.30 | 4.88 | 0.24 | 4.46 |
| 30 | | 0.08 | 6.22 | 0.06 | 5.97 | 0.05 | 5.49 |
| | | 0.17 | 6.21 | 0.13 | 5.73 | 0.10 | 5.38 |
| | | 0.26 | 6.15 | 0.19 | 5.68 | 0.15 | 5.22 |
| | | 0.35 | 6.06 | 0.26 | 5.54 | 0.21 | 5.09 |
| 35 | | 0.07 | 6.94 | 0.05 | 6.63 | 0.04 | 6.19 |
| | | 0.15 | 6.61 | 0.11 | 6.43 | 0.09 | 6.03 |
| | | 0.23 | 6.50 | 0.17 | 6.25 | 0.14 | 5.86 |
| | | 0.31 | 6.39 | 0.23 | 6.11 | 0.19 | 5.73 |

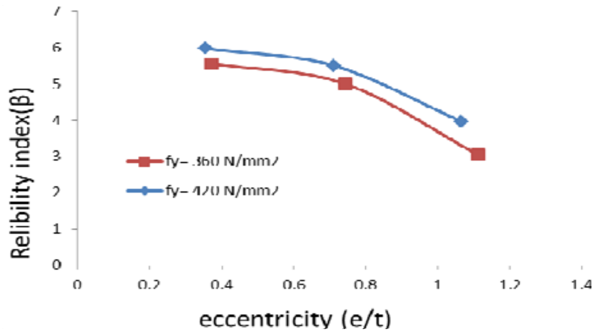

Fig. 6 Relation between reliability index (β) and eccentricity (e/t) for 500*500 mm$^2$ Concrete cross section, μ = 1.6% and $f_{cu}$=30 MPa

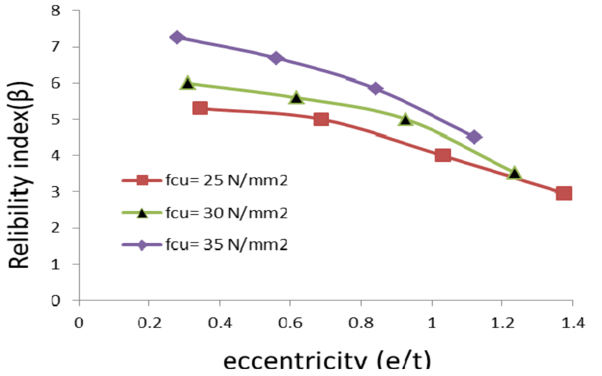

Fig. 7 Relation between reliability index (β) and eccentricity (e/t) for 500*500 mm$^2$ Concrete cross section, μ = 2.80% and fy=360 MPa

TABLE VI
RELIABILITY INDEX (Β) FOR 500*1000 MM R.C. CROSS SECTION, μ= 2.8%

| Fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | B |
| 25 | 360 | 0.08 | 6.15 | 0.06 | 5.87 | 0.05 | 5.51 |
| | | 0.17 | 5.95 | 0.12 | 5.86 | 0.10 | 5.40 |
| | | 0.25 | 5.98 | 0.19 | 5.53 | 0.15 | 5.32 |
| | | 0.34 | 5.87 | 0.25 | 5.42 | 0.20 | 4.96 |
| 30 | | 0.07 | 6.95 | 0.05 | 6.47 | 0.04 | 6.19 |
| | | 0.15 | 6.81 | 0.11 | 6.41 | 0.09 | 5.93 |
| | | 0.23 | 6.70 | 0.17 | 6.19 | 0.13 | 5.81 |
| | | 0.30 | 6.70 | 0.23 | 6.08 | 0.18 | 5.61 |
| 35 | | 0.07 | 7.63 | 0.05 | 7.28 | 0.04 | 6.84 |
| | | 0.14 | 7.54 | 0.10 | 7.04 | 0.08 | 6.67 |
| | | 0.21 | 7.56 | 0.15 | 6.93 | 0.12 | 6.43 |
| | | 0.28 | 7.43 | 0.21 | 6.74 | 0.16 | 6.29 |
| 25 | 420 | 0.08 | 6.71 | 0.06 | 6.35 | 0.04 | 5.94 |
| | | 0.16 | 6.57 | 0.12 | 6.04 | 0.09 | 5.86 |
| | | 0.24 | 6.17 | 0.18 | 5.96 | 0.14 | 5.71 |
| | | 0.32 | 6.09 | 0.24 | 5.81 | 0.19 | 5.42 |
| 30 | | 0.07 | 7.47 | 0.05 | 6.98 | 0.04 | 6.61 |
| | | 0.14 | 7.22 | 0.10 | 6.79 | 0.08 | 6.51 |
| | | 0.21 | 7.14 | 0.16 | 6.56 | 0.13 | 6.22 |
| | | 0.29 | 7.06 | 0.21 | 6.53 | 0.17 | 6.09 |
| 35 | | 0.06 | 8.01 | 0.04 | 7.77 | 0.03 | 7.10 |
| | | 0.13 | 8.00 | 0.09 | 7.63 | 0.07 | 7.12 |
| | | 0.19 | 7.83 | 0.14 | 7.39 | 0.11 | 6.94 |
| | | 0.26 | 7.72s | 0.19 | 7.23 | 0.15 | 6.77 |

TABLE VII
RELIABILITY INDEX (Β) FOR 500*1000 MM R.C. CROSS SECTION, μ= 4%

| Fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | B |
| 25 | 360 | 0.07 | 3.44 | 0.05 | 3.02 | 0.04 | 2.59 |
| | | 0.14 | 3.28 | 0.10 | 2.84 | 0.08 | 2.40 |
| | | 0.21 | 3.09 | 0.16 | 2.68 | 0.13 | 2.22 |
| | | 0.28 | 2.85 | 0.21 | 2.44 | 0.17 | 2.04 |
| 30 | | 0.06 | 4.21 | 0.04 | 3.74 | 0.03 | 3.27 |
| | | 0.13 | 4.02 | 0.09 | 3.55 | 0.07 | 3.11 |
| | | 0.19 | 3.86 | 0.14 | 3.40 | 0.11 | 2.91 |
| | | 0.26 | 3.62 | 0.19 | 3.16 | 0.15 | 2.73 |
| 35 | | 0.06 | 4.97 | 0.045 | 4.46 | 0.03 | 3.95 |
| | | 0.12 | 4.78 | 0.091 | 4.29 | 0.07 | 3.77 |
| | | 0.18 | 4.64 | 0.13 | 4.09 | 0.10 | 3.58 |
| | | 0.24 | 4.40 | 0.18 | 3.90 | 0.14 | 3.44 |
| 25 | 420 | 0.06 | 3.42 | 0.04 | 2.97 | 0.03 | 2.50 |
| | | 0.13 | 3.22 | 0.09 | 2.77 | 0.07 | 2.31 |
| | | 0.19 | 3.08 | 0.14 | 2.62 | 0.11 | 2.12 |
| | | 0.26 | 2.83 | 0.19 | 2.38 | 0.15 | 1.95 |
| 30 | | 0.06 | 4.17 | 0.04 | 3.68 | 0.03 | 3.18 |
| | | 0.12 | 3.99 | 0.09 | 3.49 | 0.07 | 2.99 |
| | | 0.18 | 3.83 | 0.13 | 3.30 | 0.11 | 2.80 |
| | | 0.24 | 3.61 | 0.18 | 3.13 | 0.14 | 2.65 |
| 35 | | 0.05 | 4.94 | 0.04 | 4.40 | 0.03 | 3.86 |
| | | 0.11 | 4.74 | 0.08 | 4.20 | 0.06 | 3.67 |
| | | 0.17 | 4.63 | 0.12 | 4.04 | 0.10 | 3.48 |
| | | 0.22 | 4.39 | 0.17 | 3.87 | 0.13 | 3.30 |

TABLE VIII
RELIABILITY INDEX (Β) FOR 500*1500 MM R.C. CROSS SECTION, μ= 1.6%

| Fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | β |
| 25 | 360 | 0.04 | 5.35 | 0.03 | 5.00 | 0.02 | 4.75 |
| | | 0.09 | 5.23 | 0.07 | 4.98 | 0.056 | 4.66 |
| | | 0.14 | 5.18 | 0.10 | 4.86 | 0.08 | 4.67 |
| | | 0.18 | 5.14 | 0.14 | 4.84 | 0.11 | 4.49 |
| 30 | | 0.04 | 6.03 | 0.03 | 5.70 | 0.02 | 5.47 |
| | | 0.08 | 5.98 | 0.06 | 5.65 | 0.04 | 5.30 |
| | | 0.12 | 5.95 | 0.09 | 5.57 | 0.07 | 5.25 |
| | | 0.16 | 5.85 | 0.12 | 5.51 | 0.09 | 5.21 |
| 35 | | 0.03 | 6.80 | 0.02 | 6.47 | 0.02 | 6.07 |
| | | 0.07 | 6.72 | 0.05 | 6.40 | 0.04 | 5.98 |
| | | 0.11 | 6.64 | 0.08 | 6.27 | 0.06 | 5.92 |
| | | 0.14 | 6.43 | 0.11 | 6.13 | 0.08 | 5.83 |
| 25 | 420 | 0.04 | 5.61 | 0.03 | 5.31 | 0.02 | 5.03 |
| | | 0.08 | 5.51 | 0.06 | 5.21 | 0.05 | 4.89 |
| | | 0.13 | 5.47 | 0.10 | 5.16 | 0.08 | 4.84 |
| | | 0.17 | 5.25 | 0.13 | 5.14 | 0.10 | 4.73 |
| 30 | | 0.03 | 6.37 | 0.02 | 5.99 | 0.02 | 5.68 |
| | | 0.07 | 6.19 | 0.05 | 5.92 | 0.04 | 5.57 |
| | | 0.11 | 6.18 | 0.08 | 5.84 | 0.07 | 5.56 |
| | | 0.15 | 6.07 | 0.11 | 5.79 | 0.09 | 5.50 |
| 35 | | 0.03 | 7.12 | 0.02 | 6.77 | 0.02 | 6.30 |
| | | 0.07 | 7.05 | 0.05 | 6.62 | 0.04 | 6.20 |
| | | 0.10 | 6.92 | 0.07 | 6.56 | 0.06 | 6.22 |
| | | 0.14 | 6.86 | 0.10 | 6.46 | 0.084 | 6.13 |

TABLE IX
RELIABILITY INDEX (Β) FOR 500*1500 MM R.C. CROSS SECTION, μ= 2.8%

| Fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | B | e/t | β |
| 25 | 360 | 0.03 | 6.33 | 0.02 | 6.01 | 0.02 | 5.57 |
| | | 0.07 | 6.20 | 0.05 | 5.99 | 0.04 | 5.59 |
| | | 0.11 | 6.15 | 0.08 | 5.77 | 0.06 | 5.52 |
| | | 0.15 | 6.11 | 0.11 | 5.69 | 0.09 | 5.29 |
| 30 | | 0.03 | 7.01 | 0.02 | 6.62 | 0.02 | 6.25 |
| | | 0.06 | 6.99 | 0.05 | 6.55 | 0.04 | 6.19 |
| | | 0.10 | 6.97 | 0.07 | 6.44 | 0.06 | 6.14 |
| | | 0.13 | 6.85 | 0.10 | 6.33 | 0.08 | 6.04 |
| 35 | | 0.03 | 7.80 | 0.02 | 7.42 | 0.01 | 6.85 |
| | | 0.06 | 7.63 | 0.04 | 7.37 | 0.03 | 6.79 |
| | | 0.09 | 7.59 | 0.07 | 7.37 | 0.05 | 6.75 |
| | | 0.12 | 7.53 | 0.09 | 7.22 | 0.07 | 6.64 |
| 25 | 420 | 0.03 | 6.75 | 0.02 | 6.38 | 0.02 | 6.13 |
| | | 0.07 | 6.67 | 0.05 | 6.27 | 0.04 | 5.98 |
| | | 0.10 | 6.66 | 0.08 | 6.22 | 0.06 | 5.93 |
| | | 0.14 | 6.50 | 0.10 | 6.17 | 0.08 | 5.89 |
| 30 | | 0.03 | 7.40 | 0.02 | 7.10 | 0.01 | 6.69 |
| | | 0.06 | 7.35 | 0.04 | 7.01 | 0.03 | 6.71 |
| | | 0.09 | 7.34 | 0.07 | 6.91 | 0.05 | 6.52 |
| | | 0.12 | 7.10 | 0.09 | 6.86 | 0.07 | 6.29 |
| 35 | | 0.02 | 8.21 | 0.02 | 7.94 | 0.01 | 7.32 |
| | | 0.05 | 8.09 | 0.04 | 7.67 | 0.03 | 7.29 |
| | | 0.08 | 8.07 | 0.06 | 7.54 | 0.05 | 7.28 |
| | | 0.11 | 7.95 | 0.08 | 7.49 | 0.07 | 7.21 |

TABLE X
RELIABILITY INDEX (Β) FOR 500*1500 MM R.C. CROSS SECTION, μ= 4%

| fcu N/mm$^2$ | fy N/mm$^2$ | P= 0.15 Pult KN | | P= 0.20 Pult KN | | P= 0.25 Pult KN | |
|---|---|---|---|---|---|---|---|
| | | e/t | β | e/t | β | e/t | B |
| 25 | 360 | 0.03 | 7.25 | 0.02 | 7.25 | 0.01 | 6.30 |
| | | 0.06 | 7.15 | 0.04 | 7.15 | 0.03 | 6.22 |
| | | 0.09 | 7.02 | 0.07 | 7.02 | 0.05 | 6.16 |
| | | 0.12 | 6.93 | 0.09 | 6.93 | 0.07 | 6.07 |
| 30 | | 0.02 | 7.71 | 0.02 | 7.71 | 0.01 | 7.09 |
| | | 0.05 | 7.70 | 0.04 | 7.70 | 0.03 | 7.02 |
| | | 0.08 | 7.63 | 0.06 | 7.63 | 0.05 | 6.68 |
| | | 0.11 | 7.58 | 0.08 | 7.58 | 0.07 | 6.62 |
| 35 | | 0.02 | 8.58 | 0.02 | 8.58 | 0.01 | 7.59 |
| | | 0.05 | 8.31 | 0.04 | 8.31 | 0.03 | 7.46 |
| | | 0.08 | 8.09 | 0.06 | 8.09 | 0.04 | 7.37 |
| | | 0.10 | 8.05 | 0.08 | 8.05 | 0.06 | 7.24 |
| 25 | 420 | 0.02 | 8.24 | 0.02 | 8.24 | 0.01 | 6.95 |
| | | 0.05 | 7.68 | 0.04 | 7.68 | 0.03 | 6.94 |
| | | 0.08 | 7.66 | 0.06 | 7.66 | 0.05 | 6.82 |
| | | 0.11 | 7.51 | 0.08 | 7.51 | 0.07 | 6.67 |
| 30 | | 0.02 | 8.46 | 0.02 | 8.46 | 0.01 | 7.45 |
| | | 0.05 | 8.42 | 0.04 | 8.42 | 0.03 | 7.31 |
| | | 0.08 | 8.25 | 0.06 | 8.25 | 0.04 | 7.25 |
| | | 0.10 | 8.10 | 0.08 | 8.10 | 0.06 | 7.22 |
| 35 | | 0.02 | 9.27 | 0.01 | 9.27 | 0.01 | 8.20 |
| | | 0.05 | 8.87 | 0.03 | 8.87 | 0.03 | 8.09 |
| | | 0.07 | 8.86 | 0.056 | 8.86 | 0.04 | 7.89 |
| | | 0.10 | 8.54 | 0.07 | 8.54 | 0.06 | 7.75 |

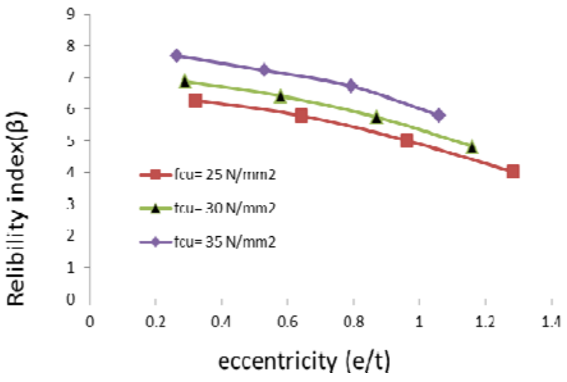

Fig. 8 Relation between reliability index (β) and eccentricity (e/t) for 500*500 mm$^2$ Concrete cross section, μ = 2.80% and fy=420 MPa

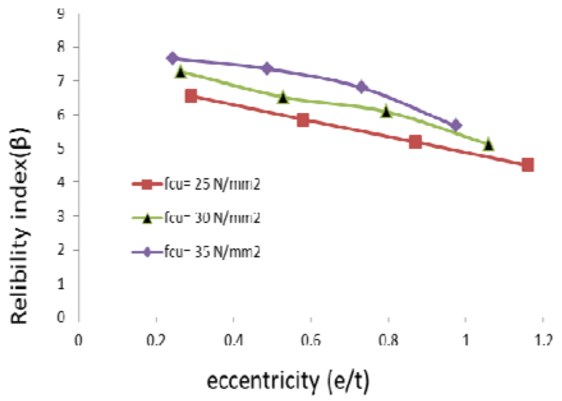

Fig. 9 Relation between reliability index (β) and eccentricity (e/t) for 500*500 mm$^2$ Concrete cross section, μ = 4.0% and fy=360 Mpa

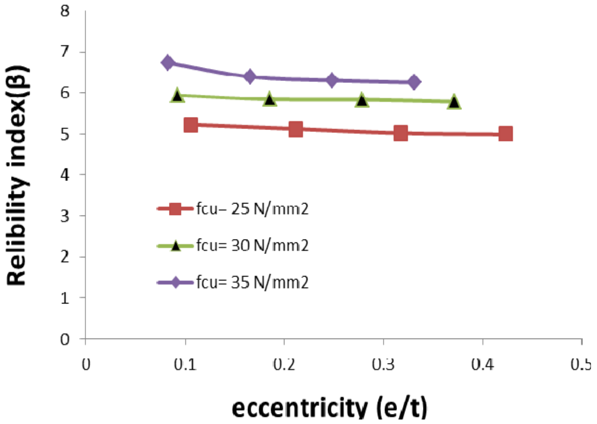

Fig. 10 Relation between reliability index (β) and eccentricity (e/t) for 500*1000 mm$^2$ Concrete cross section, μ = 1.6% and fy=360 MPa

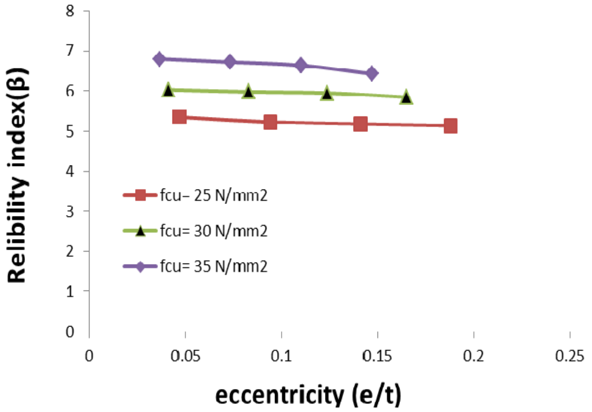

Fig. 11 Relation between reliability index (β) and eccentricity (e/t) for 500*1500 mm$^2$ Concrete cross section, μ = 1.6% and fy=360 MPa

## IV. CONCLUSIONS

The following conclusions could be obtained from the work done in this study:

- Increasing both the loads eccentricity and the column slenderness ratio (λ), causes a decrease of the reliability index. The reliability index (β) will be slightly decreased in case of higher column thicknesses.
- Increasing the concrete compressive strength (fcu) causes an increase in reliability index (β). This effect was significant in case of columns with small reinforcement ratio.
- Increasing the yield steel strength (fy) causes an increase in reliability index (β). This effect was significant in case of columns with small reinforcement ratio.
- The results of this study indicate that the good quality control has significant effects on decreasing the coefficient of variation for the concrete compressive strength and yield steel strength which in turn improve the performance of slender reinforced columns through increasing the reliability index β.

## REFERENCES

[1] Frangopol, D.M., Ide, Y. and Iwaki, I., “Effects of Loads Path and Load Correlation on The Reliability of Concrete Columns”, Probabilistic Mechanics and Structural Reliability. Proceeding of the seventh specialty conference, Worcester, Massachusetts, USA, pp. 9-206, 1996.

[2] Frangopol, D.M., Ide, Y., Spacone, E. and Iwaki, I., “A New Look at Reliability of Reinforced Concrete Columns”, Structural Safety, Vol. 18, No.2/3, pp. 123-150, 1996.

[3] Jiang, Y., Yang, Y. and Yang, W., “Reliability Analysis Based on Random Correlative Characteristics Between Moment and Axial Force for RC Member Subjected to Eccentric Compression”, J. Building Struct., Vol. 32, No.8, pp. 106-112, 2011.

[4] Hong, H.P. and Zhou, W., “Reliability Evaluations of RC Columns”, J. Building Struct., Vol. 125, No. 7, PP.784-790, 1999.

[5] Bazant, E.T, “The Reliability Estimation of Short and Slender Columns Under Random Loads”, ACI , Vol. 88, PP. 391-401, 1991.

[6] Holicky, M. and Vrouwenvelder, T., “Time Variant Reliability of a Reinforced Concrete Column”, In: Advances in Safety and Reliability. Proceeding of the ESREL 97 International Conference on Safety and Reliability, Lisbon, Portugal, PP. 14-1307, 1997.

[7] Mohamed, A., Soares, R. and Venturini, W.S., “ Partial Safety Factors for Homogeneous Reliability of Non liner Reinforcement Concrete Columns”, Structural Safely, Vol. 23, PP. 137-156, 2001.

[8] Sofia, M.C. and Frangopol, D.M., “Safety Evaluation of Slender High-Strength Concrete Columns under Sustained Loads”, Computers and structures, Vol. 81, PP.1475-1486, 2003.

[9] Youbo, J. and Weijun. Y., “An Approach Based on Theorem of Total Probability for Reliability Analysis of RC Columns with Random Eccentricity”, Structural Safely, Vol. 41, PP. 37-46, 2013.

[10] Mostafa, A.M.H., “Probabilistic Analysis of Reinforced Concrete Short Columns”. A Thesis of Master Degree in Structural Engineering. Ain Shams University, 1989.

[11] Jose, M.A., “Probbilistic analysis of reinforced concrete columns”, Advances in Engineering Software, Vol. 32, PP. 871-879, 2001.

[12] Kwak, H. and Kim, J., “Nonlinear Behavior of Slender RC Columns (2). Introduction of Design Formula”, Construction and Building Materials, Vol. 20, PP. 538-553, 2006.