

# Gas Pressure Evaluation through Radial Velocity Measurement of Fluid Flow Modeled by Drift Flux Model

Aicha Rima Cheniti, Hatem Besbes, Joseph Haggege, Christophe Sintès

**Abstract**—In this paper, we consider a drift flux mixture model of the blood flow. The mixture consists of gas phase which is carbon dioxide and liquid phase which is an aqueous carbon dioxide solution. This model was used to determine the distributions of the mixture velocity, the mixture pressure, and the carbon dioxide pressure. These theoretical data are used to determine a measurement method of mean gas pressure through the determination of radial velocity distribution. This method can be applicable in experimental domain.

**Keywords**—Mean carbon dioxide pressure, mean mixture pressure, mixture velocity, radial velocity.

## I. INTRODUCTION

FOR security or clinical purposes, it remains usually interesting to measure the pressure of the gas in mixture of gas and liquid in many kinds of fluid flows. For the clinical case for example, the measurement of carbon dioxide arterial pressure is extremely needed in intensive care units.

The two-phase flow phenomena are observed in different systems like transport systems (pump and ejectors), process systems (chemical reactors, phase separators), heat transfer systems (evaporators, condensers), power systems (boiling water, two-phase propulsors), biological systems (cardiovascular system, blood flow), and so on [1]. The possible phases' combinations of two-phase flow in pipe are gas and liquid phases, solid and liquid phases, gas and solid phases, and two immiscible liquid phases [1]. Two-phase flow is described by three models as homogeneous equilibrium mixture model [2], two-fluid model [1]-[3], and drift flux model [1]. These models are governed by the same physical laws of transport of mass, momentum, and energy.

The combination of gas and liquid phases is the most frequent for the two-phase flow in biological system for example plasmatic solution and carbon dioxide. In the case of miscible gas, suitable model is the Drift flux [4]. The study of

Aicha Rima Cheniti is with the National Engineering School of Tunis, Tunisia, BP, 37, Le Belvedere 1002 Tunis (e-mail: arima\_mer@yahoo.fr).

Hatem Besbes is with the Laboratory of Biophysics and Medical Technologies in Higher Institute of Medical Technologies of Tunis (ISTMT). He is with Faculty of Medicine of Tunis, 15 Rue Djebel Lakhthar. La Rabta. 1007, Tunis, Tunisia (besbesh@gmail.com).

Joseph Haggege is with the Laboratory LARA, Electrical Engineering Department, National Engineering School of Tunis, BP, 37, Le Belvedere 1002 Tunis (e-mail: joseph.haggege@enit.rnu.tn).

Christophe Sintès is with Lab-STICC, ITI Department, Telecom Bretagne, Technopole Brest-Iroise, CS 83818, 29238 Brest Cedex 3, France (e-mail: christophe.sintes@telecom-bretagne.eu).

gas phase in a mixture by the considered model brings important data to analyze and understand the problems of a particular blood system. In this work, we are interested in the study of a system consisting of a two-phase incompressible Newtonian mixture moving horizontally through a constricted rigid cylindrical tube with a constant cross-section. For this, we consider an aqueous carbon dioxide solution with carbon dioxide gas. This mixture is used to simulate the behaviour of the blood flow and to determine the carbon dioxide pressure existing in the considered mixture. In fact, the theoretical study of this system affords distributions of mixture velocities, mixture pressure, and carbon dioxide pressure [4]. This study can allow the evaluation of mean gas pressure through the determination of radial velocities distribution.

## II. METHODS AND MATERIALS

### A. Calculation

For the simulation of physical parameters of the mixture, MATLAB is used. Thus, the calculation undergoes the evaluation of the mean mixture pressure and the mean carbon dioxide pressure knowing radial velocities distribution.

### B. Mathematical Model

In the first step, the arterial blood was modelled as an aqueous solution of carbon dioxide mixed with the carbon dioxide gas, and the arterial wall is modelled as a rigid canalization [4]. The Drift flux model allows the correlation between the flow velocities and the mixture pressure. The governing equations describing physical parameters in the considered model are mass and momentum conservation equations (1)-(3).

1) Mixture continuity equation

$$\nabla \cdot (\rho_m U_m) = 0 \quad (1)$$

2) Continuity equation for dispersed phase

$$\nabla \cdot (\alpha_2 \rho_2 U_m) = -\nabla \cdot \left( \frac{\alpha_2 \rho_1 \rho_2}{\rho_m} V_{2j} \right) \quad (2)$$

3) Mixture momentum equation

$$\nabla \cdot (\rho_m U_m) = -\nabla p_m + \nabla \cdot (\tau) + \rho_m g_m - \nabla \cdot \left( \frac{\alpha_2 \rho_1 \rho_2}{1 - \alpha_2} V_{2j} V_{2j} \right) \quad (3)$$

where, phase1 presents liquid phase, phase2 presents gas phase,  $U_m$  is the mixture velocity vector,  $\rho_m$  is the mixture density,  $\alpha_2$

is the volumetric fraction of gas,  $\rho_2$  is the gas density,  $\rho_1$  is the liquid density,  $p_m$  is the mixture pressure,  $g_m$  is the gravitational acceleration vector,  $V_{2j}$  is the drift velocity vector of the gas phase with respect to the volume center of the mixture, and  $\tau$  is the viscous stress [1].

The mixture pressure is related to the phase pressures by

$$p_m = \sum_{k=1}^2 \alpha_k p_k \quad (4)$$

where  $\alpha_k$  presents the volumetric fraction of  $k^{\text{th}}$  phase and  $p_k$  presents the pressure of  $k^{\text{th}}$  phase.

The Young-Laplace equation is added to this model to relate both gas pressure and liquid pressure. The equation is defined by

$$p_2 - p_1 = \frac{2\sigma}{R} \quad (5)$$

where  $p_1$  presents liquid pressure,  $p_2$  presents gas pressure,  $R$  presents bubble's radius, and  $\sigma$  presents the surface tension [5], [6].

The drift flux model described previously and the Young-Laplace equation permit the determination of the gas pressure knowing the mixture velocity and pressure distributions [4].

### C. Physical Parameters Determination

The resolution of equations system already mentioned in the last section gives the distribution of radial velocity variation with the mean mixture pressure. The mean mixture pressure  $P_{moy}$  is calculated by the integration of pressure values in different cells. The mean gas pressure  $P_{gmoy}$  is deduced from

the mean mixture pressure by using (4) and (5). The choice of mean value is related to the measurement processes which usually give mean results.

The calculations are done in finite-volume discretization model in rigid canalisation with 10 cm length and 4 mm radius with reference to the radial artery dimensions [7]. The measurement domain is divided symmetrically to x-axis in cells ranges with 10 mm x 0.4 mm x 0.4 mm. The number of velocities' samples  $U_{zm}$  (along z axis),  $U_{ym}$  (along y axis), and  $U_{xm}$  (along x axis) are chosen respectively as  $9 \times 9 \times 10$ ,  $9 \times 10 \times 9$ , and  $10 \times 9 \times 9$  in the x-y-z space.

### III. RESULTS AND DISCUSSION

Numerical simulations permitted to calculate the distribution of radial velocities  $U_{zm}$  for different mean mixture pressures  $P_{moy}$  (Fig. 1). The calculated results show an incoherence in the two first cell ranges and the last cell range. This incoherence is manifested by the non-derivability of the representative functions in these intervals. Thus, the study interests the cell ranges between the third and the eighth relatively to x direction ( $x=3$  and  $x=8$ ). The radial velocities  $U_{zm}$  distribution is calculated for different z values (z from 2 to 9) relatively to different calculated mean mixture pressure values (Fig. 2).

From the curves visualised on Figs. 2-9 it is possible to deduce the functions families  $P_{moy} = f(U_{zm})$  for different x positions. So, the determination of a given  $U_{zm}$  value for known positions z and x (in the selected interval) permits to deduce  $P_{moy}$ .

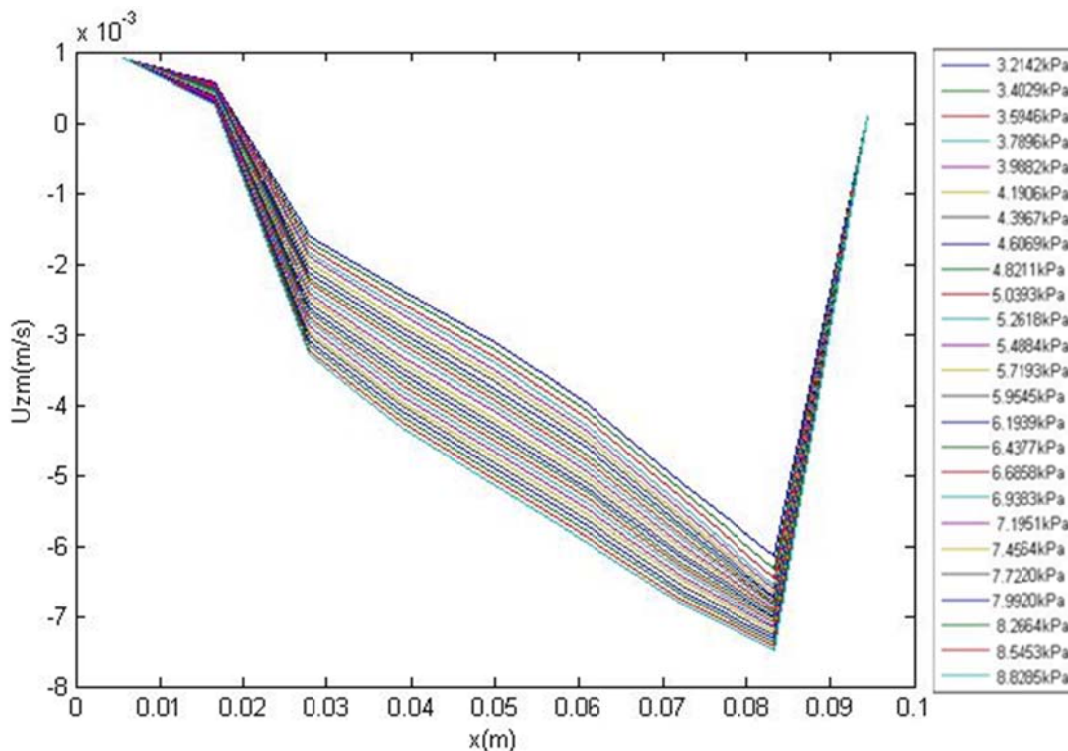


Fig. 1 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$  for  $z=2$

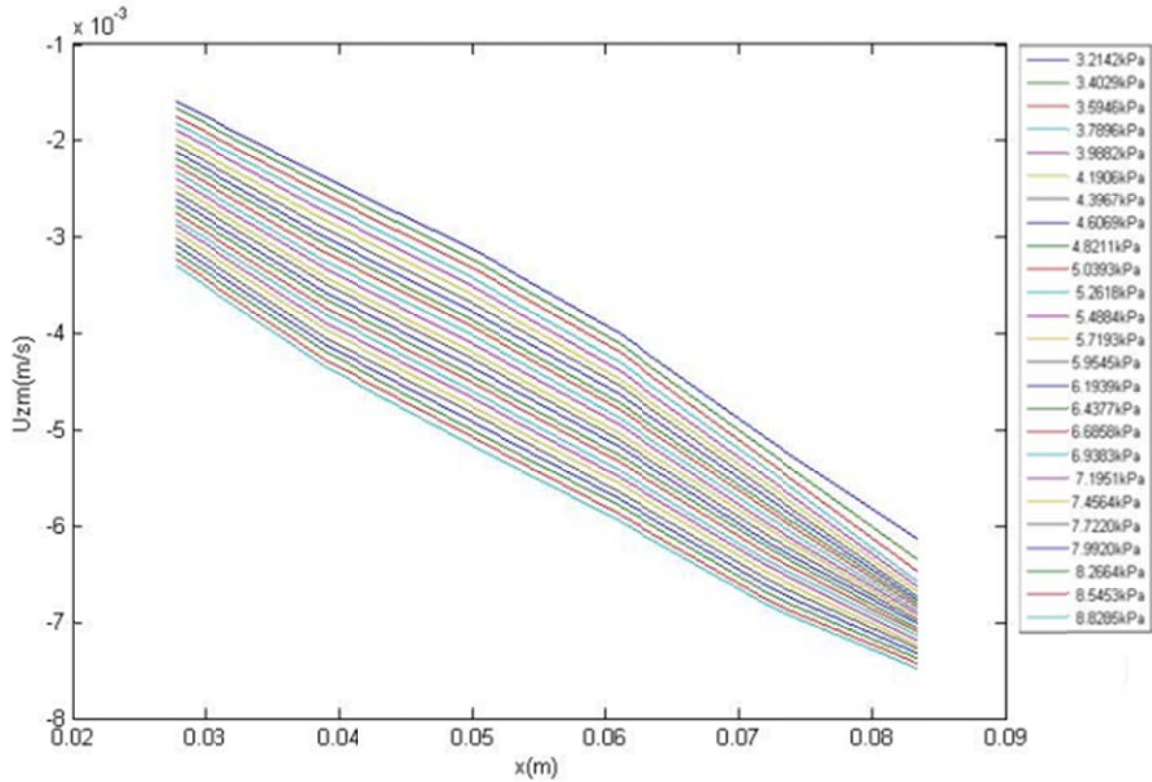


Fig. 2 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$ , between  $x=3$  and  $x=8$  for  $z=2$

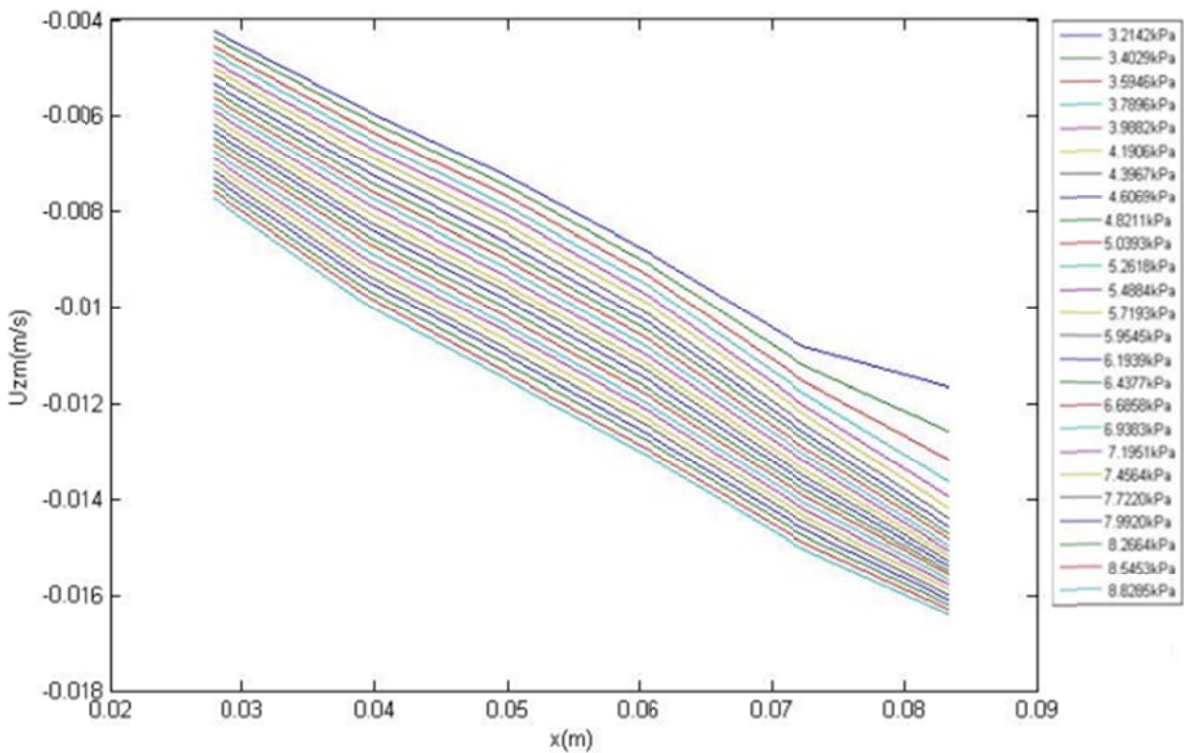


Fig. 3 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$ , between  $x=3$  and  $x=8$  for  $z=3$

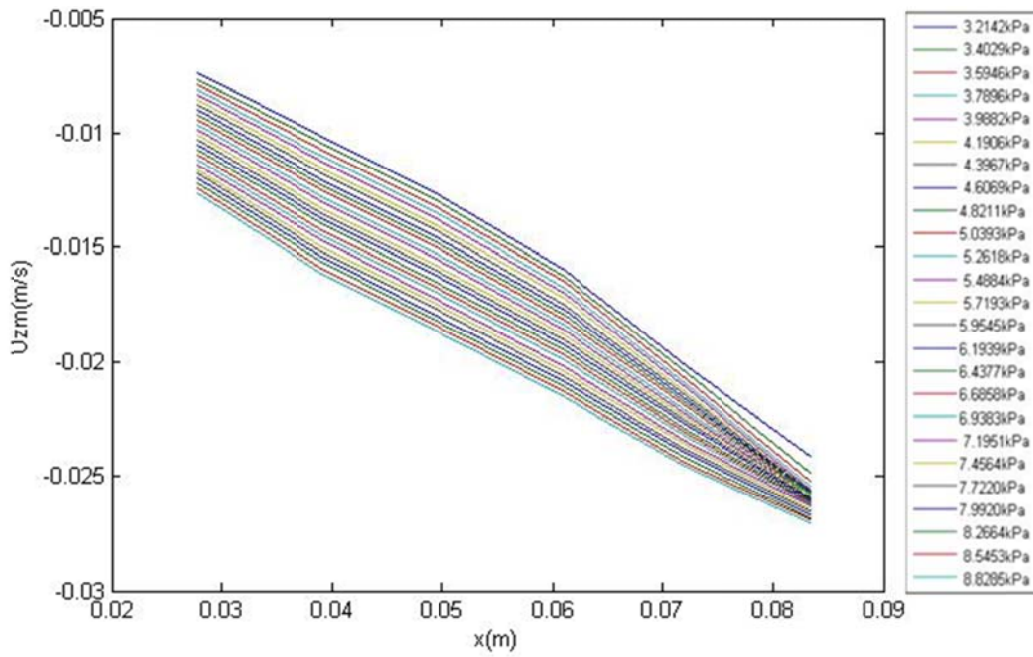


Fig. 4 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$  between  $x=3$  and  $x=8$  for  $z=4$

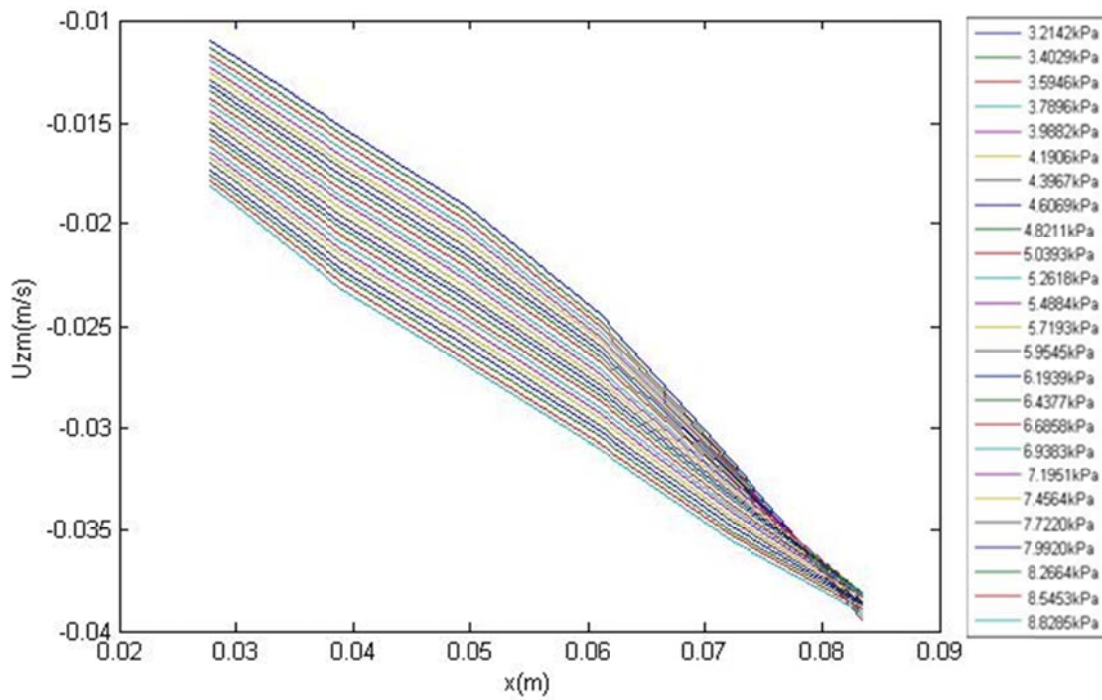


Fig. 5 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$  between  $x=3$  and  $x=8$  for  $z=5$

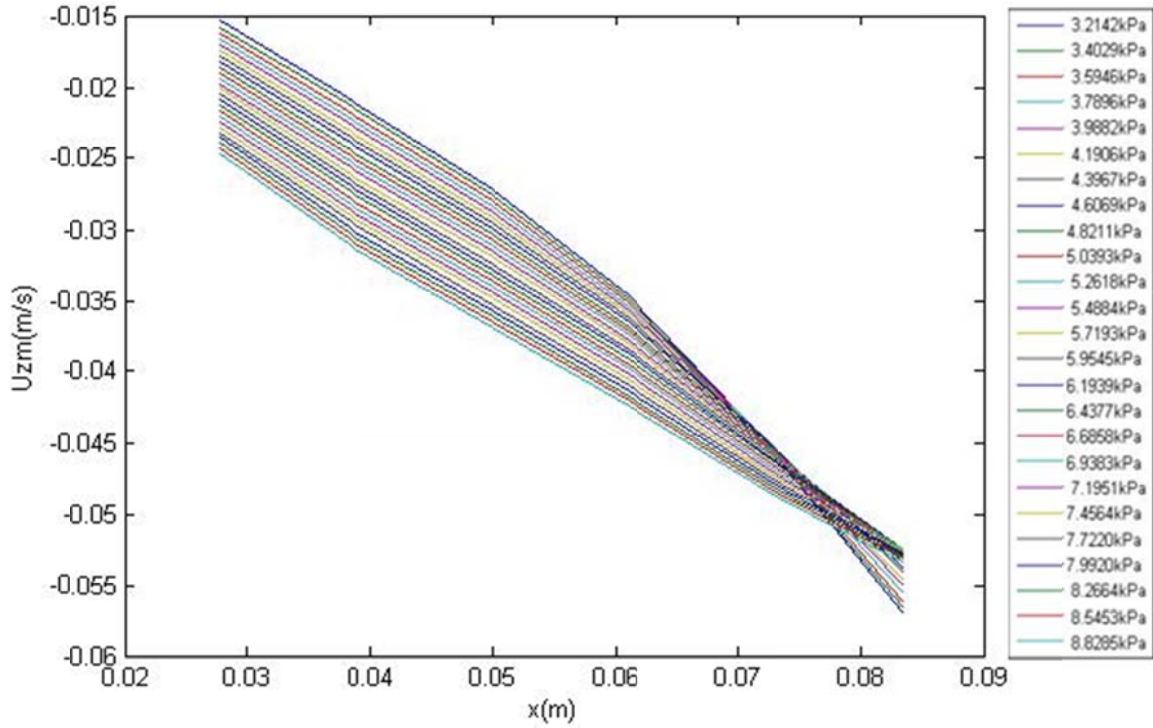


Fig. 6 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$  between  $x=3$  and  $x=8$  for  $z=6$

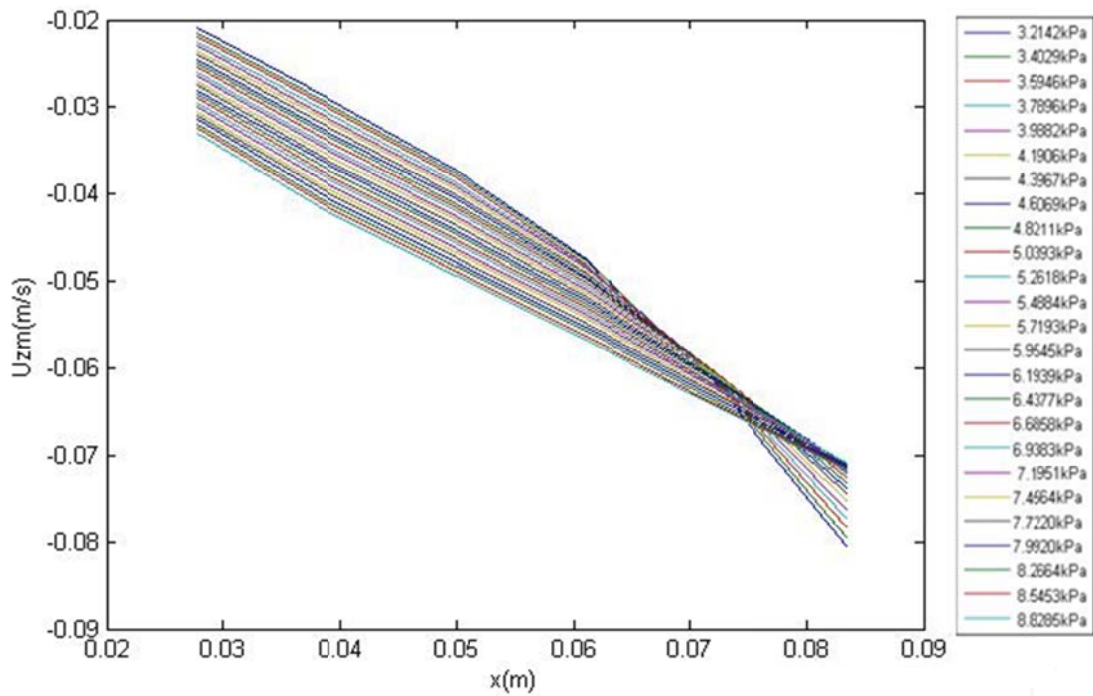


Fig. 7 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$  between  $x=3$  and  $x=8$  for  $z=7$

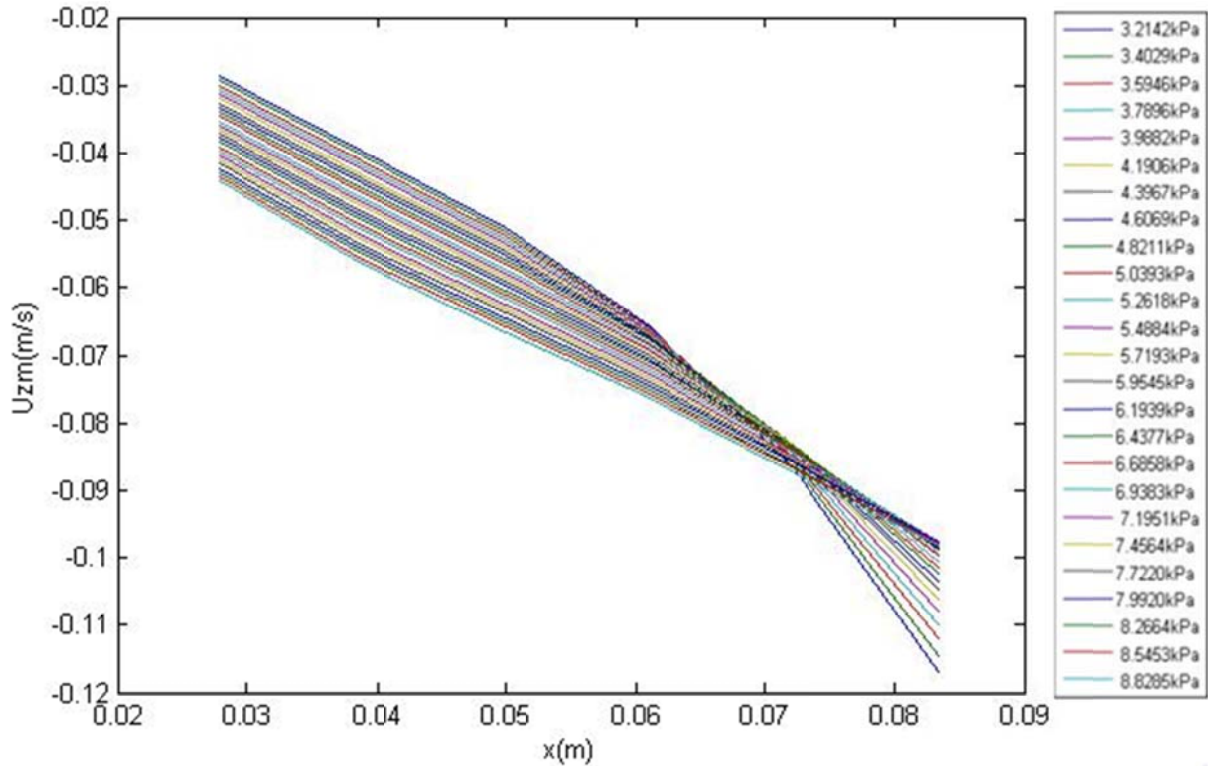


Fig. 8 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$ , between  $x=3$  and  $x=8$  for  $z=8$

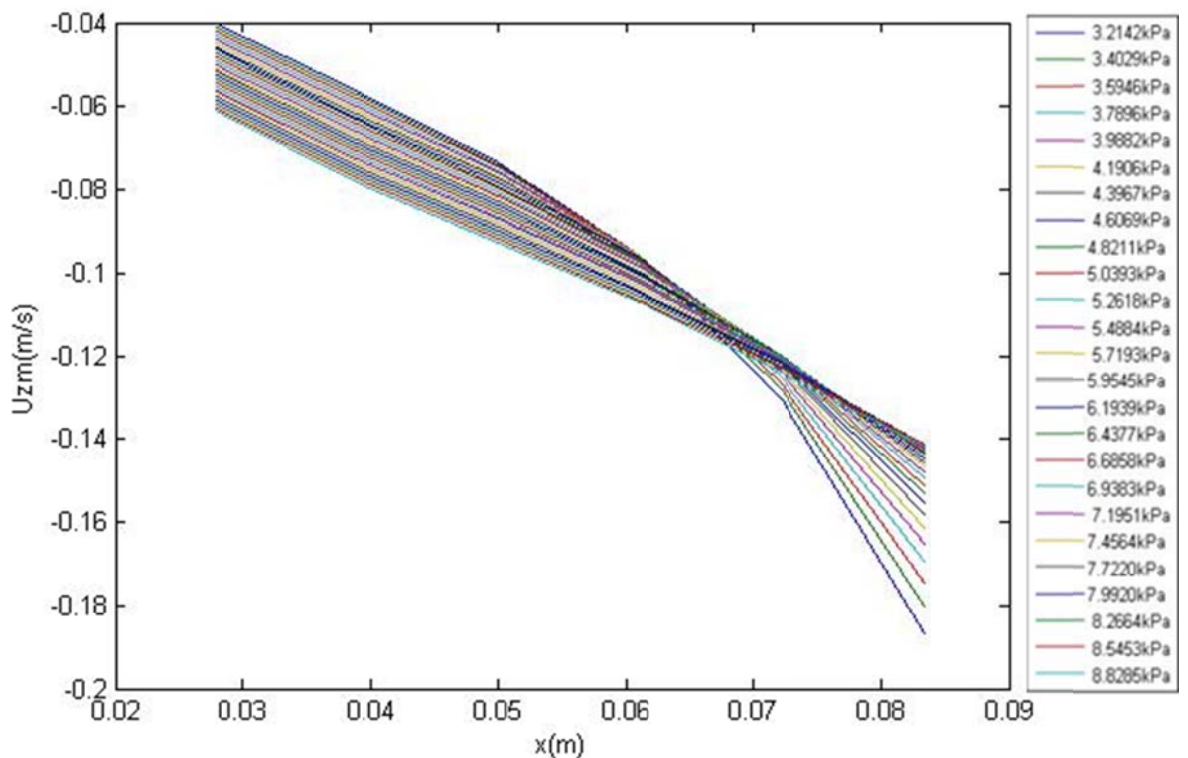


Fig. 9 Variation of  $U_{zm}$  along x-axis with different values of  $P_{moy}$ , between  $x=3$  and  $x=8$  for  $z=9$

On Figs. 10-15, we can observe the clear variation in the distribution of mean mixture pressures from the position  $x=3$  to the position  $x=8$ , but there is generally a similarity of curve

shapes at different  $z$  level. Then, these characteristic functions can be used as a pressure scales.

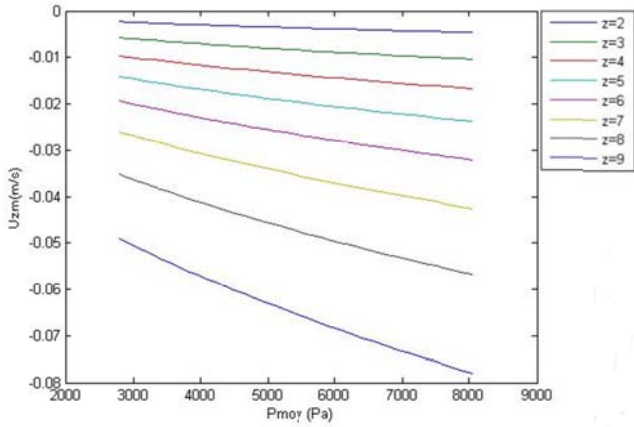


Fig. 10 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=3$

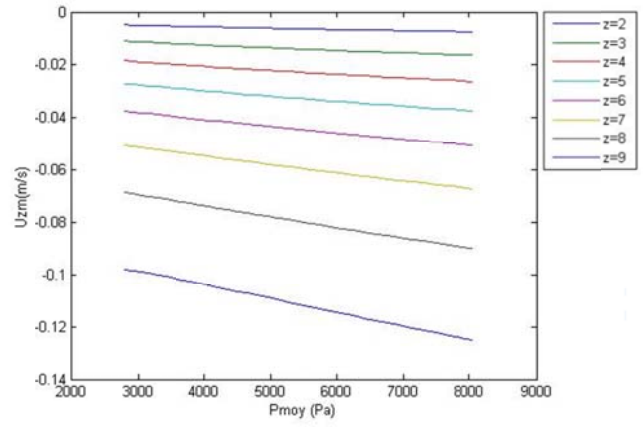


Fig. 13 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=6$

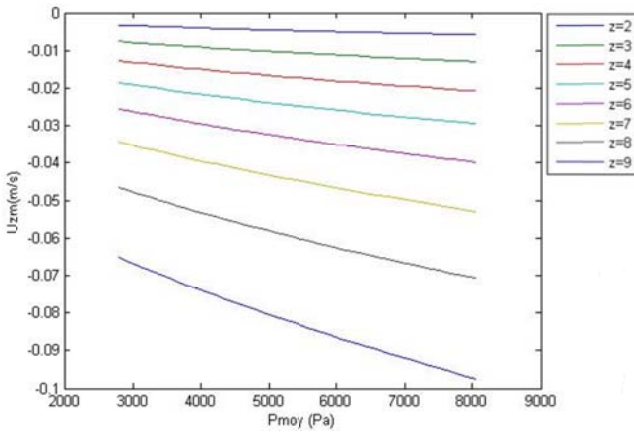


Fig. 11 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=4$

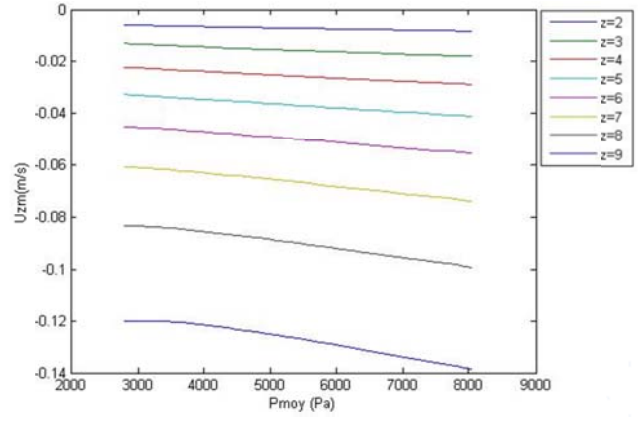


Fig. 14 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=7$

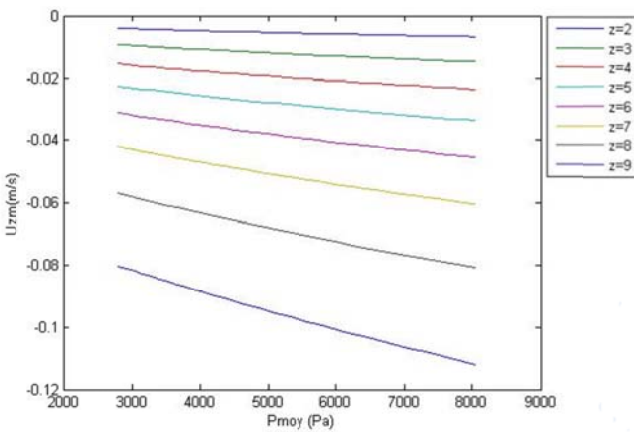


Fig. 12 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=5$

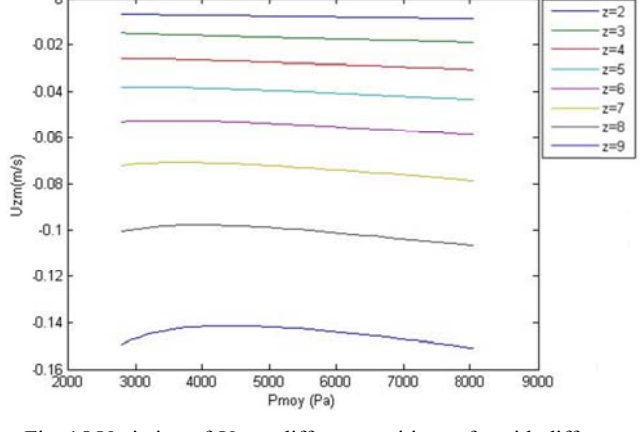


Fig. 15 Variation of  $U_{zm}$  at different positions of  $z$  with different values of  $P_{moy}$  for  $x=8$

As the mean gas pressure is related to the mean mixture pressure by (4) and (5), Fig. 16 shows a curve that permits to deduce  $P_{g moy}$  from  $P_{moy}$ .

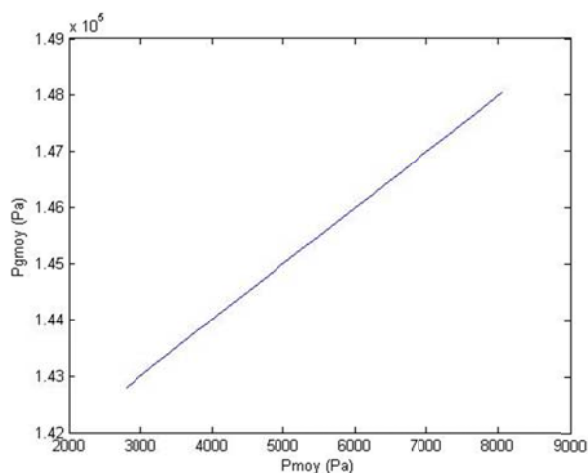


Fig. 16 Variation of  $P_{gmoy}$  with  $P_{moy}$

#### IV. CONCLUSION

The numerical study done in this work allows the determination of pressure scale that can be used to determine the gas pressure. This operation needs a device that permits the determination of radial velocities values  $U_{zm}$  for each  $(x,z)$  position. In this case, it is possible to use ultrasound waves.

#### REFERENCES

- [1] Ishii M., and Hibiki T., Thermo-Fluid Dynamics of Two-Phase Flow. New York: Springer, 2011.
- [2] Randy S. Lagumbay, Oleg V. Vasilyev, and Andreas Haselbacher, "Homogeneous Equilibrium Mixture Model for Simulation of Multiphase/ Multicomponent Flows," International Journal for Numerical Methodes in Fluids, 2007, pp. 1-32.
- [3] Ishii M., and Kakac S., Advances in Two-Phase Flow and Heat Transfer, Fundamentals and Applications. Germany: Martinus Nijhoff Publishers, 1983.
- [4] Aicha Rima CHENITI, Hatem BESBES, Joseph HAGGEGE, and Christophe SINTES, "Toward ex-situ measurement of arterial carbon dioxide pressure," in Proc. 35th IASTED International Conference Modelling, Identification and Control, Innsbruck, 2016, pp. 14-19.
- [5] M. A. Rodriguez-Valverde, M. A. Cabrerizo-Vilchez, and R. Hidalgo-Alvarez, "The Young-Laplace equation links capillarity with geometrical optics," European Journal of Physics, 2003, pp. 159-168.
- [6] Saul Goldman, "Generalizations of the Young-Laplace equation for the pressure of a mechanically stable gas bubble in a soft elastic material," The Journal of Chemical Physics, 2009, 184502.
- [7] Joseph D. Bronzino, The biomedical engineering handbook. Boca Raton: CRC Press LLC, 2000.