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Evaluation of Carbon Dioxide Pressure through Radial Velocity Difference in Arterial Blood Modeled by Drift Flux Model

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***Abstract*—**In this paper, we are interested to determine the carbon dioxide pressure in the arterial blood through radial velocity difference. The blood was modeled as a two phase mixture (an aqueous carbon dioxide solution with carbon dioxide gas) by Drift flux model and the Young-Laplace equation. The distributions of mixture velocities determined from the considered model permitted the calculation of the radial velocity distributions with different values of mean mixture pressure and the calculation of the mean carbon dioxide pressure knowing the mean mixture pressure. The radial velocity distributions are used to deduce a calculation method of the mean mixture pressure through the radial velocity difference between two positions which is measured by ultrasound. The mean carbon dioxide pressure is then deduced from the mean mixture pressure.

***Keywords*—**Mean carbon dioxide pressure, mean mixture pressure, mixture velocity, radial velocity difference.

I. Introduction

THE arterial blood gas (ABG) analysis consists of a blood sampling by a puncture of the radial artery (systemic artery bringing the oxygenated blood to the hand) and a measurement of three vital parameters which are partial pressure of oxygen PO_2, partial pressure of carbon dioxide PCO_2 and arterial pH. This allows the evaluation of haemostasis, the confirmation of the respiratory failure diagnosis, and the monitoring of a treatment's effectiveness in the intensive care unit (ICU), pneumology and operating room [1]. The sample Blood is analyzed by electrometric procedures to determine arterial blood pH and PCO_2 [2].

ABG analysis is needed for monitoring the respiration of patients under artificial respiration in ICU. Without this information, it becomes hard to physicians to control perfectly the mechanical ventilation. Knowing that the classical sampling process of the arterial blood using a syringe is not only painful for the patient but mostly cannot be repeated frequently. However, frequent blood sampling can cause anemia, due to the relatively excessive amount of collected blood (usually 5 to 6 mL per sample including losses), infections and contaminations. Also, it can occasionally cause necrosis due to a decrease or local blockage of blood flow [3]. For all these reasons, a non-invasive method is required to take real time measurements of arterial blood pH and PCO_2. This requires necessarily an ex-situ measurement procedure. When it is question to measure a pressure quantity, ultrasounds can be exploited as a safe gas pressures measurement method. The work described in this paper, is interested to procedure determination of the carbon dioxide pressure.

The setting of the measurement procedure firstly needs a mathematical modeling of the arterial blood. In this case, the blood solution is modeled as an incompressible Newtonian mixture of two phases. Mainly, it is about an aqueous carbon dioxide solution with carbon dioxide gas. The suitable model to describe the flow of this mixture is Drift flux model. The Drift flux and the Young-Laplace equations are used to simulate the fluid flow and to deduce the carbon dioxide pressure when calculating the mixture radial velocity [4]. In this paper, the method used for the assessment of mean carbon dioxide pressure is based on the determination of radial velocity distributions.

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II. Methods and Materials

A. Mathematical Model

The arterial blood contains various cells in plasma solution (water) which are red blood cells, white blood cells and platelets as well as gases including carbon dioxide and oxygen. Since the work is interested to determine the carbon dioxide pressure, the considered fluid is modeled as a two phase mixture flowing in a rigid canalization by the Drift flux model. It is about a gas phase (microbubbles of carbon dioxide) and a liquid phase (aqueous carbon dioxide solution) [4].

The drift flux model considers the whole mixture taking into account a drift velocity between the gas phase (dispersed phase) and the liquid phase (continuous phase). Experimentally, the measurement of the mixture velocity permits to deduce another characteristic of the two phases especially the gas phase. Both phases are considered to be incompressible. The first three equations of the considered model are used to express the mass and momentum conservations as

1) Mixture continuity equation

$$\nabla.(\rho_m U_m) = 0 \tag{1}$$

2) Continuity equation for dispersed phase

$$\nabla.(\alpha_2 \rho_2 U_m) = -\nabla.\left(\frac{\alpha_2 \rho_1 \rho_2}{\rho_m} V_{2j}\right) \tag{2}$$

3) Mixture momentum equation

$$\nabla.(\rho_m U_m) = -\nabla p_m + \nabla.(\tau) + \rho_m g_m - \nabla.\left(\frac{\alpha_2}{1-\alpha_2}\frac{\rho_1 \rho_2}{\rho_m} V_{2j} V_{2j}\right) \tag{3}$$

where, U_m: mixture velocity, ρ_m: mixture density, ρ_1: liquid density, ρ_2: gas density, α_2: volumetric fraction of gas, p_m: mixture pressure, τ: viscous stress, g_m: gravitational acceleration vector, V_{2j}: drift velocity vector of the gas phase [5].

The mixture pressure is expressed by the liquid pressure and the gas pressure as:

$$p_m = \alpha_1 p_1 + \alpha_2 p_2 \tag{4}$$

α_1 presents volumetric fraction of liquid phase, p_1 presents liquid pressure, and p_2 presents gas pressure.

Equations (1)-(3) permit to determine distributions of mixture velocities, the volumetric fraction of gas, and mixture pressure [4]. Equation (4) allows to relate pressures of gas and liquid phases to the mixture pressure but it is not sufficient to calculate the carbon dioxide pressure. To evaluate the gas pressure distribution, it is necessary to use a fifth equation. This equation is needed to determine the mentioned pressure distribution, knowing the mixture pressure distribution and the phase fractions. Then, the Young- Laplace equation is used to correlate the pressures of the two phases (5). The considered equation is written as:

$$p_2 - p_1 = \frac{2\sigma}{R} \tag{5}$$

The term σ presents the surface tension and the term R presents the radius of the microbubble of the gas [6], [7].

Then, the use of the two later equations (4) and (5) allows the assessment of gas pressure distribution through the mixture pressure and velocities.

B. Calculation Method

For a given fixed initial conditions, the resolution and the numerical simulations of (1)-(5) permit to calculate the spatial distribution of the three velocity's components U_{xm}, U_{ym}, and U_{zm} respectively along x axis (flow axis), y axis, and z axis, as well as the spatial distribution of the mixture pressure p_m and carbon dioxide pressure p_2. This constitutes the first step of the study [4].

The theoretical results allow understanding of the behavior and the characteristics of the considered mixture. Through these results, a calculation method is established to determine the mean gas pressure in the mixture knowing the mean mixture pressure. The later parameter is deduced from the mixture radial velocity distribution.

The blood sampling is done orthogonally to the flow axis, to avoid the effect of the heart pressure. Inspired by this technique, the mean carbon dioxide pressure is calculated by the radial velocity distribution values. Practically, mean gas pressure is determined through mean mixture pressure. According to (3), the mixture velocity values (U_{zm}, U_{ym}, and U_{xm}) depend on mixture pressure p_m. Thus, several distributions of U_{zm} in (x,z) plan are calculated for different values of mean mixture pressure. These later values are calculated by the integration of pressure values in all canalization's cells.

In practice, ultrasound is exploited in Doppler technique for measurement of linear blood flow. In this case, it is concevable to use two ultrasonic probes fixed on the canalization perpondicularly to the flow direction. Both probes must be relatively one far from the other to avoid measurement errors. In these positions, probes are able to measure the radial velocity of the mixture in each point on the probes fields of view. This method permits to deduce the mean mixture pressure knowing the radial velocity difference values between the two considered directions as

$$P_{moy} = f(\Delta U_{zm}) \tag{6}$$

C. Calculation

The numerical simulations of the mixture parameters are made by MATLAB. Therefore, the calculation permits the assessment of the mean mixture pressure and the mean carbon dioxide pressure through the differences of radial velocity distributions.

III. Results and Discussion

The fluid moves horizontally through a cylindrical rigid canalization with constant cross-section (4mm radius and 10cm length). Samples of the velocities U_{zm}, U_{ym}, and U_{xm} are chosen respectively as 9x9x10, 9x10x9, and 10x9x9 in the xyz space. Numerical simulations of the mixture allowed to calculate the distribution of radial velocity U_{zm} for different values of mean mixture pressure P_{moy}.

The calculation of velocity distributions U_{zm} along x axis for different values of z between 2 and 9 and different values of mean mixture pressure between 3.2 and 8.8 kPa, gives families of curves (Fig. 1) that changes remarkably with z position, but they are also shown coherence in the x interval between 3 and 8. In fact, in this interval, all the curves are continuous and differentiable. Then, from the curve families presented in Fig. 1, new curve families are deduced expressing the mean mixture pressure through the velocity distribution U_{zm} (Fig. 2).

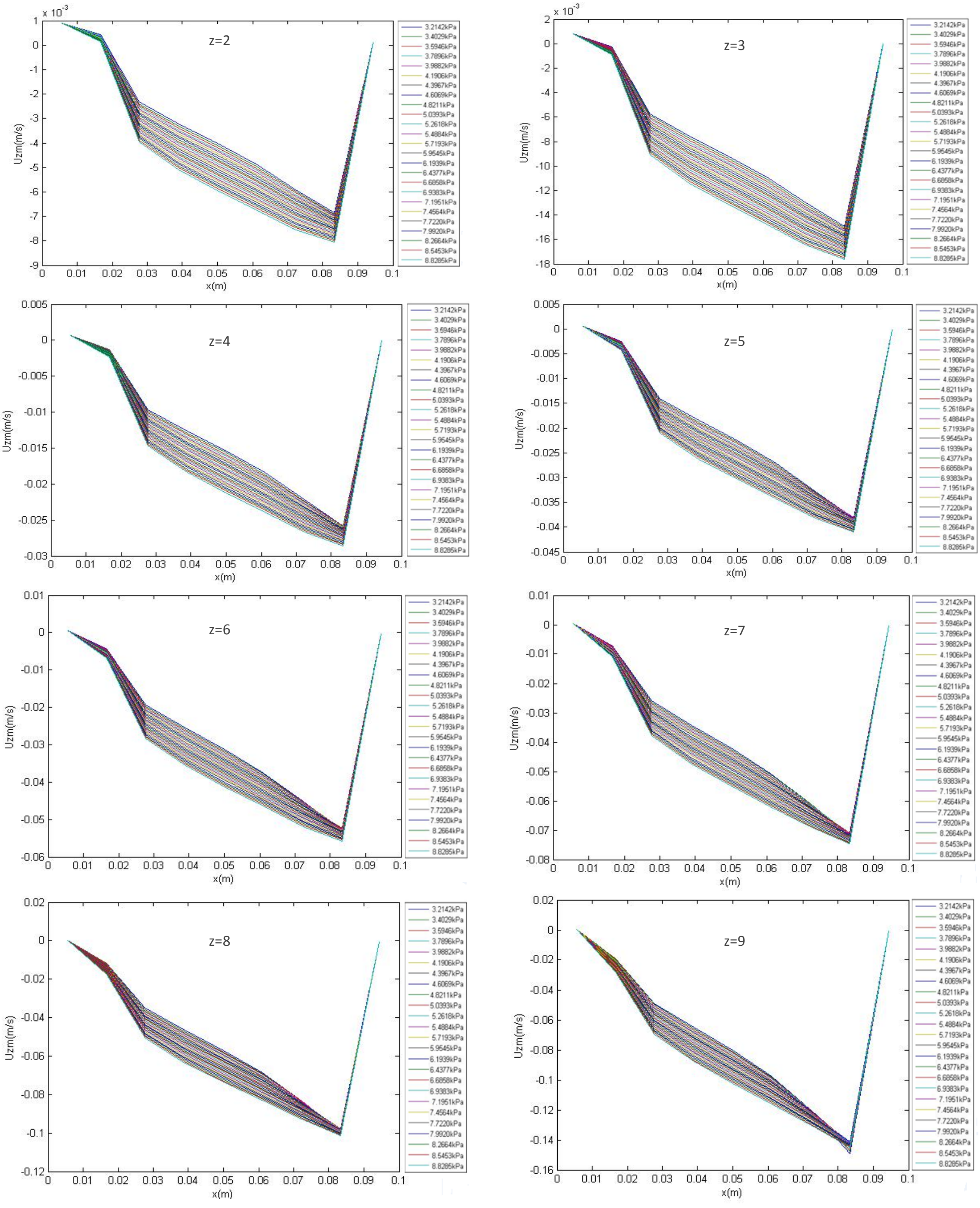

Fig. 1 Distribution of U_{zm} with different values of P_{moy} at different z positions

Contrary to the first families of curves (Fig. 1), the second families show independent and generally linear curves which can give options to deduce calculation methods of mean mixture pressure by ultrasound measurement of radial velocity U_{zm}. Practically, it is not suitable to do simultaneous measurements of velocities in all considered positions along the x axis (from the first position to the ninth last position) because of confusion in ultrasound detection when two probes are nearby. This can constitute a source of measurement errors. To circumvent this difficulty, it is advisable to calculate the mean mixture pressure by the measurement of radial velocities differences between just two important positions along the x axis chosen according to results, as shown in Fig. 1.

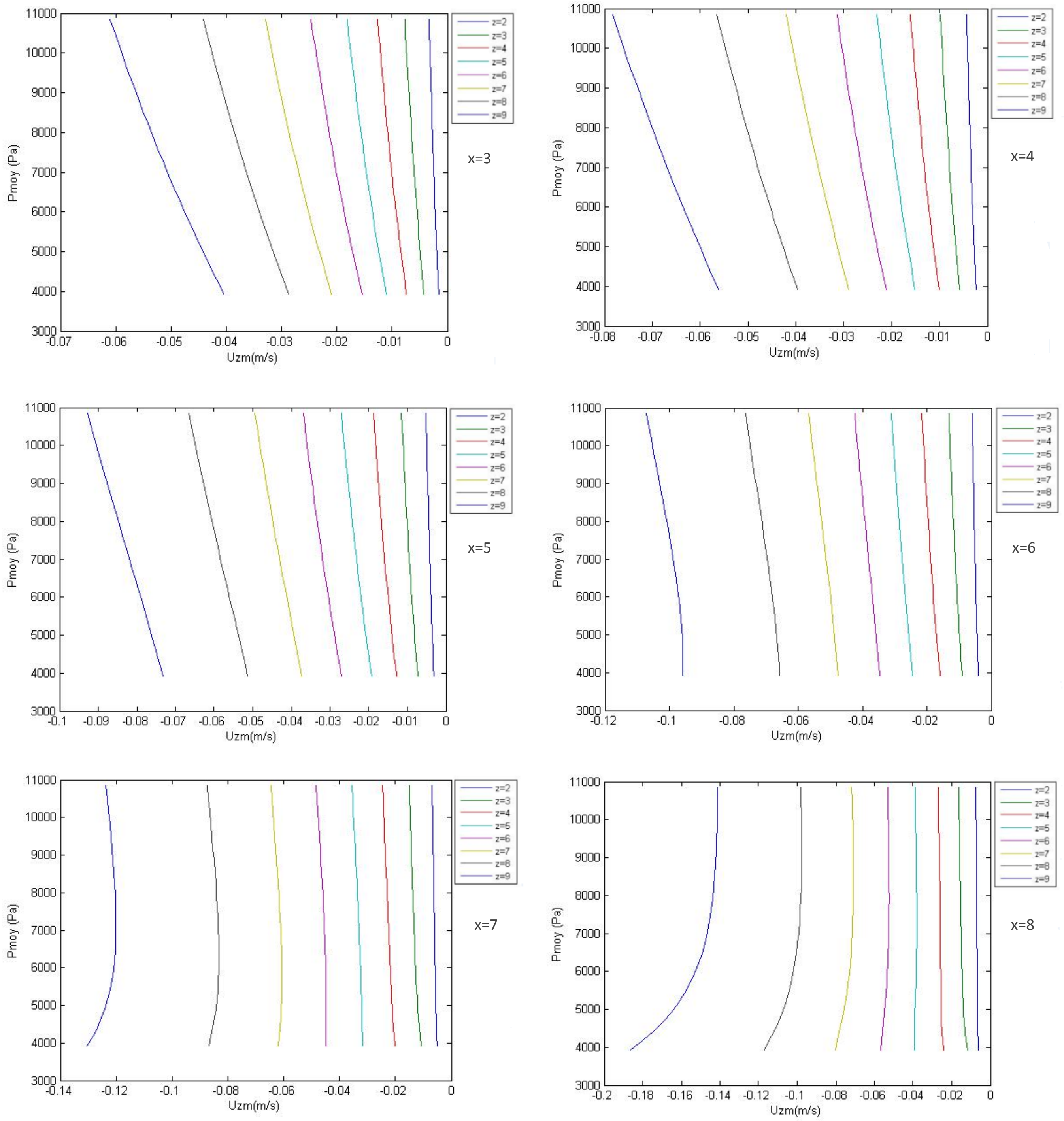

Fig. 2 Distribution of U_{zm} with P_{moy} for different x positions

Then, the radial velocities differences are calculated between positions 3 and 8 on the x axis for the z positions (z axis presents the radial axis) from 2 to 9 (Fig. 3). Thus, it is possible to use two ultrasonic probes put on the two considered x positions and measure radial velocity.

The radial velocity's differences calculated between these two positions is relatively important and then measurable. Fig. 3 represents the distribution of radial velocity's differences with different values of mean mixture pressure from the second position to the ninth position along z axis. Every curve corresponds to the variation of P_{moy} with the difference of U_{zm} for a z position (away from the center and boundary). At the beginning, the function P_{moy}= f (ΔU_{zm}) takes a linear profile for the first positions then the profile becomes increasingly nonlinear while approaching the walls. This nonlinearity is caused by the effects of boundaries on the mixture velocity.

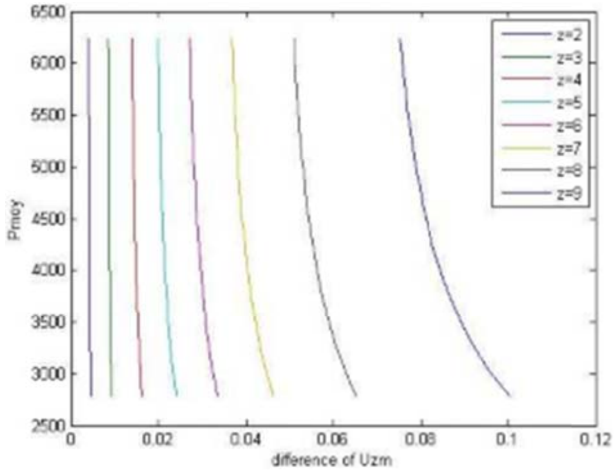

Fig. 3 Distribution of U_{zm} difference between x=3 and x=8 with P_{moy} at z positions

According to the results shown in Fig. 3, it is appropriate to determine the mean mixture pressure using linear curves. For this, the functions P_{moy}= f (ΔU_{zm}) for z=2, 3, and 4 are established mathematically by linear fittings as P_{moy}=aΔU_{zm}+b. After comparison between the curves and fittings (Fig. 4), it turns out that only curves related to both z=2 and z=3 are linear and the linearity starts to disappear since z=4. Then, the mean mixture pressure is calculated with radial velocity's difference in positions z=2 and z=3. The two linear curves of positions z=2 and z=3 are described by the following linear equations:

For position z=2:

$$P_{moy} = -7.6685e^{06}\Delta U_{zm} + 0.0371e^{06} \tag{7}$$

For position z=3:

$$P_{moy} = -4.8493e^{06}\Delta U_{zm} + 0.0471e^{06} \tag{8}$$

The calculated function at radial positions z=2 and z=3 will be operated to define the mean mixture pressure. The first function will be used to measure the P_{moy} and the second one will be employed to verify the measured value of P_{moy}.

Equations (4) and (5) relate to the mean gas pressure P_{gmoy} to the mean mixture pressure. Fig. 5 shows the variation P_{gmoy} with the P_{moy}. This will be able to determine the mean carbon dioxide pressure in the mixture through the difference of radial velocity.

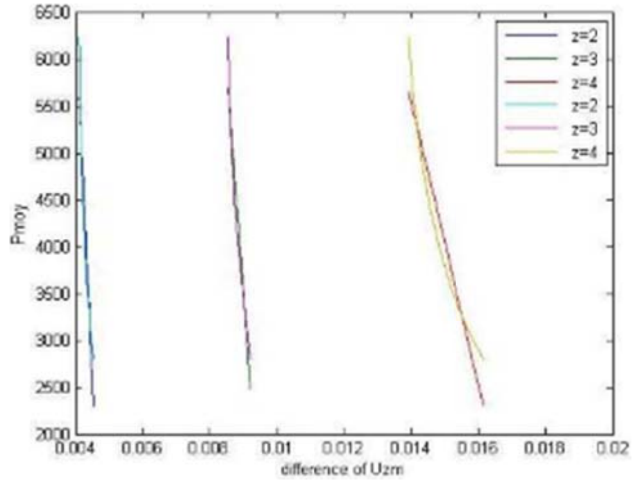

Fig. 4 Superposition of real curves and fittings of U_{zm} difference at z=2, z=3, and z=4

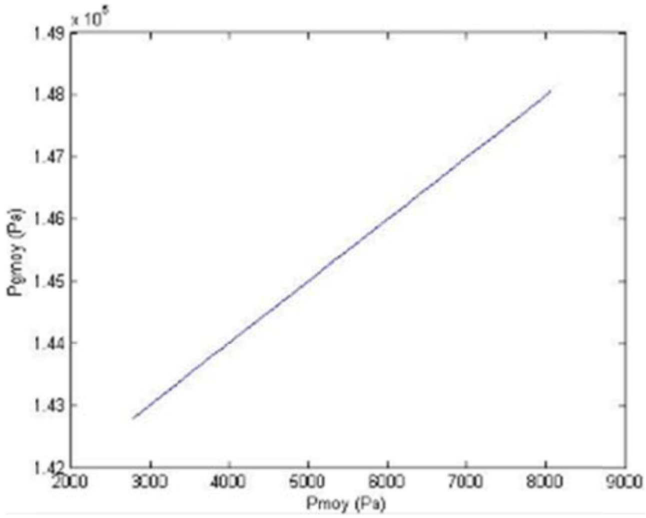

Fig. 5 Variation of P_{gmoy} with P_{moy}

IV. Conclusion

The arterial carbon dioxide pressure presents an indispensable parameter to control the mechanical ventilation of patients in ICUs. To measure this parameter in a non-invasive way, the arterial blood was modeled by the Drift flux model that relates the fluid velocity and the fluid pressure and the Young-Laplace equation that involves the gas pressure. The numerical simulations of this model were exploited to the calculation of mixture radial velocity distribution in x and z positions with different values of mean mixture pressure P_{moy}. These distributions allow to determine the calculation methods of mean mixture pressure by ultrasound measurement of radial velocity U_{zm} at different z and x positions. To make this method practical, it is suitable to determine the mean mixture pressure through radial velocity differences along z axis at two x positions. The use of two probes permits to avoid measurement errors due to the confusion in ultrasound detection. According to the calculated distribution of U_{zm} differences between the two considered x positions, two linear curves are chosen to determine the mean mixture pressure at z=2 and z=3. The fitting of these curves are realized as P_{moy}=aΔU_{zm}+b. Then, these functions are used to determine P_{moy} through measured radial velocity differences and consequently to deduce the mean carbon dioxide pressure.

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