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Modeling and Simulations of Surface Plasmon Waveguide Structures

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***Abstract*—**This paper presents an investigation of the fabrication of the optical devices in terms of their characteristics based on the use of the electromagnetic waves. Planar waveguides are used to examine the field modes (bound modes) and the parameters required for this structure. The modifications are conducted on surface plasmons based waveguides. Simple symmetric dielectric slab structure is used and analyzed in terms of transverse electric mode (TE-Mode) and transverse magnetic mode (TM-Mode. The paper presents mathematical and numerical solutions for solving simple symmetric plasmons and provides simulations of surface plasmons for field confinement. Asymmetric TM-mode calculations for dielectric surface plasmons are also provided.

***Keywords*—**Surface plasmons, optical waveguides, semiconductor lasers, refractive index, slab dialectical.

I. INTRODUCTION

OPTICAL waveguides are the most common structures which are used to analyze the field component and introduce the ways of confining the optical field to the desired direction. The dielectric waveguides are the simple case used to demonstrate the behavior of the field and the mechanisms of achieving the bound modes. To enhance the modification, surface plasmons (SPs) have recently emerged providing the metals with a sophisticated characterization to be fabricated to nanometer scale. The constitution of SP is based on the interacting of the optical waves with the surface charge (free electrons). Therefore, SPs assist in concentrating the optical waves which lead to a considerable enhancement in the field. This improvement can be used to manipulate the interactions of the optical waves. For much smaller metallic structures compared to the optical wavelength, the vitality for a hug signal improvement can be achieved.

Majority of optical components depends on dielectrics for high speed as well as bandwidth. SP is one of the modern candidates in which the electromagnetic waves radiate along the surface of the metal-cladding interface. This mechanism has introduced a vast development of the photonic devices, which results in the miniaturization of the optical circuits with much smaller dimensions. To clarify more, when the free electrons of the metal have the interaction with the optical waves, this will lead to the field to trap on the metal surface [1].

SPs were massively introduced in surface science field following the pioneering work of Ritchie in the 1950s [2]. SPs are the waves that travel along the conductor surface, often a metal. These are crucially the optical waves that are trapped on the surface because of their interaction with the free electrons of the conductor. In this case of interaction, the response of free electrons is obtained collectively by oscillating in resonance with the wave of light. The resonant interaction between the surface charge oscillation and the electromagnetic field of the light constitutes the SP and gives rise to its unique properties. Since SP was considered, there have been improvements in the capability of guiding electromagnetic waves at optical frequencies along the metal-dielectric interfaces [2], [3].

For optical frequencies, particular metals such as gold [Au] and silver [Ag] represent negative relative electrical permittivity of the metal, meant $\varepsilon_m = -j|\varepsilon|$. This in turn will result in having imaginary refractive index (n_m) in a negative direction, since $\varepsilon_m = n_m^2 = -|\varepsilon|^2 < 0$.

SP wave mode can be excited as the field decays exponentially, and therefore this mode is very tightly confined to the interface and forms more of a basis for more compact semiconductor lasers. SP is basically a two dimensional excitation where the field exponentially decays with distance from the surface. It can be used as a means of enhancing the performance and minimizing the photonic network size. The use of SP has allowed the resonator size to be minimized, which is required for achieving a nano-laser [4].

The dimensions of SP allow for the possibility of localizing electromagnetic radiation under the diffraction limit. However, absorption loss can have a serious impact on the operation of the device, due to waves travelling along the surface of the metal. To tackle this issue, some studies were conducted regarding this interesting problem and showed that the gain of the medium, which resides in close proximity, can compensate the loss of the metal [5]. Many experiments have been conducted in this area, the first notable one was to use pump-probe configuration on the surface, between a silver (Ag) film and an optically pumped polymer dye, where SPs loss deactivation was demonstrated [5].

The optical surface plasmon performance is basically restricted by the absorption loss in metal, which is addressed as a reason for decreasing in the factor of quality of SPs and limiting the length of the propagation. To resolve this matter, it is useful to consider the optical gain to a dielectric adjacent to metal. It has been evidenced that, almost 420 cm^{-1}, representing gain value, was achieved, which is suitable to compensate the loss of substantial SP [6]. The same result was achieved using gold. Fig. 1 demonstrates SPs at the interface

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between dialectic and metal materials and how the electromagnetic waves oscillate along the interface [7].

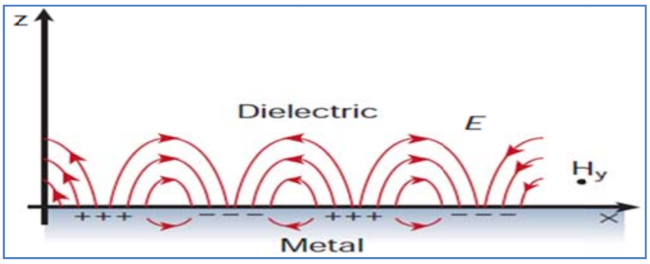

Fig. 1 SPs at the interface between a metal and a dielectric material

II. PROPAGATION OF SPS

When the light has been changed into SPs mode on a metal surface, it can propagate, while will experience a gradual attenuation resulting in losses arising from absorption in the metal. This attenuation based on the dielectric function of the metal at the oscillating frequency of SP [8]. Therefore, as the dielectric function of the metal is complex containing real and imaginary part, it will play a major role for evaluating the performance in terms of the amount of losses and propagation length. In visible spectrum, silver is known as a metal which has the lowest losses compared to the other metals. It has propagation range typically from 10 to 100μm [8].

Previously, the absorption by using metal was considered as a serious issue which SPs were not introduced for being capable working successfully for photonic components because its propagation length was very small compared to the optical structure components that period. Such structures have recently been experiencing a considerable change in their characteristics to be significantly smaller than the length of propagation. These improvements have shifted the attention to integrate several SPs based structures into smaller photonic circuits [8].

III. MATHEMATICAL METHOD OF SOLVING SIMPLE SYMMETRIC SPS

The symmetric case structure is shown in Fig. 2 and can be analyzed as follows:

A. Solving Core Region

In this case the consideration is to use the hyperbolic function instead;

$$\frac{d{f_b}^2(y)}{dy^2} + k_0^2\varepsilon_b f_b(y) = 0 \tag{1}$$

The solution will be derived in terms of using the symmetry property, and then the equation will be

$$f_b(y) = B\cosh(|k_b|y). \qquad y < b \tag{2}$$

$$\varepsilon_b = -|\varepsilon_b|$$

$${k_b}^2 = -{k_0}^2|\varepsilon_b| - \beta^2 \tag{3}$$

$$j\omega\varepsilon_0\varepsilon_a\mathrm{E}{=}\nabla\times H$$

$$-j\omega\varepsilon_0|\varepsilon_b|\mathrm{E} = \begin{bmatrix} u_x & u_y & u_z \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_{ax} & 0 & 0 \end{bmatrix}$$

$$-j\omega\varepsilon_0\varepsilon_b E_z(y) = -\frac{\partial}{\partial y}H_{ax} = -\frac{\partial}{\partial y}(B\cosh(|k_b|y))$$

$$E_z(y) = \frac{-|k_b|}{j\omega\epsilon_0\epsilon_b}\beta\sinh(|k_b|y) \tag{4}$$

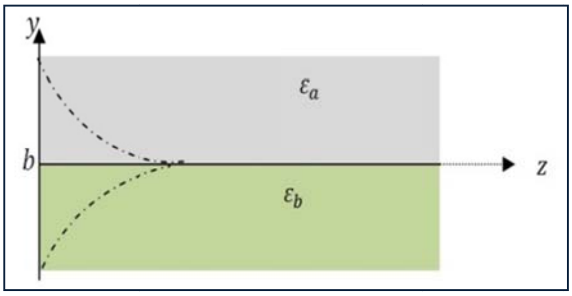

Fig. 2 A symmetric case of SPs

B. Solving the Upper Region

$$f_a(y) = Ae^{-|k_a|y}, \qquad y > b. \tag{5}$$

$${k_a}^2 = {k_0}^2\varepsilon_a - \beta^2 \qquad k_a = j|k_a|$$

$$H_{ax}(y) = Ae^{-|k_a|y} \tag{6}$$

$$E_x(y) = 0$$

$$j\omega\varepsilon_0\varepsilon_a E_y = \partial z H_{ax} = -j\beta H_{ax}$$

$$j\omega\varepsilon_0\varepsilon_a E_z(y) = -\partial y H_{ax}$$

$$E_z(y) = \frac{|k_a|}{j\omega\varepsilon_0\varepsilon_a}Ae^{-|k_a|y} \tag{7}$$

Applying the boundary condition for fields at $y = +b$ where the fields must be continuous

$$H_a(y = b) = H_b(y = b)$$

$$D_a(y = b) = D_b(y = b)$$

$$Ae^{-|k_a|b} = B\cosh(|k_b|b) \tag{8}$$

$$\frac{|k_a|}{j\omega\varepsilon_0\varepsilon_a}Ae^{-|k_a|b} = \frac{-|k_b|}{j\omega\varepsilon_0\varepsilon_b}B\sinh(|k_b|b) \tag{9}$$

Applying the division of (9) by (8),

$$\frac{|k_a|}{\varepsilon_a} = \frac{-|k_b|}{\varepsilon_b}\tanh(|k_b|b) \tag{10}$$

$$\varepsilon_b = -|\varepsilon_b|$$

$$\frac{|k_a|}{\varepsilon_a} = \frac{|k_b|}{|\varepsilon_b|}\tanh(|k_b|b) \tag{11}$$

$$b|k_b|\tanh(|k_b|b) = \frac{|\varepsilon_b|}{\varepsilon_a}|k_a|b \qquad (12)$$

$$u = b|k_b| \quad , \quad v = b|k_a| \qquad (13)$$

$$u\tanh(u) = \frac{|\varepsilon_b|}{\varepsilon_a}|k_a|b \qquad (14)$$

$$w^2 = u^2 - v^2 \qquad (15)$$

$$v^2 = \frac{\varepsilon_a}{|\varepsilon_b|}^2 u^2\tanh(u) \qquad (16)$$

$$= u^2(1 - \frac{\epsilon_a}{|\epsilon_b|}^2 u^2\tanh^2(u))$$

$$f(t) = w^2 - u^2(1 - \frac{\varepsilon_a}{|\varepsilon_b|}^2 u^2\tanh^2(u)) \qquad (17)$$

This equation is used in the simulation to search for value of u.

IV. Simulations and Results

The test is made to have a metal core and a dielectric cladding with permittivity of the metal $\varepsilon_m < 0$. The metal has permittivity (-45), while the permittivity of the dielectric $\varepsilon_a > 0 = (3.6)^2$ and the thickness of the core is $12.5nm$ with free space wavelength $(1\mu m)$. The simulation is implemented based on the use of TM-Mode. The main distinction between this structure and the ordinary slab wave guide is the use of the metal with negative permittivity since the field is represented by applying TM-Mode. The results are displayed as in Table I.

Using the values of k_b, k_a to be substituted in the following equations to plot the field

$$H_{ax}(\mathrm{y}) = B_a e^{-|k_a|y}$$

$$H_{ax}(\mathrm{y}) = A_m\cosh(|k_m|y)$$

TABLE I
SP Metal-Cladding Structure ($\varepsilon_m = -45$, $\varepsilon_a = (3.6)^2$)

TE-Mode						
$d(nm)$	u	Q	$k_b = \frac{u}{d}$	$k_a = \lvert\frac{v}{d}\rvert$	$\beta\ (\mu m)^{-1}$	$\varepsilon_{eff} = (\frac{\beta}{k_0})^2$
12.5	0.6	0.59	48.8	9.657	47.8348	57.960

Due to the design at the optical frequencies, the loss becomes a significant factor that might face the modern modifications. It leads to considering SP which works better in case of using lossy materials. The simulation is done for both symmetric and asymmetric cases. The considerable point that is noticed using the negative dielectric constant of the metal, the propagation constant mode β is placed out of the range compared to that which was resulted from the ordinary dielectric waveguide. Therefore, the effective refractive index will be higher than the core and cladding indexes as introduced in Table I.

Fig. 3 represents the excitation of the field at the interface where SP is placed. The shape of the field in the core region has hyperbolic function, thereby the electromagnetic waves travel close to the interface. This shows improvements in the field confinement since there is an interaction between the optical waves and the free electrons at the surface. This kind of structure satisfies both of the conditions that are mentioned above. The first condition is the permittivity of the utilized metal is negative, while it is witnessed form the result in Fig. 3, there is an exponential decaying for the perpendicular field to the surface. The previous simulation was to examine the field mode in case of single interface (symmetric slab waveguide), while the following figure represents asymmetric waveguide

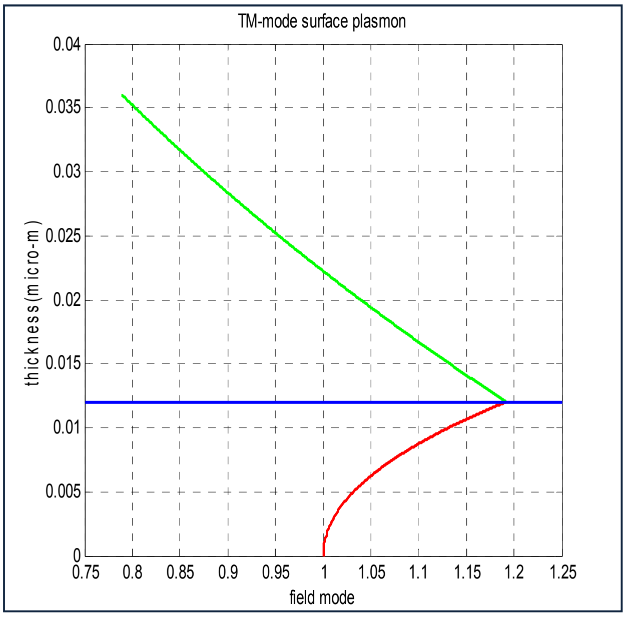

Fig. 3 The use of SP for field confinement

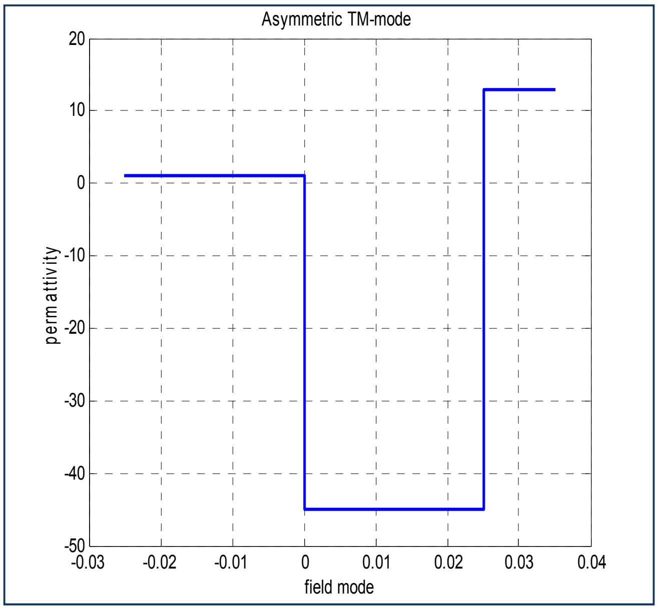

Fig. 4 General representation of asymmetric slab waveguide

In the asymmetric slab structure as shown in Fig. 4, the permittivity of the metal and the top layer is still the same as given above, while the permittivity of the substrate is $\varepsilon_b = 1$.

For such structure, the solution can be simply obtained since β must have similar value in both interfaces to satisfy the boundary condition. Table II shows the results of the asymmetric structure. Such structure provides mode confinement in both sides to manipulate more fields in the desirable region.

TABLE II
SP METAL-CLADDING STRUCTURE ($\varepsilon_m = -45$, $\varepsilon_a = (3.6)^2$ $\varepsilon_b = 1$)

TE-Mode					
$d(nm)$	k_a	k_m	k_b	$\beta\ (\mu m)^{-1}$	$n_{eff} = (\frac{\beta}{k_0})^2$
25	36	95.93	42.14	42.148	45.999

Fig. 5 shows the decaying in the field in case of TM-Mode to be bounded to the surface; this is one of the conditions that SPs depend on. Another noticeable point form the figure is that there is continuity in the field at both interfaces while the core region has a hyperbolic shape. This in turn, supports the condition for the necessity of having field continuity at the interface to apply the boundary conditions.

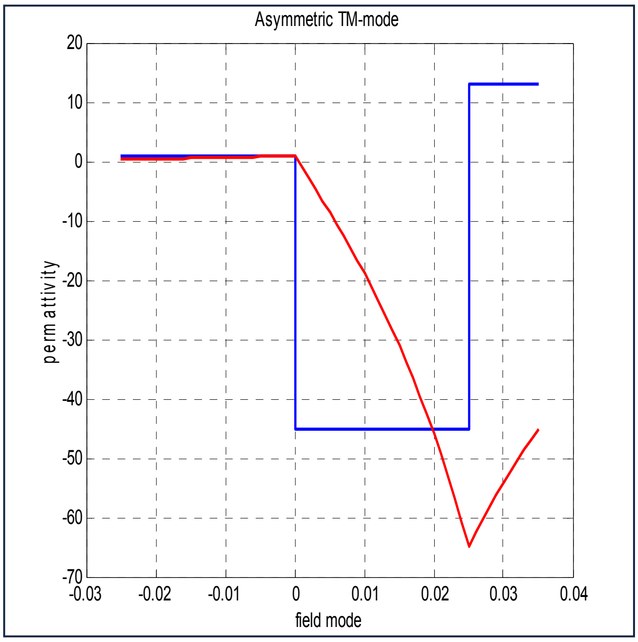

Fig. 5 TM -Mode in asymmetric structure in the presence of the SP

V. DISCUSSIONS AND CONCLUSIONS

To sum up, a significant progression has been made in modeling the waveguide structure in both mathematical and numerical methods. The results that are introduced provide the whole picture of dealing with the fabrication of optical structures. In this part a specific emphasis has been given to the SPs waveguides because they are comparatively easy to analyze. This modeling is based on the propagation constant mode (β) and effective refractive index $(neff)$ which are the main parameters to design or modify any structure. The challenge of having advanced circuits is not limited at this stage, but the purpose becomes more worthy for modifying the structure with more advanced methods. The interest in developing the optical geometries has witnessed more advances since SPs were introduced. When SP is presented to the structure, it has shown that the permittivity of the core is negative because the materials are lossy at the optical frequencies. In this particular case, to search for (β) is not similar as in ordinary dielectric waveguide. The effective refractive index is obtained out of the interval between the both indexes. It results in $n_{eff} = 7.3131$, which is placed out of both indexes 3.6 and 6.707. The shape of bound mode has the action of hyperbolic function. Fig. 3 shows a decaying of the field towards the interface of the metal supporting the definition of SPs. Fig. 5 demonstrates that the continuity of the field for the use of the SP is achieved based on the TM-Mode.

REFERENCES

[1] R. Zia, M. D. Selker, P. B. Catrysse, and M. L. Brongersma, "Geometries and materials for subwavelength surface plasmon modes" *JOSA A,* vol. 21, pp. 2442-2446, 2004.

[2] R. H. Ritchie. "Plasma losses by fast electrons in thin films" Physical Review, 106(5):874–881, 1957

[3] A. V. Zayats, I. I. Smolyaninov, and A. A. Maradudin, "Nano-Optics of surface Plasmon polaritons", *Physics reports,* vol. 408, pp. 131- 314, 2005.

[4] R. Buckley and P. Berini, "Figures of merit for 2D surface plasmon waveguides and application to metal stripes", *Optics express,* vol. 15, pp. 12174-12182, 2007.

[5] Noginov, M. A., et al. "Stimulated emission of surface Plasmon polaritons", *Physical Review Letters* 101.22 (2008): 226806.

[6] Z. Abdul Sattar, K. Shore, and Z. Wang, "Wave-Guiding Analysis of Annular Core Geometry Metal-Clad Semiconductor Nano-Lasers", 2014.

[7] Liu, Liu, Zhanghua Han, and Sailing He. "Novel surface plasmon Waveguide for high integration" *Optics Express* 13.17 (2005): 6645-6650.

[8] Barnes, William L., Alain Dereux, and Thomas W. Ebbesen. "Surface plasmo subwavelength optics" *Nature* 424.6950 (2003): 824-830.