

Mathematical Modeling of Wind Energy System for Designing Fault Tolerant Control

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Abstract—This paper addresses the mathematical model of wind energy system useful for designing fault tolerant control. To serve the demand of power, large capacity wind energy systems are vital. These systems are installed offshore where non planned service is very costly. Whenever there is a fault in between two planned services, the system may stop working abruptly. This might even lead to the complete failure of the system. To enhance the reliability, the availability and reduce the cost of maintenance of wind turbines, the fault tolerant control systems are very essential. For designing any control system, an appropriate mathematical model is always needed. In this paper, the two-mass model is modified by considering the frequent mechanical faults like misalignments in the drive train, gears and bearings faults. These faults are subject to a wear process and cause frictional losses. This paper addresses these faults in the mathematics of the wind energy system. Further, the work is extended to study the variations of the parameters namely generator inertia constant, spring constant, viscous friction coefficient and gear ratio; on the pole-zero plot which is related with the physical design of the wind turbine. Behavior of the wind turbine during drive train faults are simulated and briefly discussed.

Keywords—Mathematical model of wind energy system, stability analysis, shaft stiffness, viscous friction coefficient, gear ratio, generator inertia, fault tolerant control.

I. INTRODUCTION

THE wind power is considerably cheap, clean and non-polluting source of energy. The conventional power generation sources use fossil fuels leading to environmental pollution. These fossil fuels are reducing day by day. To meet the demand of power, the considerable growth has been seen in wind energy conversion systems. The growth is mainly focused on large capacity wind energy systems. These are remotely located. The stochastic nature of the wind causes power fluctuations and frequent faults. The faults may lead to the major failure, if not treated in time. To study the impact of these faults and design a fault tolerant control system, an appropriate mathematical model needs to be developed.

In recent literature, the techniques of mathematical modeling for wind energy systems are researched well. The detailed nonlinear mathematical model of wind energy system is discussed in [1]. The aerodynamic model of wind energy system is simulated in [2] shows that the wind shear and tower shadow causes 3p pulses in the aerodynamic torque and affect the power quality. In literature, the wind energy system is

modeled as six mass, three mass, two mass and one mass model and the transient response of it is studied in [3]-[5]. The research shows that the two-mass model can be effectively used with sufficient accuracy. The effects of the parameters such as inertia constants of rotor and generator, spring constant, damping constant and gear ratio on transient stability is studied in [6]-[8].

The goal of this paper is to develop the mathematical model of wind energy system which can be used for designing fault tolerant control system. The wind energy system is divided in sub-models which can be suitably modeled separately. The drive train represents the set of components necessary to transmit the power from rotor to generator. The structure of the large capacity wind energy systems is heavier and the components used are more flexible. Due to the stochastic nature of the wind and complex assembly of the drive train, the varying stresses and significant vibrations are created. The mechanical faults due to misalignment or the bearing faults are very common. It causes the frictional losses which are considered in mathematical modeling. The mathematical model derived in literature does not consider these losses.

In wind energy plant, the advance control systems are used. In the close loop control system the input variables used are pitch angle, rotor speed, generator speed, generator torque and output power which are measured by the sensors. In the presented wind energy system model, these parameters are used as the input states which can be estimated using derived model. The developed model is suitable for designing fault tolerant control.

II. WIND ENERGY SYSTEM MODELING

The wind energy system is divided into small sub-models represented by Fig. 1. It includes the wind model, aerodynamics, pitch actuator, tower, drive train and generator. The wind model includes the effects of wind shadow, shear and turbulence. The aerodynamic model calculates the aerodynamic torque and thrust with the rotor effective wind. The pitch actuator adjusts the pitch angle to maintain the rated speed in high wind region. The drive train increases the speed of the rotor necessary for the generator to yield the maximum power. In figure, V_r is the rotor effective wind speed in [m/s], $\beta(t)$ is pitch angle in $^\circ$, $\beta(t)_{ref}$ is reference pitch angle in $^\circ$, T_a is the aerodynamic torque which is input to the drive train in [N], T_g is generator torque in [N], w_r and w_g are rotor and generator speed in [rad/s], x_t is displacement of nacelle from its equilibrium position in [m].

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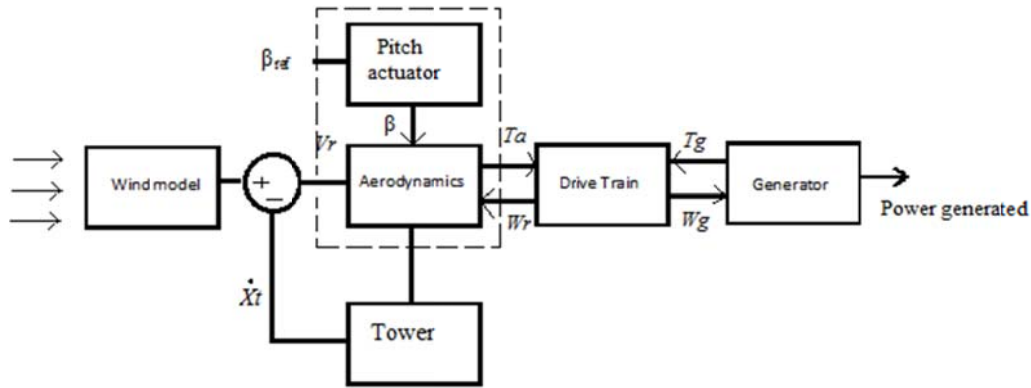


Fig. 1 Wind energy system divided into sub-models

A. Wind Model

The wind speed is influenced by the components, wind shear, shadow and turbulence. The obstacles in the path of the wind causes frictional forces to act on the wind called as wind shear. The rotor effective wind speed for individual blade decreases, when blade comes in front of the tower. The wind flow is redirected due to the presence of the tower called wind shadow effect. The wind turbulence depends on the environmental factors like temperature, pressure, humidity as well as the motion of wind itself in three dimensions. Fig. 2 shows that effective wind is the collective effect of wind shear, wind shadow and turbulence.

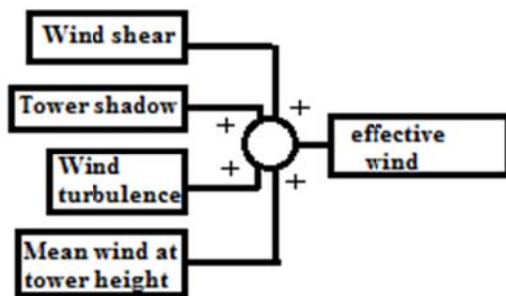


Fig. 2 Effective wind considering wind shear, shadow and turbulence

B. Aerodynamic Model

Aerodynamic torque is transferred from the rotor to the generator through drive train. It is given by (1):

$$T_a = \frac{1}{2w_r} \rho A V_r^3 C_p \quad (1)$$

where, ρ is the density of the air, A is rotor swept area exposed to the air, C_p is the power coefficient of a turbine which is a function of pitch angle and tip speed ratio. The pitch angle can be changed to control the rotor speed. In case of higher wind speed the pitch angle has to increase to maintain the rated rotor speed. Any fault occurred in case of pitch angle position may cause the system to go in runaway condition which is the failure of the system.

In partial load region maximum value of C_p is maintained by adjusting very small pitch angle where the maximum power can be captured. In full load region, the rotor speed is

maintained constant by adjusting the pitch angle and the rated power is maintained.

The wind acting on the rotor causes aerodynamic thrust F_{th} in [N] on the tower which makes the tower to sway back and forth. It reduces the rotor effective wind. It is given by (2):

$$F_{th} = \frac{1}{2} A V_r C_t \quad (2)$$

C_t is thrust coefficient, which is a function of pitch angle and tip speed ratio.

C. Pitch Actuator

The role of the pitch actuator is in full load region. Pitch angle is adjusted by the pitch actuator of blade to maintain the rated rotor speed. It is given by (3):

$$\ddot{\beta}(t) = -2\zeta\omega\dot{\beta}(t) - \omega^2\beta(t) + \omega^2\beta(t)ref \quad (3)$$

where, ω is natural frequency and ζ is damping ratio of pitch actuator model.

State space model for the pitch actuator is given by (4). In the close loop control system, the pitch sensor is used to measure the pitch angle and give the feedback. If fault occurs in sensor, the wrong feedback causes major consequences. The state model derived can be used for pitch angle estimation. The estimated parameter can be used instead of sensor measurement. The step response and root locus are plotted in Fig. 3.

$$\begin{bmatrix} \dot{\beta}(t) \\ \ddot{\beta}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{bmatrix} \begin{bmatrix} \beta(t) \\ \dot{\beta}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -\omega^2 \end{bmatrix} [\beta(t)ref] \quad (4)$$

D. Drive Train Model

The drive train is the complex mechanical structure and susceptible to the faults. It consists of the rotor, main gear box and the generator. The main gear box is used to increase the speed of low speed rotor shaft to high speed generator shaft. There are fluctuations in the aerodynamic torque from rotor because of variable input wind. The drive train modifies the torque transmitted. This modified torque can be assessed by analyzing drive train model response thoroughly.

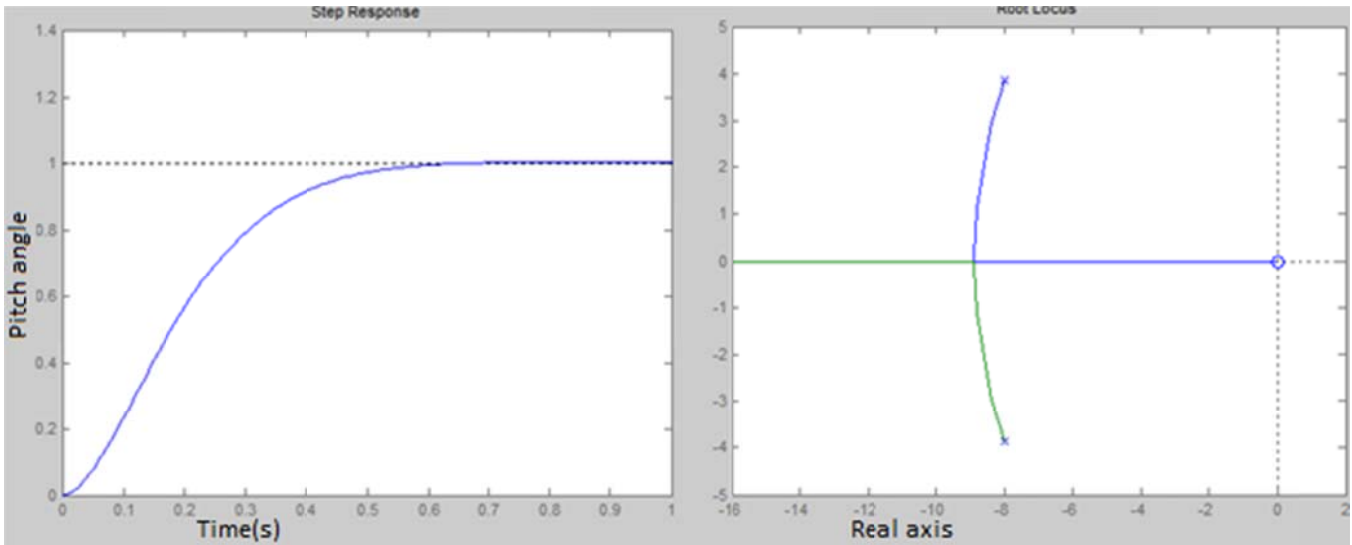


Fig. 3 Step response and root locus for pitch actuator

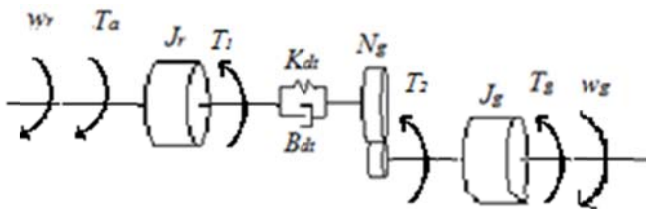


Fig. 4 Drive train model

Fig. 4 shows the drive train is divided into three parts as low speed shaft, gear box and high speed shaft. High speed shaft is connected to generator. The drive train one mass model considers that, all the drive train components are lumped together and work as single rotating mass. The two-mass model considers the generator and rotor inertias connected by the single spring shaft. The three-mass model considers the generator, rotor and gear box inertias. Literature reveals that two mass model is sufficiently accurate [6]. The aerodynamic torque and the electrical torque are the inputs to the drive train from the rotor and the generator side. The differential equations take the form as

$$J_r \dot{w}_r = T_a - K_{dt} \theta_\delta - B_{dt} \dot{\theta}_\delta \quad (5)$$

$$J_g N_g \dot{w}_g = -T_g N_g + K_{dt} \theta_\delta + B_{dt} \dot{\theta}_\delta \quad (6)$$

$$\dot{\theta}_\delta = w_r - \frac{w_g}{N_g} \quad (7)$$

where, N_g is gear ratio, J_g is inertia of generator and high speed shaft in $[Kgm^2]$, J_r is the inertia of rotor and low speed shaft, B_{dt} is viscous friction coefficient, K_{dt} shaft stiffness in $[Nm/rad]$ and θ_δ is torsion angle of drive train in $[rad]$.

The state space model of the drive train is formed as (8)

$$\begin{aligned} \dot{X} &= AX + Bu \\ Y &= CX. \end{aligned}$$

where,

$$A = \begin{bmatrix} \frac{-B_{dt}}{J_r} & \frac{B_{dt}}{J_r N_g} & \frac{-K_{dt}}{J_r} \\ \frac{B_{dt}}{J_r N_g} & \frac{-B_{dt}}{J_g N_g^2} & \frac{K_{dt}}{J_g N_g} \\ 1 & \frac{-1}{N_g} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{J_r} & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with,

$$X = \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix}, u = \begin{bmatrix} T_a \\ T_g \end{bmatrix}, Y = \begin{bmatrix} w_r \\ w_g \\ \theta_\delta \end{bmatrix} \quad (8)$$

The state space model is controllable and observable. The input states of the state space equation are rotor speed, generator speed and the torsion angle of drive train. These states can be estimated using the derived state space model. The states-rotor speed and generator speed are useful in close loop control system for controlling the pitch angle and output power. These are measured by the sensors and given as feedback to the controller. In case of faulty sensor measurement, the estimated states can be used in control system.

There are multiple inputs and multiple outputs in the drive train model. To analyze the characteristics of the drive train, rotational speed of high speed shaft must be studied. To study it in detail the transfer functions of generator speed for two inputs aerodynamic torque, w_g/T_g and generator torque, w_g/T_a are analyzed by plotting its step responses and root loci. The aerodynamic torque is the input torque generated by the rotor with input wind. Generator torque is an opposing torque generated by the generator.

The step response, root locus for w_g/T_a and w_g/T_g are plotted in Fig. 5 (a).

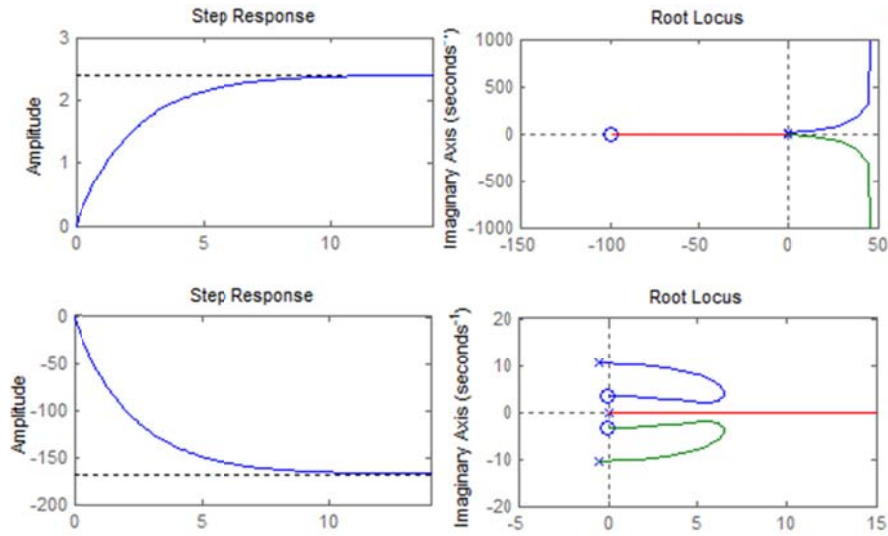


Fig. 5 (a) Step response and root locus for the transfer functions of w_g/T_a and w_g/T_g

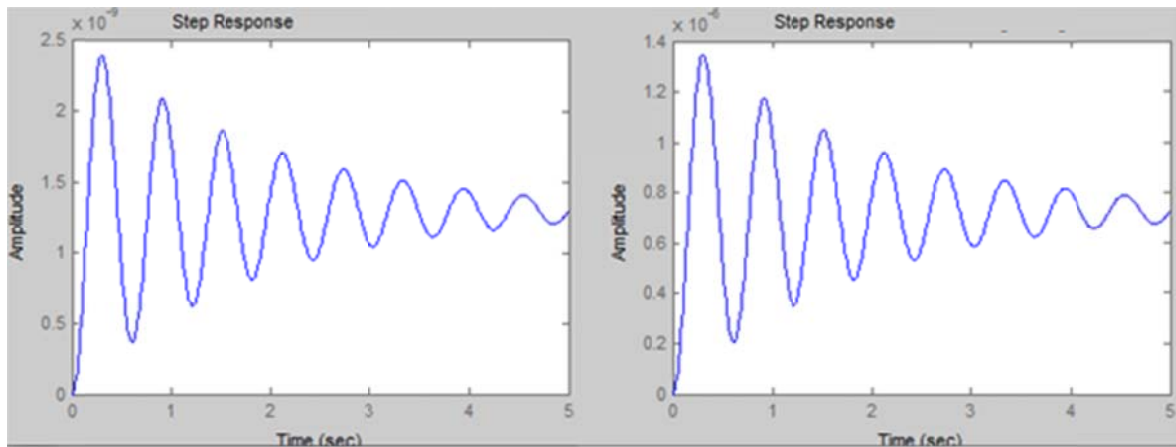


Fig. 5 (b) Closed look of Step response for the transfer functions of w_g/T_a and w_g/T_g

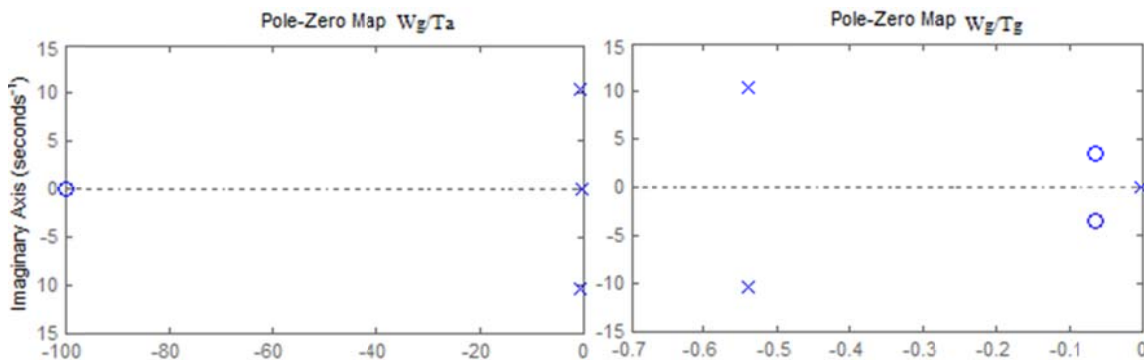


Fig. 6 Pole-zero plot for the transfer functions of w_g/T_a and w_g/T_g

If we have a close look of the step responses of w_g/T_a and w_g/T_g plotted in Fig. 5 (b), oscillations are observed initially. Also, the curves take more time to settle. These oscillations may cause the fatigue damage to the system when mechanical or electrical fault occurs. So any fault in a system may lead to wear and tear of mechanical part and also a major damage.

Looking at the pole-zero plot of the system in Fig. 6, there are two dominant poles and third real pole which is near the origin in the left hand plane. The system is stable but takes more time to settle due to the pole very near the origin. Due to any mechanical or electrical fault, if pole moves at right hand plane, system will be unstable.

E. Tower

The tower accelerations caused by thrust force reduces the effective wind speed of rotor shown in Fig. 7. The tower gets displaced from the equilibrium due to thrust by distance x_t . Mathematically the tower can be modeled as (9)

$$M\ddot{x}_t = F_{th} - K_t x_t - B_t \dot{x}_t \quad (9)$$

where, M is mass of tower in [Kg], K_t is tower spring coefficient in [Nm], B_t is tower damping coefficient.

State space function for tower is given by (10)

$$\begin{bmatrix} \dot{x}_t \\ \ddot{x}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K_t}{M} & -\frac{B_t}{M} \end{bmatrix} \begin{bmatrix} x_t \\ \dot{x}_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix} [F_{th}] \quad (10)$$

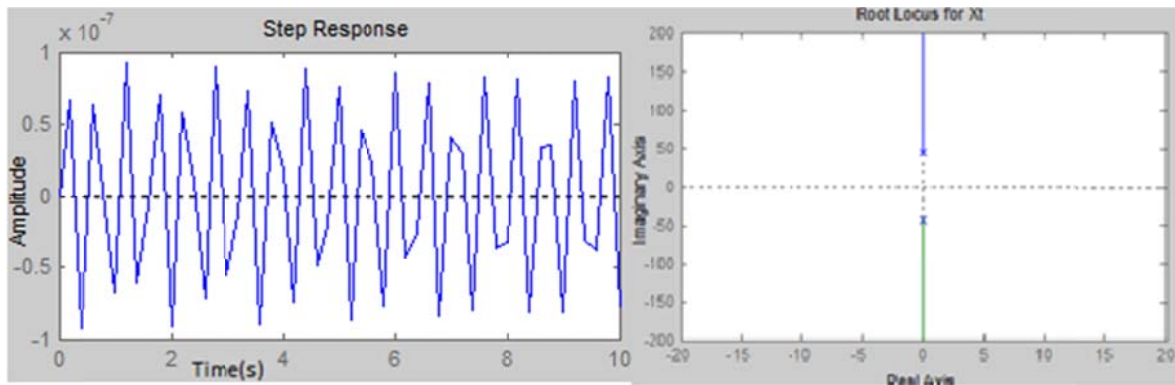


Fig. 8 Step response and root locus for tower acceleration

F. Generator

Various types of generators, e.g. induction type, synchronous type, and popularly used doubly fed type induction generator, are used in wind energy system. Electric power is generated by the generator, and to enable variable-speed operation, currents in the generator are controlled using power electronics. Therefore, power electronic converters interface the wind turbine generator output with the utility grid. The first order model of generator with converter can be represented by (11):

$$\frac{T_g}{T_{gref}} = \frac{\tau}{\tau s + 1} \quad (11)$$

where τ is time constant of the system.

The power produced by the generator depends on the rotational speed of the rotor and the applied load. It is described by (12):

$$P_g = \eta_g w_g T_g \quad (12)$$

where η_g is the efficiency of the generator.

III. STABILITY ANALYSIS OF WIND TURBINE

In wind energy systems, the drive train is a very complex mechanical assembly. In drive train, there are frequent mechanical faults due to misalignment and damaged bearing.

The step response and root locus of tower acceleration to step change in F_{th} are plotted in Fig. 8.

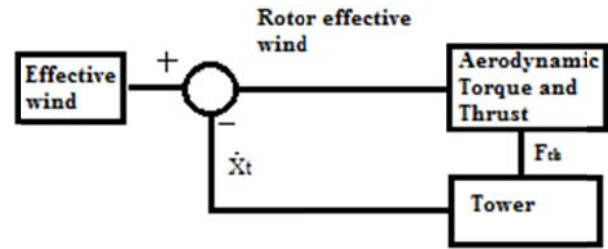


Fig. 7 Rotor effective wind speed affected by tower acceleration

Any such fault may cause the transients shown in Fig. 9. The system finally stabilizes with some steady state error in the output generator speed.

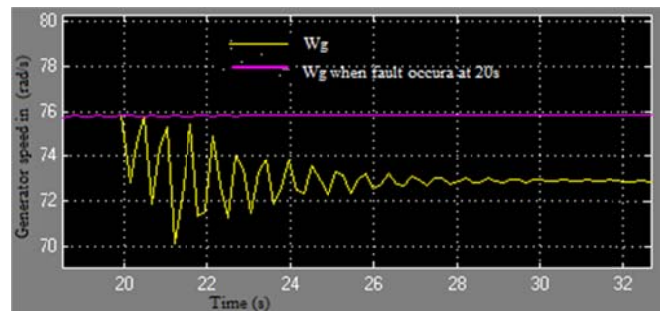


Fig. 9 Generator speed when fault occurs at 20 second

At variable rotational speed, these losses and faults vary strongly with torque transmission. In partial load region, the rotational speed is varying. Considering this type of fault the T_{loss} is additional term must be considered in the above model to design controller in future which can take care of steady state error. So, (6) takes the form as (13)

$$J_g N_g \dot{w}_g = -T_g N_g - T_{loss} N_g + K_{at} \theta_\delta + B_{at} \dot{\theta}_\delta \quad (13)$$

Further, it is converted into the state space form as (14)

$$\dot{X} = AX + B1u + B2v$$

$$Y = CX.$$

where,

$$A = \begin{bmatrix} \frac{-B_{dt}}{J_r} & \frac{B_{dt}}{J_r N_g} & \frac{-K_{dt}}{J_r} \\ \frac{B_{dt}}{J_r N_g} & \frac{-B_{dt}}{J_g N_g^2} & \frac{K_{dt}}{J_g N_g} \\ 1 & \frac{-1}{N_g} & 0 \end{bmatrix}, B1 = \begin{bmatrix} \frac{1}{J_r} & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix}, B2 = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-1}{J_g} \\ 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

with,

$$X = \begin{bmatrix} W_r \\ W_g \\ \theta_\delta \end{bmatrix}, u = \begin{bmatrix} T_a \\ T_g \end{bmatrix}, v = [T_{loss}], Y = \begin{bmatrix} W_r \\ W_g \\ \theta_\delta \end{bmatrix} \quad (14)$$

This modified mathematical model is used to study the detail stability analysis of drive train system when disturbances occur. Four cases have been considered for detailed transient stability analysis of drive train model from (14)

Case1: Change in Viscous Friction Coefficient

The objective of this case is to study the effect of viscous friction coefficient on the stability of wind energy system. Here the mutual viscous friction between generator and rotor is considered.

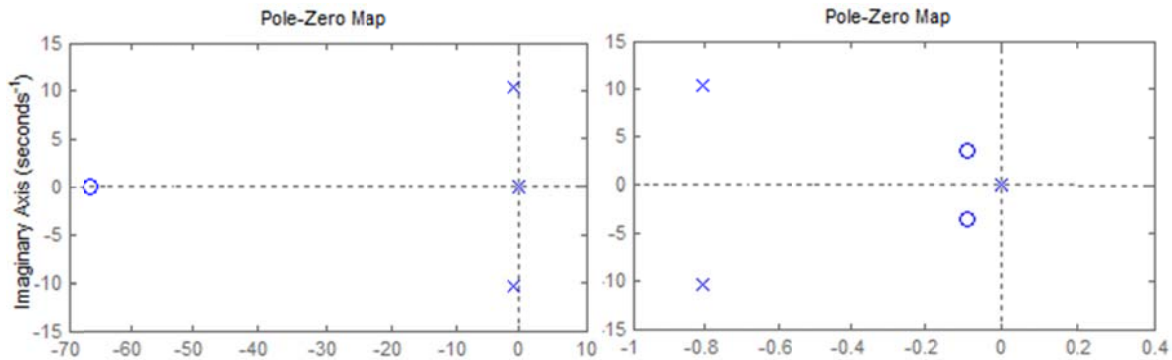


Fig. 10 Pole –zero plot for the transfer functions of w_g/T_a and w_g/T_g with 50% increase in B_{dt}

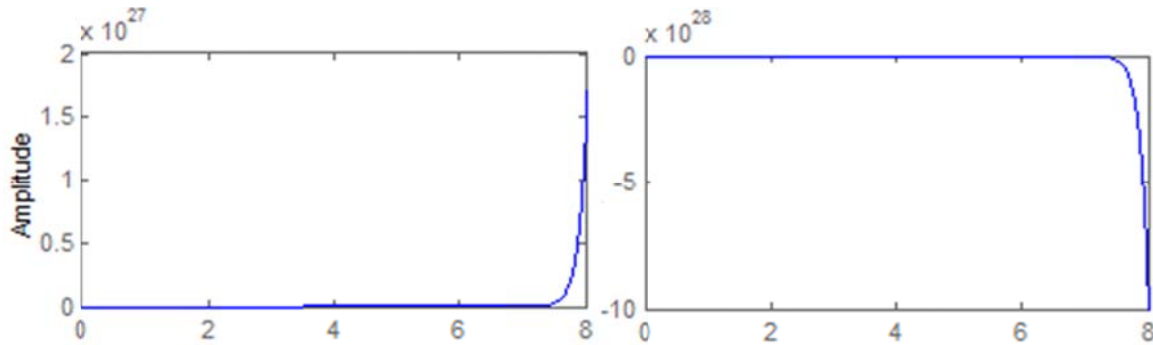


Fig. 11 Step response for w_g/T_g and w_g/T_a with 50% increase in B_{dt}

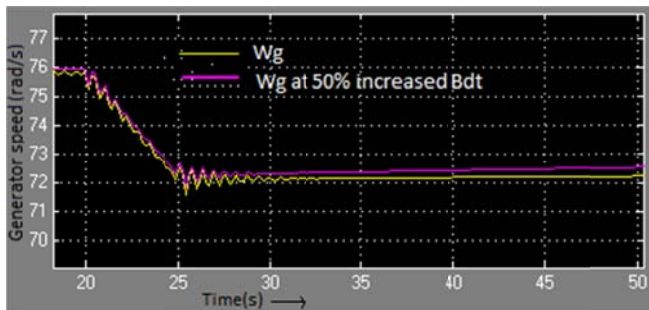


Fig. 12 Generator speed when fault occur in drive train with actual B_{dt} and increased B_{dt} by 25%

If we analyze the pole-zero location of the transfer function w_g/T_a , the real zero moves towards the dominant poles as B_{dt} increases and thus the oscillatory response improves. But,

looking at the poles of the system shown in pole-zero plot in Fig. 10, when B_{dt} is increased by 25%, one pole moves towards right hand plane. This causes the instability of the system. The component T_{loss} does not affect the pole-zero location. So it does not affect the transient stability.

Fig. 11 shows the step responses of both the transfer function are unstable at B_{dt} more by 25%. Fig. 12 shows the transient response when fault occurred at 20s.

Case 2: Change in Shaft Stiffness

The objective of this case is to study the effect of shaft stiffness on the transient stability of the wind energy system.

The difference in the inertia constant of the generator and wind turbine is very large. Input to the wind energy system is wind which is variable. Due to the continuous variations in the wind, flexibility must be maintained in these parts. Otherwise

mechanical faults or sometimes failure may occur. The less shaft stiffness of wind energy system results in the oscillation in the wind turbine torque during the faults.

During electrical faults, the electrical torque is significantly reduced, and therefore the drive train acts like a torsion spring that gets untwisted. Due to the torsion spring characteristic of the drive train, the transients are observed in generator speed.

Looking at the pole-zeroes shown in Fig. 13, there is no effect of change in stiffness constant on poles. But, zero in transfer function w_g/T_a is moving away from dominant poles

as K_{dt} increases. So the oscillations are reduced. The component T_{loss} does not affect the pole-zero location. So it does not affect the transient stability.

Case 3: Drive Train Gear Ratio

The objective of this case is to study the effect of gear ratio on the transient stability of the wind energy system. The gear ratio of the drive train must be selected which produces the greatest amount of torque and does not exceed the maximum amount allowed by the generator.

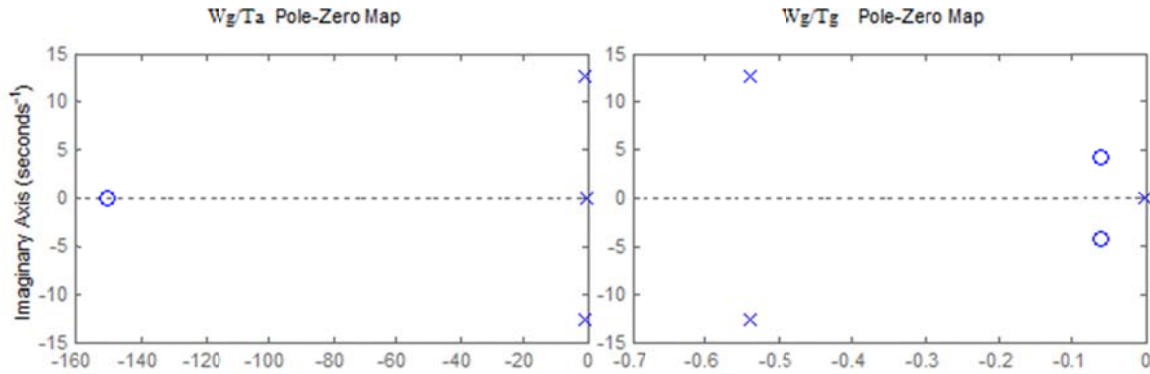


Fig. 13 Pole-zero plot for w_g/T_g and w_g/T_a with 25% increase in K_{dt}

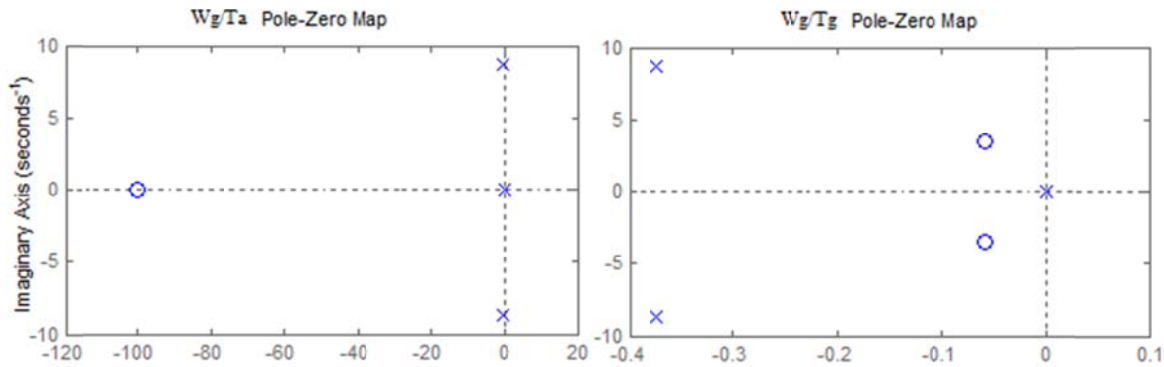


Fig. 14 Pole-zero plot for w_g/T_g and w_g/T_a with gear ratio increase by 20

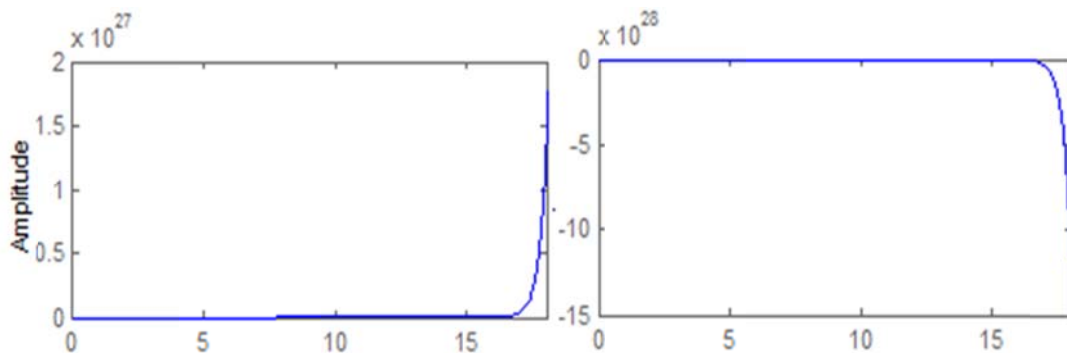


Fig. 15 Step response for w_g/T_g and w_g/T_a with gear ratio more by 20

With increase in the gear ratio, the efficiency is increased. But looking at the pole-zero, if we increase the gear ratio, one of the poles moves towards right hand plane as shown in Fig.

14. And so, the system moves towards instability. With lesser gear ratio the system becomes more stable. Fig. 15 shows that the increased gear ratio makes the step responses unstable.

Fig. 16 shows that when gear ratio is high, the fault may lead the system to the instability. The component T_{loss} does not affect the pole-zero location. So it does not affect the transient stability.

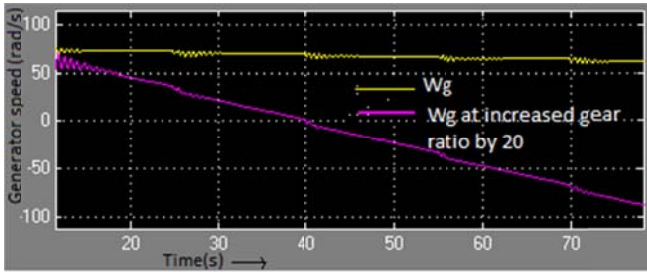


Fig. 16 Generator speed when fault occur in drive train with actual gear ratio and increased gear ratio by 20

Case 4: Inertia Constant

The objective of this case is to study the effect of inertia constant of rotor. Inertia constant of both turbine and generator has significant effect on transient stability. When there is any fault, it leads to the drive train distortion. The large total inertia constant makes the system more stable during power system disturbance or fault condition.

The poles-zero map in Fig. 17 shows that the stability of the system can be increased with the increased generator inertia. If this is decreased, one of the pole moves towards right hand plane and system becomes unstable. Fig. 18 shows the step response of the w_g/T_a and w_g/T_g is unstable when inertia constant of generator is decreased by 25%. The component T_{loss} does not affect the pole-zero location. So it does not affect the transient stability.

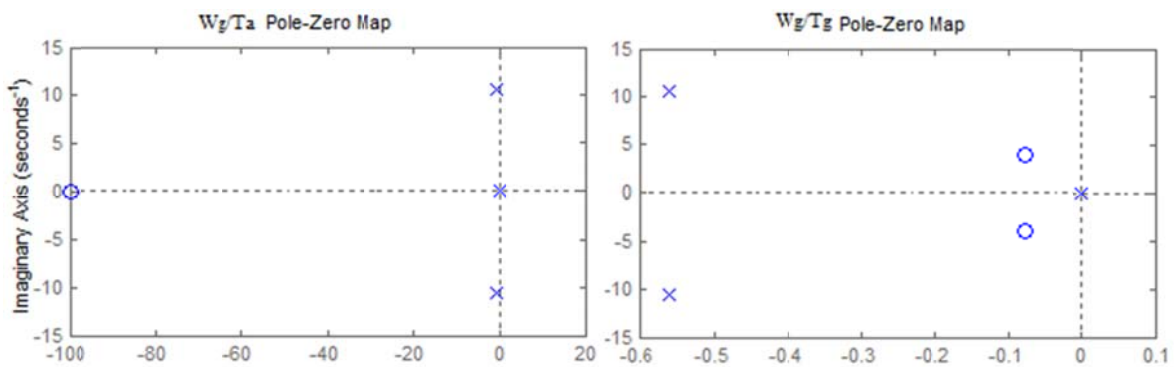


Fig. 17 Pole-zero plot for w_g/T_g and w_g/T_a with with rotor inertia less by 25%

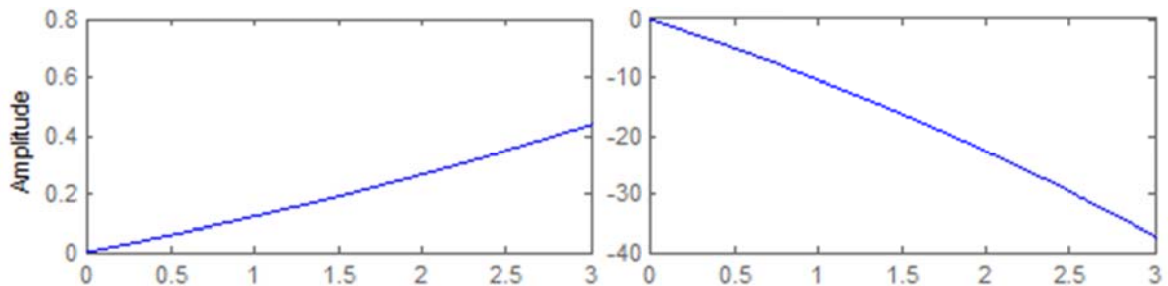


Fig. 18 Step response for w_g/T_g and w_g/T_a with rotor inertia less 25%

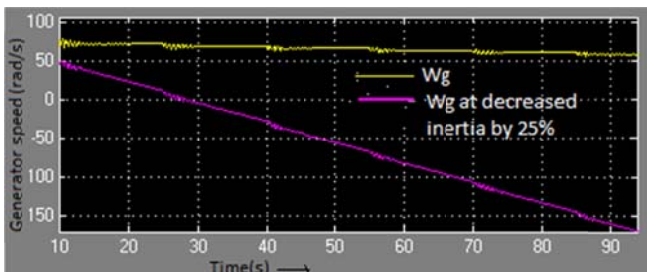


Fig. 19 Generator speed when fault occur in drive train with actual and decreased generator inertia constant by 25%

Fig. 19 shows the instability of the system when fault occurs at 25% decreased generator inertia constant. This fault

causes transients which also reflect in the power fluctuations. To minimize these oscillations and improve the performance, there are two ways. One way to mitigate these is to design perfect filtering while designing control system. Another way is to add a drive train stress damper to minimize these effects. It acts as a band pass filter.

IV. CONCLUSION

In this work, the mathematical model of wind energy system is developed. The two mass model is modified by considering the frequent drive train faults. With the help of developed model, the input variables which are the states of the state space models, can be estimated and are useful for fault tolerant control system. The model has been used for

stability analysis by considering the sensitivity of the parameters-generator inertia constant, spring constant, viscous friction coefficient and gear ratio. This study further is related with the physical design of the wind turbine. From the stability analysis, it has been concluded that the spring constant has less effect on stability. The increased viscous friction coefficient, gear ratio and decreased generator inertia can make the system unstable. Increased generator inertia constant improves the transient response. The considered T_{loss} in the mathematical model is useful in designing fault tolerant control system. It will help the controller to remove the offset in the final output generator speed. The perfect generator torque control facilitates in good tracking of reference torque which minimizes the fatigue stresses.

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