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Fractional-Order PI Controller Tuning Rules for Cascade Control System

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Abstract**—**The fractional–order proportional integral (FOPI) controller tuning rules based on the fractional calculus for the cascade control system are systematically proposed in this paper. Accordingly, the ideal controller is obtained by using internal model control (IMC) approach for both the inner and outer loops, which gives the desired closed-loop responses. On the basis of the fractional calculus, the analytical tuning rules of FOPI controller for the inner loop can be established in the frequency domain. Besides, the outer loop is tuned by using any integer PI/PID controller tuning rules in the literature. The simulation study is considered for the stable process model and the results demonstrate the simplicity, flexibility, and effectiveness of the proposed method for the cascade control system in compared with the other methods.

Keywords**—**Fractional calculus, fractional–order proportional integral controller, cascade control system, internal model control approach.

I. Introduction

The performance of cascade control system largely depends on tuning of both inner and outer loops. For the design method based on the frequency response, the tuning rules given by [1]-[3] are usually recommended to design the controllers in terms of higher order dynamics and/or time delay in the open loop transfer function of outer loop. However, the frequency response methods also have a major weakness due to many trial and error graphical calculations. Krishnaswamy [4] introduced the tuning charts, which can be predicted the primary controller parameters by using the integral time absolute error (ITAE) criterion for the load disturbance on the secondary loop. However, this method is also limited to use for the first-order plus dead time (FOPDT) model and just focused on the proportional integral/ proportional (PI/P) configuration. Therefore, the overall performance of control system can be poor for higher order process models. To overcome this problem, [5] introduced the tuning rules to obtain desired closed loop responses for the cascade control system and showed the enhanced overall performance. In general, there are two steps for the tuning of cascade control system: the secondary controller is firstly tuned based on the inner process model and the primary controller is then tuned based on the outer process model. Accordingly, if the secondary controller is retuned for some uncertainties, an additional identification step is essential for retuning the primary controller, which is often unwieldy in the industry.

Recently, fractional calculus [6], [7] that has been an increasing attention paid to fractional-order processes, which are really useful to represent the different stable physical phenomena with anomalous decay, both from the academic and control engineers for the modeling and control issues due to its flexibility and advancement in terms of computation power. Besides, the fractional-order differential equations (FODE) can be obtained by using fractional calculus and is also a generalization of the ordinary differential equations (ODE). The generalization of the PID controller, which is so-called the $PI^{\lambda}D^{\mu}$ [8], is involved two extra parameters as the fractional-order integrator (λ) and fractional-order differentiator (μ). The fractional-order PID controller affords more flexibility in PID controller design due to the selection of five controller parameters that include the proportional gain, the integral gain, the derivative gain, the integral order, and the derivative order. However, the tuning rules of FOPI/PID controller are much more complex in compared with standard (integer) PID controller that has only three parameters [8], [9]. In order to pose the same ease of use of standard PID controller, there are many different ways to design the FOPI/PID controller. The first mention involving the use of fractional structure in a feedback-loop was early made by [10] and then it was extended by [11], where a feedback amplifier was obtained by considering a feedback-loop in terms of the performance of the closed-loop that was invariant to changes in amplifier gain. However, this idea was not concretized and remained for decades as a simple proposition. Oustaloup [12] introduced the fractional-order algorithms for the control of dynamic system based on non-integer derivative and demonstrated the significant improvement of the CRONE (French abbreviation of Commande Robuste d'Ordre Non Entier) controller in compared with the integer PID controller. In general, due to its two extra parameters (λ and μ), the fractional order PID controller can be achieved better performance in compared with the classical PID controller and it has been become a new trend to solve many industrial control problems [13], [14]. In accordance with the literature, the tuning method of $PI^{\lambda}D^{\mu}$ can be generally classified as analytic [9] and heuristic methods [15]. In fact, most of the analytic methods are often tuned by considering the nonlinear objective function, which is depended on the specification imposed by the users [15].

In this paper, our aim is to design an analytic method of generalized FOPI controller for enhanced performances of both integer and fractional processes with time delay for

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cascade control systems. It is mainly based on the concepts of fractional calculus [6] and IMC approach [16]. By using the frequency domain, the proposed PI tuning rules can be directly derived for many typical process models without introducing any nonlinear objective function.

II. Preliminaries

Some basic fundamentals of fractional calculus, together with the problem statement that need to understand the fractional system, as well as the controller are briefly introduced in this section.

A. Fractional Calculus

Fractional calculus [6] is generalization of the ordinary calculus, which is developed a functional operator *D*, associated to an order *v* that is not restricted to integer numbers. It generalizes usual notions of derivative for a positive *v* and integrals for a negative *v*.

It is clear that there are various kinds of definitions for fractional derivative. However, the most commonly use is the Riemann-Liouville definition [6], which is generalized two equalities easily proved for integer orders:

$$ {}_0D_x^{-n} f(x) = \int_c^x \frac{(x-t)^{n-1}}{(n-1)!} f(t)\,dt, \quad n \in N \tag{1}$$

It is important to note that the generalized definition of *D* becomes ${}_cD_x^v f(x)$. The Laplace transform of *D* pursues the well-known rule for zero initial condition as $L\left[{}_0D_x^v f(x)\right] = s^v F(s)$. It is implied that under initial condition, the system with a dynamic behavior described by differential equations involving fractional derivative give rise to transfer functions with fractional power of *s*. More details are given in [6].

B. Fractional Linear Model

According to a single-input, single-output (SISO) linear time invariant (LTI) system, the FODE, provided both input and output signals u(t) and y(t) that is relaxed at t = 0, can be expressed by differential equation:

$$\sum_{i=0}^{n} a_i D_0^{\alpha_i} y(t) = \sum_{j=0}^{m} b_j D_0^{\beta_j} u(t) \tag{2}$$

As a result, (2) can be described in the Laplace domain by the following transfer function:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_0 s^{\alpha_0}} \tag{3}$$

where α_i and α_i are arbitrary real positive.

III. Analytical Tuning Rules of FOPI Controllers for Cascade Control System

A. Design of FOPI Controller in Frequency Domain

The fractional integro-differential equation of the FOPI controller is described by

$$u(t) = \mathrm{K_C} e(t) + \mathrm{K_I} D_t^{-\lambda} e(t), \quad (\lambda > 0) \tag{4}$$

where $\mathrm{K_C}, \mathrm{K_I}$, and λ denote the proportional term, integral term, and fractional order in the FOPI controller, respectively.

The continuous transfer function of the FOPI controller can be obtained through Laplace transformation as:

$$G_C(s) = K_C + \frac{K_I}{s^\lambda} \tag{5}$$

From (5), it is clear that the FOPI controller involves three parameters (K_C, K_I, and λ) to tune, since the fractional order λ is not necessarily integer.

The FOPI controller is represented in the frequency domain by substituting $s = j\omega$ into (5):

$$\mathrm{G_C}(j\omega) = \mathrm{K_C} + \frac{\mathrm{K_I}}{(j\omega)^\lambda} \tag{6}$$

Hence, the convenient form is given as:

$$(j\omega)^\lambda = \omega^\lambda (\cos\gamma_I + j\sin\gamma_I), \quad \gamma_I = \frac{\pi\lambda}{2} \tag{7}$$

The FOPI controller in terms of the complex equation is established by substituting (7) into (6):

$$G_C(j\omega) = \left(K_C + \frac{K_I \cos\gamma_I}{\omega^\lambda}\right) - j\frac{(K_I \sin\gamma_I)}{\omega^\lambda} \tag{8}$$

B. FOPI Controller Design Procedure for General Process Models

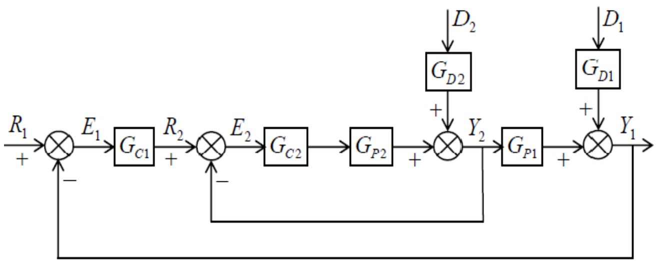

Fig. 1 Block diagram of cascade control system

The cascade control system is shown in Fig. 1; the closed loop transfer functions for inner and outer loops are obtained by:

$$\frac{Y_2}{R_2} = \frac{G_{C2} G_{P2}}{1 + G_{C2} G_{P2}} \tag{9}$$

$$\frac{Y_1}{R_1}=\frac{G_{C1}G_{C2}G_{P1}G_{P2}}{1+G_{C2}G_{P2}+G_{C1}G_{C2}G_{P1}G_{P2}} \tag{10}$$

Here, the controllers G_{C1} and G_{C2} have to be designed to satisfy set-point tracking ($R1$) and disturbance ($D1$, $D2$) regulating requirements.

C. Design of Secondary Controller

A secondary controller has to be designed to such that set-point tracking (Y_2/R_2) gives a stable overdamping response. In accordance with the IMC parameterization introduced by [16], the process model $\tilde{G}_P(s)$ is factored into two parts: $\tilde{G}_P(s)=p_m(s)p_A(s)$, where $p_m(s)$ is the portion of the model inverted by the controller (minimum phase), $p_A(s)$ is the portion of the model not inverted by the controller (it is the non-minimum phase that may be included the dead time and/or right half plane zeros and chosen to be all-pass), and the requirement that $p_A(0)=1$ is necessary for the controlled variable to track its set-point. Then, the IMC controller $q(s)$ can be designed as $q(s)=p_m^{-1}(s)f(s)$. For the 1DOF control structure, the IMC filter $f(s)$ is chosen for enhanced performance as:

$$f(s)=\frac{1}{(\tau_c s+1)^r} \tag{11}$$

where τ_c is an adjustable parameter, which can be utilized for the tradeoffs between the performance and robustness. The integer r is selected to be large enough for the IMC controller proper. Then, the IMC controller is obtained by

$$q(s)=p_m^{-1}(s)\frac{1}{(\tau_c s+1)^r} \tag{12}$$

Substituting (11) into (9), the closed-loop transfer functions for the desired set-point is simplified as:

$$\frac{Y_2}{R_2}=p_{A2}(s)\frac{1}{(\tau_{c2}s+1)^{r2}} \tag{13}$$

The ideal feedback controller G_{C2} that yields the desired loop responses can be constituted by

$$G_{C2}=\frac{q_2}{1-\tilde{G}_{P2}q_2} \tag{14}$$

Therefore, the ideal feedback controller can be found by

$$G_{C2}=\frac{p_{m2}^{-1}}{(\tau_{c2}s+1)^{r2}-p_{A2}} \tag{15}$$

The resulting controller given by (15) does not have the FOPI-type controller form despite that it is physically realizable. Therefore, it should be transformed into the complex form, and then compared with (8). Finally, the analytical tuning rules can be simply derived for a number of process models.

Consider the FOPDT process model as following:

$$G(s)=\frac{K}{\tau s+1}e^{-\theta s} \tag{16}$$

In accordance with the above-mention procedure for the design of IMC-based controller, the ideal feedback controller equivalent to the IMC controller can be found by

$$G_{c2}(s)=\frac{\tau s+1}{K\left[(\tau_{c2}s+1)-e^{-\theta s}\right]} \tag{17}$$

By substituting $s=j\omega$ into (17), it yields

$$G_{c2}(j\omega)=\frac{1-\cos(\gamma)+(\tau_{c2}\omega+\sin(\gamma))\tau\omega}{K\left(4\sin^2\left(\frac{\gamma}{2}\right)+\tau_{c2}\omega\sin(\gamma)+\tau_{c2}^2\omega^2\right)}+ j\frac{\left[\tau\omega(1-\cos(\gamma))-\tau_{c2}\omega-\sin(\gamma)\right]}{K\left(4\sin^2\left(\frac{\gamma}{2}\right)+\tau_{c2}\omega\sin(\gamma)+\tau_{c2}^2\omega^2\right)} \tag{18}$$

where,

$$e^{-j\theta\omega}=\cos(\gamma)-j\sin(\gamma),\quad \gamma=\theta\omega \tag{19}$$

By comparing (18) with (6), the analytical tuning rules can be found as:

$$K_{I2}=\frac{\omega^{\lambda}\left[\sin(\gamma)+\tau_{c2}\omega-\tau\omega(1-\cos(\gamma))\right]}{K\omega\sin(\gamma_I)\left(4\sin^2\left(\frac{\gamma}{2}\right)+\tau_{c2}\omega\sin(\gamma)+\tau_{c2}^2\omega^2\right)} \tag{20}$$

$$K_{c2}=\frac{1-\cos(\gamma)+(\tau_{c2}\omega+\sin(\gamma))\tau\omega}{K\left(4\sin^2\left(\frac{\gamma}{2}\right)+\tau_{c2}\omega\sin(\gamma)+\tau_{c2}^2\omega^2\right)}-\frac{K_{I2}\cos(\gamma_I)}{\omega^{\lambda}} \tag{21}$$

D. Design of Primary Controller

The closed loop transfer function for the outer loop can be approximately represented by

$$\frac{Y_1}{R_1}=\frac{G_{C1}G_{P1}\frac{P_{A2}}{(\tau_{c2}s+1)^{r2}}}{1+G_{C1}G_{P1}\frac{P_{A2}}{(\tau_{c2}s+1)^{r2}}} \tag{22}$$

Therefore, the process model of the outer loop is considered as

$$G_1=G_{P1}\frac{P_{A2}}{(\tau_{c2}s+1)^{r2}} \tag{23}$$

Now, consider a stable process model of the outer loop of the form:

$$G_1(s) = p_{m1}(s)\, p_{A1}(s) \tag{24}$$

Here, our purpose is also to design the controller, G_{C1}, so that the closed loop transfer function of the outer loop, $Y1/R1$, has the form given by

$$\frac{Y_1}{R_1} = p_{A1}(s)\frac{1}{(\tau_{c1}s+1)^{r1}} \tag{25}$$

Then, the controller transfer function of the outer loop is represented by:

$$G_{C1}(s) = \frac{q_1}{1-G_1 q_1} = \frac{P_{m1}^{-1}(s)(\tau_{c2}s+1)^{r2}}{P_{A2}(s)\left[(\tau_{c1}s+1)^{r1} - P_{A1}(s)\right]} \tag{26}$$

The primary controller G_{C1} can be approximated to the PI/PID controller form as shown in [5].

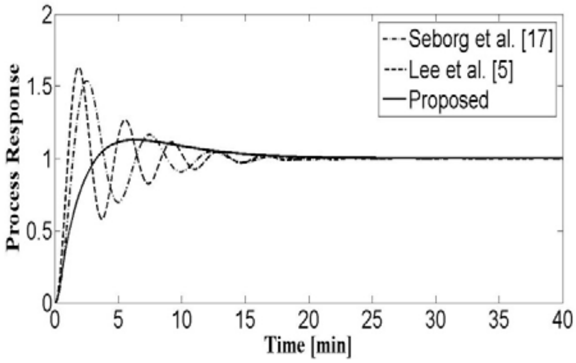

Fig. 2 Closed loop response due to set point change (Y_1/R_1) for the illustrative example

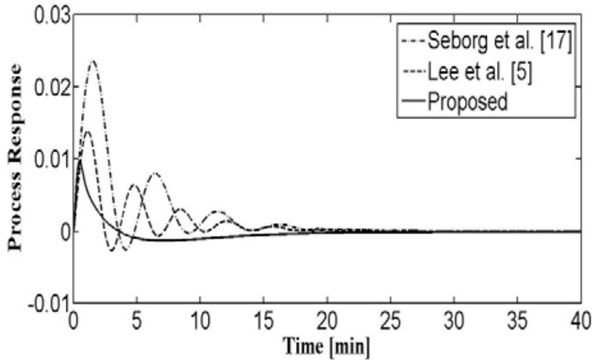

Fig. 3 Closed loop response due to set point change (Y_1/D_2) for the illustrative example

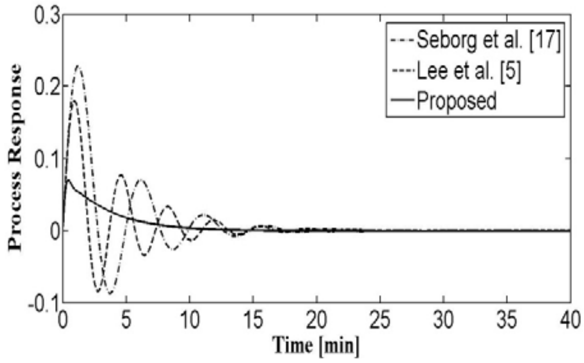

Fig. 4 Closed loop response due to set point change (Y_1/D_1) for the illustrative example

IV. Simulation Study

In order to have a fair comparison, the IAE criterion is considered here for the set-point tracking [17]:

$$IAE = \int_0^\infty |e(t)|dt \tag{27}$$

In this section, the following process model introduced by [17] and [5] was studied as the illustrated example to demonstrate the performance of the proposed method in comparison with those of other well-known methods.

TABLE I
TUNING VALUES BY ALL OF COMPARATIVE DESIGN METHODS FOR THE ILLUSTRATIVE EXAMPLE

Controller parameters	Proposed	Seborg *et al*. [17]	Lee *et al*. [5]
	Outer loop controller		
K_{C1}	6.2	3.5	6.2
τ_{I1}	6.2	5.3	6.2
τ_{D1}	1.48	-	-
	Inner loop controller		
K_{C2}	1.68	4	5
τ_{I2}	0.82	-	-
λ_2	0.9	-	-
IAE	2.42	2.75	2.77

$$\begin{aligned} G_{P1} &= \frac{4}{(2s+1)(4s+1)} \\ G_{P2} &= \frac{5}{(s+1)} \\ G_{D1} &= \frac{1}{(3s+1)} \\ G_{D2} &= 1 \\ G_{m1} &= 0.05 \\ G_{m2} &= 0.2 \end{aligned} \tag{28}$$

The PID controllers for inner and outer loops for the above process were tuned by the proposed tuning rules. The results were compared with those by the frequency method [17] and

[5]. The resulting PID parameters are listed in Table I. Since the PID controllers in cascade control should be tuned considering all the closed loop performances both for set-point tracking (Y_1/R_1) and disturbance rejection (Y_1/D_1 and Y_1/D_2), the tuning methods were tested in terms of all these performances. Figs. 2-4 show the closed loop responses tuned by the proposed method and the frequency response method for the unit step change in R_1, L_2, and L_1, respectively. The results shown in the figures illustrate the superior performance of the proposed method.

IV. Conclusion

An analytical design method of FOPI controller for the cascade control systems was proposed based on fractional calculus and IMC approach to provide improved performance for both disturbance rejection and set-point tracking. The simulation studies demonstrate that can be applied to a large number of dynamic models, consistently afforded the superior performance with fast and well-balanced closed-loop time responses.

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